Appendix to chapter 10. Inequality of Capital Ownership

The series used to construct figures 10.1-10.6, replicated in the book on p.340-348 are available in table S10.1, as well as in the corresponding excel file.

Table S10.1. The concentration of wealth in Europe and in the USA, 1810-2010
(series used for figures 10.1-10.6)

All the details on the sources and the methods used to construct these series are in the excel file. Only the main points are presented here. First, the estimates for the wealth inequality are calculated among the adult population, which always leads to higher inequality than if it was measured at deaths (using "mortality multiplier" techniques enable to switch from one to the other, this is explained in detail by Kopzuck-Saez 2004 and Piketty-Postel-Vinay-Rosenthal 2006).

A very interesting general reference on the historical evolution of the concentration of wealth is the book Waldenstrom 2009, Lifting all boats… In particular table 3.A1 p.120-121 (Sweden) and tables 4.A1-A3 pp.148-154 (Denmark, France, Norway, Switzerland, UK, US). These are very complete tables gathering series on the concentration of wealth in a large number of countries. Also see Ohlsonn-Roine-Waldenstrom 2008 and Waldenstrom 2012. On the United States and the United Kingdom, also see Lindert 2000 (Handbook of Income Distribution), in particular table 2 pp.181-182 and table 3 p.188. For the United Kingdom, Lindert relies specially on its article JPE 1986. The survey Lindert 2000 is however somewhat outdated and less complete than Waldenstrom 2009. On Australia, see the recent study available here.

The main sources used to construct the series presented in table S10.1 are the following ones (excel file):

France. Calculations are based on the series from Piketty-Postel-Vinay-Rosenthal 2006 for 1810-1990 (built from inheritance data), completed by the estimates of Landais-Piketty-Saez 2011 for 2010 – that are built with the combination of different types of data, in particular inheritance data and fiscal data from the ISF (Impôt de Solidarité sur la Fortune, the French wealth tax). All series were homogenized in order to relate to the wealth distribution of the livings. The huge uncertainties on the more recent estimates, which certainly under-estimate the wealth of the wealthiest, needs to be pointed out. In particular, the self-reported surveys on wealth done by INSEE, have to be taken carefully because they under-estimate a lot the top of the distribution in comparison to fiscal data (which are themselves a lower bound for the wealthiest). For instance, the surveys done by INSEE in 2004 and 2010 give a top decile share just above 50% of the total wealth (see here), whereas fiscal data (inheritance and ISF) suggest a top decile share above 60% of the total wealth, which, besides, is more coherent with the results for other countries.

Sweden. See Waldenstrom 2009, Table 3.A1, p.120-121.

United Kingdom. See Atkinson et al 1989 Table 1 for 1923-1981. Atkinson et al 1989 are the extension of Atkinson-Harrison 1978; the series only start in 1923 given the absence of cross data for the size of inherited wealth and age before this year. However, Atkinson-Harrison 1978 table 6.1 give an estimate of the top 1% share for 1911-1913 on the basis on the available imperfect data (they get 69% of the wealth for the top 1% to be compared with 61% in 1923). These series are also used in Waldenstrom 2009 pp.148-154. I completed these series with the IRS series for 1980-2010, and the Lindert's estimates for 1810-1910. See excel file.

United States. Totally homogenous and satisfactory historical series do not exist for the United States (this can be partly explained by the fact that the federal inheritance tax was created only in 1916 and has always concerned only a small part of the population). The series given here for the period 1989-2010 are based on estimates from the Survey of Consumer Finances: see Kennickell 2009 Table 4; Kennickell et al 2011 Tables 2-3; Wolff 2010 Table 2);[33] for the period 1962-1989 in Wolff 1994; and for the period 1916-1962 on the estimates of Kopczuk-Saez 2004 Table B1 (based on inheritance data) homogenized with later series (see this fichier excel for the details). These different series are also used in Waldenstrom 2009 pp.148-154, who however does not attempt to homogenize the raw estimates. For period 1810-1910, I

[33] Pour des tentatives d'amélioration de la fiabilité des estimations SCF pour le haut de la distribution pour la période récente, voir Johnson-Shreiber 2006 et Raub-Johnson-Newcomb 2010.

use the data from the Lindert 2000's survey (estimate for the total population, including slaves). Huge uncertainties exist on these estimates. See Gallman 1969's estimates, who got wealth concentrations for the United States higher than Lindert (around 60-70% for the top decile). Also see L. Soltow, *Distribution of wealth and income in the United States in 1798*, University of Pittsburgh Press, 1989

Examples of the distribution of lands in traditional societies (**p.345**)

I write in the book on p.345, that the distribution of farmland in traditional rural societies is characterized by a very high concentration, typically with 80%-90% of the total of by the top decile. I give the example of the article of R. S. Bagnall, Journal of Roman Studies, 1992, which gives an estimate of the distribution of lands for Egypt under the Roman Empire. For an estimate on the 1950s, and according to which around 80% of the farmlands were held by the top decile, refer to P. Verme "Inside Inequality in Egypt: Historical trends, recent facts, people's perceptions and the spatial dimension", World Bank 2012, table 2, p.7.

