MASTER PPD - PUBLIC ECONOMICS: TAX & TRANSFER POLICIES

FINAL EXAM, NOVEMBER 25, 2010 - SOLUTIONS

1. Exercise 1: Tax Revenues and Tax Incidence (5 points)

1) Define national income (0.5 point) and give the rough level of taxes in rich countries as a fraction of national income (0.5 point).

Answer National income = GDP - capital depreciation + net foreign factor income. T = about 40-50% of national income.

2) Give a rough estimate of the tax rates on labor, capital, and consumption as of today in rich countries. (0.5 point)

Answer Labor taxe rate = 35-40%; capital taxe rate = 25-30%; consumption taxe rate=20-25%.

3) Why is it difficult to know exactly who ends up paying taxes? What methods can shed light on this issue? (1 point)

Answer The nominal payer is not necessarily the one who really ends up paying, i.e. economic tax incidence is not necessarily the same as juridic tax incidence. Two methods: national accounts and tax incidence studies using micro-data.

4) According to macroeconomic data, who pays employer social contributions: capital, labor, both? Why? (1 point)

Answer The long-run and cross-country stability of factor shares suggests that all social contributions are ultimately paid by labor.

5) Give and present briefly an example of an incidence study using micro data. (1,5 point).

Answer See G. Fack "Are Housing Benefits An Effective Way To redistribute Income", *Labour Economics*, 2006; or C. Carbonnier, "Who Pays Sales Taxes? Evidence from French VAT Reforms", 1987-1999, *Journal of Public Economics*, 2007

2. Exercise 2: Income Tax and Income Distribution (5 points)

For incomes earned in 2009, the U.S. federal income tax schedule for a single person was the following:

Tax bracket	Tax rate
\$0 - \$8,375	10%
\$8,376 - \$34,000	15%
\$34,001 - \$82,400	25%
\$82,401 - \$171,850	28%
\$171,851 - \$373,650	33%
$>$ \$373,651	35%

1) Write the formula that gives the average tax rate of a person with income of \$100,000. Is this average tax rate smaller, equal or bigger than 28%?. (1 point)

Answer avg tax rate = [0.1 x 8,375 + 0.15 x (34,000 - 8,376) + 0.25 x (82,400 - 34,001) + 0.28 x (100,000 - 82,401)] / 100,000 = 21,709 / 100,000 = 21,7%.

2) How has the top marginal income tax rate evolved in the U.S. since the end of Word War II? (1 point)

Answer In 1945, the top marginal tax rate was around 90%. It stayed at that level until 1963, and remained at or above 70% until 1981. It declined to 50% in 1982, to 28 percent by 1988, then increased to 39.6 percent in 1993, and fell to 35 percent as of 2003.

3) Are the U.S. the only country where the top marginal income tax rate has experienced such an evolution? Give an example. (1 point)

Answer The decline of top marginal tax rates is a widespread phenomenon, though the precise timing differs across countries. In France for instance, the top marginal tax rate was also very high (70%-90%) in the post war period and until the 1970s. It is now equal to 40%.

4) What has been the trend in the share of aggregate pre-tax income accruing to top income groups in the United States since the beginning of the 1980s? (1 point)

Answer. It has increased (0.5 point). For instance (0.5 point), the share of the top 10%, including capital gains, has increased from 35% in 1980 to 50% in 2007; the share of the top 1%, including capital gains, has increased from 10% to 24%

in 2007.

5) Describe a mechanism by which lower top marginal tax rates have a causal effect on the observed share of total pre-tax income accruing to the top 1%. (1 point)

Answer 1) Lower top marginal tax rates encourage top income earners to work more (supply side). 2) Top earners may substitute taxable cash compensation with other forms of compensation such as non-taxable fringe benefits, deferred stock-option or pension compensation (tax-shifting). 3) Top earners might be able to increase their pay by exerting effort to influence corporate boards. High top tax rates might discourage such efforts aimed at extracting higher compensation.

