A Theory of Optimal Capital Taxation

Thomas Piketty, Paris School of Economics
Emmanuel Saez, UC Berkeley

November 2011
Motivation: The Failure of Capital Tax Theory

1) **Standard theory**: optimal tax rate $\tau_K=0\%$ for all forms of capital taxes (stock- or flow-based)
   - Complete suppression of inheritance tax, property tax, corporate tax, K income tax, etc. is desirable… including from the viewpoint of individuals with zero property!

2) **Practice**: EU27: tax/GDP = 39%, capital tax/GDP = 9%
   - US: tax/GDP = 27%, capital tax/GDP = 8%
     - (inheritance tax: <1% GDP, but high top rates)
   - Nobody seems to believe this extreme zero-tax result – which indeed relies on very strong assumptions

3) **Huge gap** between theory and practice on optimal capital taxation is a major failure of modern economics
This Paper: Two Ingredients

In this paper we attempt to develop a realistic, tractable K tax theory based upon two key ingredients

1) **Inheritance**: life is not infinite, inheritance is a large part of aggregate wealth accumulation

2) **Imperfect K markets**: with uninsurable return risk, use lifetime K tax to implement optimal inheritance tax

With no inheritance (100% life-cycle wealth or infinite life) and perfect K markets, then the case for $\tau_K=0\%$ is indeed very strong: $1+r =$ relative price of present consumption $\rightarrow$ do not tax $r$, instead use redistributive labor income taxation $\tau_L$ only (Atkinson-Stiglitz)
• **Key parameter:**  \( b_y = \frac{B}{Y} \)
  = aggregate annual bequest flow \( B \) /national income \( Y \)

• Huge historical variations:
\( b_y = 20-25\% \) in 19\( ^{\circ} \)C & until WW1 (=very large: rentier society)
\( b_y < 5\% \) in 1950-60 (Modigliani lifecycle) (~A-S)
\( b_y \) back up to ~15\% by 2010 → inheritance matters again

• See « On the Long-Run Evolution of Inheritance – France 1820-2050 », Piketty QJE’11

• **r\( \geq g \) story:** g small & \( r >> g \) → inherited wealth is capitalized faster than growth → \( b_y \) high

• U-shaped pattern probably less pronounced in US

→ **Optimal** \( \tau_B \) **is increasing with** \( b_y \) (or \( r-g \))
Annual inheritance flow as a fraction of national income, France 1820-2008

- Economic flow (computed from national wealth estimates, mortality tables and observed age-wealth profiles)
- Fiscal flow (computed from observed bequest and gift tax data, inc. tax exempt assets)

Annual inheritance flow as a fraction of disposable income, France 1820-2008

- Economic flow (computed from national wealth estimates, mortality tables and observed age-wealth profiles)
- Fiscal flow (computed from observed bequest and gift tax data, inc. tax exempt assets)

Result 1: Optimal Inheritance Tax Formula

- **Simple formula** for optimal bequest tax rate expressed in terms of estimable parameters:

  \[
  \tau_B = \frac{1-(1-a-\tau)s_{b0}/b_y}{1+e_B+s_{b0}}
  \]

  with: \( b_y \) = bequest flow, \( e_B \) = elasticity, \( s_{b0} \) = bequest taste
  \( \rightarrow \tau_B \) increases with \( b_y \) and decreases with \( e_B \) and \( s_{b0} \)

- For realistic parameters: \( \tau_B = 50-60\% \) (or more..or less...)
  \( \rightarrow \) **our theory can account for the variety of observed top bequest tax rates (30%-80%)**

  \( \rightarrow \) hopefully our approach can contribute to a tax debate based more upon empirical estimates of key distributional & behavioral parameters (and less about abstract theory)
Result 2: Optimal Capital Tax Mix

- **K market imperfections** (e.g. uninsurable idiosyncratic shocks to rates of return) can justify shifting one-off inheritance taxation toward lifetime capital taxation (property tax, K income tax,..)