Figure 10.7. Return to capital and growth: France 1820-1913 **(p.352)**
Figure 10.8. Capital share and saving rate: France 1820-1913 **(p.352)**

The series used to construct figures 10.7-10.8, replicated in the book on p.352, are available in table S10.2, as well as in the corresponding excel file.

Table S10.2. Capital rate of return, growth rate, capital share and savings rate in France, 1820-1910 *(series used for figures 10.7-10.8)*

This table has been taken from On the Long-Run Evolution of Inheritance..., 2010 (in particular Appendix A). For interested readers, see this research or its summarized version (QJE 2011).

Figure 10.9. Rate of return versus growth rate at the world level, from Antiquity until 2100 (**p.354**)
Figure 10.10. After tax rate of return versus growth rate at the world level from Antiquity until 2100 (**p.356**)
Figure 10.11. After tax rate of return versus growth rate at the world level from Antiquity until 2200 (**p.357**)

The series used to construct figures 10.9-10.11, replicated in the book on p.354-357, are available in table S10.3, as well as in the corresponding excel file.

Table S10.3. Capital rate of return, growth rate for the world, 0-2200
(series used for figures 10.9-10.11)

The assumptions on which these series are based are described in the book on p.353-358. All the details are available in this excel file.

Note on the “modified golden rule": $r = \theta + \gamma g$ (**p.360**)

I write on p.360 that the equilibrium rate of return to capital, in the framework of a infinite-horizon economic model, is given by the “modified golden rule" formula:

$$r = \theta + \gamma g$$

More precisely, it can be proved that this conclusion immediately stems from the hypothesis (even unlikely) according to which a unique representative agent correctly describes the behavior of the economic agents. Namely, in this model, everyone maximizes an infinite-horizon utility function with the following form:

$$U = \int_{0 \leq t \leq +\infty} e^{-\theta t} u(c_t)$$

In which θ is the time preference rate, $u(c) = c^{1-\gamma}/(1-\gamma)$ the utility function for consumption, and γ measures the concavity of the utility function (this parameter is generally assumed to be higher than 1 and usually ranging from 1,5 to 2,5).

Intuitively, $r = \theta + \gamma g$ is the unique rate of return to capital possible in the long-run for the following reason: the is the sole rate such that the agents are willing to rise their consumption at rate g, that is at the growth rate of the economy. If the return is higher, the agents will prefer to postpone their consumption and accumulate more capital, which will decrease the rate of return; and if it is lower, they will want to anticipate their consumption and borrow more, which will increase the rate of return.

For mathematical details, refer for instance to these lecture notes. Formally, it should be noted that the model requires that the “transversality condition” ($r>g$) holds (even if $\gamma<1$). This is required to make sure that the discounted present value of the future incomes is bounded.

Note on the property division regime brought in by the Civil Code (**p.576**)

For a more detailed description of the rules for property division and the new marital property regime established by the Civil Code, for example refer to T. Piketty, G. Postel-Vinay et J.L. Rosenthal, « Inherited vs Self-Made Wealth: Theory and Evidence from a Rentier Society (Paris 1872-1937) », *Explorations in Economic History*, 2013, long version. Also see the works of N. Frémeaux et M. Leturcq, « Régimes matrimoniaux et contrats de mariage en France depuis la Révolution », PSE, 2013 (available in N. Frémeaux's thesis, online here).

Note on the link between the Pareto coefficient and the gap r - g (**p.364-368**)

I write in the book on p.364-368 (and p.373-375) that the dynamic models of wealth accumulation based on multiplicative shocks generate Pareto distributions, and that the coefficient measuring the inequality of these Pareto distributions is an increasing function of the gap r - g (for a given structure of shocks).

More precisely, Pareto distribution is a particular form of distribution that follows this kind of mathematic law:

$$1-F(y)=(c/y)^a$$

In which 1-F(y) is the share of the population whose income or wealth is higher than y, c is a constant and a the coefficient of the Pareto law.