3. Exercise 3: Pigouvian Corrective Taxation (5 points)

Consider an economy with a continuum of agents i in $[0, 1]$. There are two goods: a non-energy good and an energy good. Each agent has the same utility function:

$$U_i = U_i(c_i, e_i, E) = (1-\alpha)log(c_i) + \alpha log(e_i) - \lambda log(E)$$

where:

- c_i is the individual consumption level of the non-energy good,
- e_i is the individual consumption level of the energy good,
- E is the aggregate consumption level of the energy good,
- $0 < \alpha < 1$ and $0 < \lambda < 1$

1) Explain briefly what it means that E enters negatively in the utility function U_i. (0.5 point)

Answer: It means that the consumption of the energy good is a negative externality: the consumption level of other agents affects negatively my utility.

2) In a laissez-faire economy, each agent i chooses its level of consumption of the non-energy good c_i and of the energy good e_i to maximize its utility under the budget constraint $y_i = c_i + e_i$. Compute the optimal levels of c_i and e_i. (1 point)

Answer Agent i maximizes U_i. We can replace e_i by $y_i - c_i$ in the utility function, and then maximize wrt to c_i. The first order condition is:

$$\frac{dU_i}{dc_i} = \frac{1-\alpha}{c_i} - \frac{\alpha}{y_i - c_i} = 0$$

This yields $c_i^* = (1-\alpha)y_i$ and $e_i^* = \alpha y_i$.

3) In a planned economy, a benevolent planner chooses the aggregate level of consumption of the non-energy good C and of the energy good E to maximize social welfare $U = U(C, E, E)$ under the aggregate budget constraint $Y = C + E$. Compute the socially optimal values of C and E. (1 point)

Answer The social planner maximizes $U = (1-\alpha)log(C) + (\alpha - \lambda)log(E)$. We can replace E by $Y - C$ in the utility function and maximize wrt to C. The first order condition is:

$$\frac{dU}{dC} = \frac{1-\alpha}{C} - \frac{\alpha - \lambda}{Y - C} = 0$$

This yields $C^* = \frac{1-\alpha}{1-\lambda}Y$ and $E^* = \frac{\alpha-\lambda}{1-\lambda}Y$.

4) Show and explain briefly why the consumption of energy good is lower in the planned economy than in the laissez-faire economy. (0.5 point)

Answer In the laissez-faire economy, the aggregate consumption of energy good is $\int_0^1 e_i di = \alpha Y$. Since $0 < \lambda < 1$ and $0 < \alpha < 1$, $\alpha Y > \frac{\alpha-\lambda}{1-\lambda}Y = E^*$. The social planner takes into account the negative externality caused by energy consumption, whereas individuals don't.

5) Now we introduce a corrective tax t on energy consumption. Agent's i budget constraint becomes $c_i + (1+t)e_i = y_i$. Compute the optimal levels of c_i and e_i. (0.5 point)

Answer. Replace e_i by $y_i - c_i$ in the utility function and max wrt to c_i. Same first order condition for c_i than in question 2, i.e. $c_i^* = (1-\alpha)y_i$, and the budget constraint yields $e_i^* = \frac{\alpha y_i}{1+t}$

6) Compute the tax rate t^* that allows to obtain the socially optimal value of consumption of the energy good. (1 point).

Answer. t^* is such that $E^* = \int_0^1 e_i^* di$, i.e. such that $\frac{\alpha}{1+t^*} = \frac{\alpha-\lambda}{1-\lambda}$ i.e.

$$t^* = \frac{\lambda(1-\alpha)}{\alpha - \lambda}$$

7) Show that t^* is an increasing function of λ, and explain briefly what it means. (0.5 point)

Answer

$$\frac{dt^*}{d\lambda} = \frac{\alpha(1-\alpha)}{(\alpha-\lambda)^2} > 0$$

since $0 < \alpha < 1$. The higher the externality, the higher the optimal corrective tax rate.

4. Exercise 4: Optimal Taxation of Labor Income (5 points)

This exercise is based on a simplified version of the model presented in section II of Piketty T. (1995), "Social Mobility and Redistributive Politics", *Quarterly Journal of Economics*, vol. CX, 3, p. 551-584.

Consider an economy with a continuum of agents i in $[0, 1]$. A fraction f_l of agents is lucky, a fraction f_u is unlucky. ($f_l + f_u = 1$). Each agent can obtain a high labor income y_1 or a low labor income y_0 ($y_1 > y_0$) depending on whether or not he is lucky, and on how much effort e he makes.

More precisely, the probability for an unlucky agent making effort e to have labor income y_1 is:

$$P[y_i = y_1 | e_i = e \text{ and } i \text{ is unlucky}] = \pi_0 + \theta e$$

the probability for a lucky agent making effort e to have labor income y_1 is:

$$P[y_i = y_1 | e_i = e \text{ and } i \text{ is lucky}] = \pi_1 + \theta e$$

where $\pi_0 < \pi_1$ and $\theta > 0$.

1) Explain in one sentence what the assumption $\pi_0 < \pi_1$ means (0.5 point) and in one sentence what the parameter θ captures (0.5 point).

Answer $\pi_0 < \pi_1$ means that everything else equal, it is easier for lucky agents to obtain a high labor income. θ captures the extent to which individual achievement is responsive to individual effort, i.e. the elasticity of income wrt to effort.

Let's introduce a redistributive tax system: all incomes are taxed at rate $0 < \tau < 1$ and all tax revenues are redistributed in a lump-sum way. Therefore, the after-tax income of a person with low pre-tax income is $(1-\tau)y_0 + \tau Y$ (where Y is aggregate income). The after-tax income of a person with high pre-tax income is $(1-\tau)y_1 + \tau Y$. Each agent has the same utility function:

$$U_i = y_i - \frac{e_i^2}{2}$$

2) (0.5 point) Explain in one sentence why the expected utility of an unlucky agent is:

$$U_i = (1 - \pi_0 - \theta e)(1-\tau)y_0 + (\pi_0 + \theta e)(1-\tau)y_1 + \tau Y - e_i^2/2 \tag{1}$$

Answer if i is unlucky, he as probability $\pi_0 + \theta e$ to have post-tax income $(1-\tau)y_1 + \tau Y$ and probability $(1-\pi_0-\theta e)$ to have post-tax income $(1-\tau)y_0 + \tau Y$

3) Compute the effort level e^* that maximizes the expected utility of an unlucky agent and interpret in one sentence the result. (1 point)

Answer Equation (1) can be rewritten:

$$e\theta(1-\tau)(y_1-y_0) - e^2/2 + \tau Y + (1-\tau)[(1-\pi_0)y_0 + \pi_0 y_I]$$

Maximizing with respect to e yields the first order condition:

$$e^* = \theta(1-\tau)(y_1-y_0)$$

The higher the tax rate the lower the effort, the higher the gap between high and low incomes the higher the effort, the effect depending on the elasticity of income with respect to effort.

4) Compute the optimal effort level of lucky agents. (0.5 point).

Answer Same thing than for unlucky agents.

5) Before the birth of any agent (i.e., before anyone knows if he is lucky or not), society decides to set the tax rate t at the level that maximizes the expected utility of unlucky agents (eq. (1)), given the optimal effort level e^* of lucky and unlucky agents. Society cannot observe luck nor effort.

a)(1 point). Show that the optimal tax rate is:

$$\tau = \frac{f_l(\pi_1-\pi_0)}{(y_1-y_0)\theta^2}$$

Answer First, express the aggregate income as a function of the optimal effort level:

$$Y = f_u[y_1(\pi_0+\theta e^*) + y_0(1-\pi_0-\theta e^*)] + f_l[y_1(\pi_1+\theta e^*) + y_0(1-\pi_1-\theta e^*)]$$

$$= [\pi_0 f_u + \pi_1 f_l + \theta e^*](y_1-y_0) + y_0$$

We want to maximize equation (1) given that $e^* = \theta(1-\tau)(y_1-y_0)$ and $Y = [\pi_0 f_u + \pi_1 f_l + \theta e^*](y_1-y_0) + y_0$. After some algebra, the utility of unlucky agents can be written:

$$U_i = \tau(y_1 - y_0)f_l(\pi_1 - \pi_0) + \frac{1}{2}(y_1 - y_0)^2\theta^2(1 - \tau^2) + \pi_0(y_1 - y_0) + y_0$$

The first order condition is:

$$(y_1 - y_0)f_l(\pi_1 - \pi_0) - \tau(y_1 - y_0)^2\theta^2 = 0$$

$$\tau = \frac{f_l(\pi_1 - \pi_0)}{(y_1 - y_0)\theta^2}$$

b) Interpret this formula. (1 point)

Answer The optimal tax rate is an increasing function of $\pi_1 - \pi_0$: the larger the inequality of opportunity, the higher the optimal tax rate. It is a decreasing function of θ: the higher the elasticity of income wrt to effort, the lower the optimal tax rate.