The idiosyncrasy of the Pareto laws is that if we calculate the average income or wealth y* of all the people whose income or wealth is higher than y, then the y*/y ration is equal to a constant b. This coefficient, called "inverted Pareto coefficient", is simply linked to the coefficient:

$$b = a/(a-1)$$

Intuitively, the higher is b, the thicker is the top of the distribution, and thus the stronger is the concentration of wealth. As a consequence, the coefficient b measures the distribution inequality (whereas the coefficient a varies in the opposite direction, and thus measures the distribution equality). On the mathematical link between a and b, and the way they can be empirically measured, see for instance, L*es hauts revenus en France au 20*[e] *siècle…*, 2001, Annexe B.

Pareto laws are generated by dynamic process with multiplicative shocks, for instance because portfolios and wealth are multiplied by a random multiplicative shock from a period or a generation to another. Intuitively, the bigger the gap r – g is, the more this shock generates a high concentration of wealth and thus the higher the coefficient b is. The mathematical equations to determine this coefficient b depending on r – g, as well as the corresponding technical references are available in these lecture notes. The corresponding models were developed by Champernowne in 1953 and extended by several authors, such as Stiglitz, Cowell and Nirei. Meade developed a similar intuition in his book published in 1964[34]. What needs to be emphasized is that small variations in the gap r-g (due to differences in the tax rate for the wealthiest for example) can explained huge variations in the coefficient b and thus in the concentration of wealth. For numerical simulations see Dell 2005

In the real life, inverted Pareto coefficient b typically range from barely 1.5 (low inequality) to 3-3.5 (very high inequality). To consult charts representing the evolution of the Pareto coefficients for the income distribution in the different countries by the WTID over the past century, see Atkinson-Piketty-Saez, "Top Incomes in the Long Run of History", *Journal of Economic Literature*, 2011 (in particular figures 12-15 p.50-55 on the evolution of coefficient b and table 3 p.14 for the link between a et b).

The Pareto coefficients link in a simple manner the shares of the top decile, centile, thousandth and so on (discussion in the book on p.367-368). In concrete terms, the share of the population above y is equal to $1\text{-}F(y)=(c/y)^a$. Then, for 2 percentiles p<q (for instance, p=0,1%, q=1%), we have $p=(c/y_p)^a$ and $q=(c/y_q)^a$, hence $y_p/y_q = (q/p)^{1/a}$ (in which y_p et y_q are the thresholds corresponding to the percentiles p and q). For q/p=10 (for example we want to know the top 0.1% into the top 1% or the top 0.01% into the top 0.1%, etc.), we get the following orders of magnitude:

- if a=3 (b=1,5, low inequality), then $y_p/y_q = 10^{1/3} = 2{,}15$
- if a=2 (b=2, medium inequality), then $y_p/y_q = 10^{1/2} = 3{,}16$
- if a=1,5 (b=3, high inequality), then $y_p/y_q = 10^{1/1,5} = 4{,}64$
- if a=1,4 (b=3,5, very high inequality), then $y_p/y_q = 10^{1/1,4} = 5{,}18$

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[34] Meade does not go as far as studying the Parto equilibrium law, but he insists on the fact the dynamic of inequality becomes more and more explosive when the "internal rate of reproduction" (the product of the savings rate with the rate of return) of the largest wealth is higher compared to the growth rate.

Given that the average income or wealth above the threshold are proportional to the threshold, it stems from it that a group 10 times smaller owns (as a share of the total) 20% of what a group 10 times bigger owns in a distribution with low inequality, 30% in a distribution with medium inequality, and more than 50% with very high inequality.

Note on the end of the hyper-inegalitarian equilibrium (**p.368-370**)

I mention on p.368-370 the end of the hyper-inegalitarian "rentier" equilibrium is due to the shocks of World War I. While the heirs of the period 1872-1912 managed to leave enough wealth as inheritance to finance the standard of living they experienced themselves, this equilibrium broke between the two wars. On this specific point, see « Inherited vs Self-Made Wealth...», 2013 (in particular long version, figure 12).

Table 10.1. The composition of Parisian portfolios, 1872-1912 (**p.371**)
Table S10.4. The composition of Parisians portfolios in 1872-1912

Table 10.1, replicated in the book on p.371, of which a detailed version (with the decomposition of the different foreign assets according to the level of wealth) is available in the supplementary table S10.4, have been taken from « Inherited vs Self-Made Wealth...», 2013 (table 4 and technical appendix, table B11). All the detailed calculations are given in the corresponding excel file.