- **Intuition**: what matters is capitalized bequest, not raw bequest; but at the time of setting the bequest tax rate, there is a lot of uncertainty about what the rate of return is going to be during the next 30 years → so it is more efficient to split the tax burden

→ **our theory can explain the actual structure & mix of inheritance vs lifetime capital taxation**

(& why high top inheritance and top capital income tax rates often come together, e.g. US-UK 1930s-1980s)
Link with previous work

1. Atkinson-Stiglitz JPuPE’76: No capital tax in life-cycle model with homogenous tastes for savings, consumption-leisure separability and nonlinear labor income tax

2. Chamley EMA’86-Judd JPuE’85: No capital tax in the long run in an infinite horizon model with homogenous discount rate

3. Precautionary Savings: Capital tax desirable when uncertainty about future earnings ability affect savings decisions

4. Credit Constraints can restore desirability of capital tax to redistribute from the unconstrained to the constrained

5. Time Inconsistent Governments always want to tax existing capital → here we focus on long-run optima with full commitment (most difficult case for $\tau_K>0$)
Atkinson-Stiglitz fails with inheritances

A-S applies when sole source of lifetime income is labor:
\[ c_1 + \frac{c_2}{(1+r)} = \theta l - T(\theta l) \]  
(\( \theta = \) productivity, \( l = \) labor supply)

Bequests provide an additional source of life-income:
\[ c + \frac{b(\text{left})}{(1+r)} = \theta l - T(\theta l) + b(\text{received}) \]

\( \Rightarrow \) conditional on \( \theta l \), high \( b(\text{left}) \) is a signal of high \( b(\text{received}) \)  
[and hence low \( u_c \) \( \Rightarrow \) “commodity” \( b(\text{left}) \) should be taxed even with optimal \( T(\theta l) \)]

\( \Rightarrow \) two-dimensional heterogeneity requires two-dim. tax policy tool

**Extreme example:** no heterogeneity in productivity \( \theta \) but pure heterogeneity in bequests motives \( \Rightarrow \) bequest taxation is desirable for redistribution

Note: bequests generate positive externality on donors and hence should be taxed less (but still >0)
Chamley-Judd fails with finite lives

C-J in the dynastic model implies that inheritance tax rate $\tau_K$ should be zero in the long-run

(1) If social welfare is measured by the discounted utility of first generation then $\tau_K=0$ because inheritance tax creates an infinitely growing distortion but…
this is a crazy social welfare criterion that does not make sense when each period is a generation

(2) If social welfare is measured by long-run steady state utility then $\tau_K=0$ because supply elasticity $e_B$ of bequest wrt to price is infinite but…
we want a theory where $e_B$ is a free parameter
A Good Theory of Optimal Capital Taxation

Should follow the optimal labor income tax progress and hence needs to capture key trade-offs robustly:

1) **Welfare effects**: people dislike taxes on bequests they leave, or inheritances they receive, but people also dislike labor taxes → interesting trade-off

2) **Behavioral responses**: taxes on bequests might (a) discourage wealth accumulation, (b) affect labor supply of inheritors (Carnegie effect) or donors

3) **Results should be robust** to heterogeneity in tastes and motives for bequests within the population and formulas should be expressed in terms of estimable “**sufficient statistics**”
Part 1: Optimal Inheritance Taxation

- Agent $i$ in cohort $t$ (1 cohort = 1 period = $H$ years, $H \approx 30$)
- Receives bequest $b_{ti} = z_i b_t$ at beginning of period $t$
- Works during period $t$
- Receives labor income $y_{Lt_i} = \theta_i y_{Lt}$ at end of period $t$
- Consumes $c_{ti}$ & leaves bequest $b_{t+1i}$ so as to maximize:

  \[
  \text{Max } V_i(c_{ti}, b_{t+1i}, b_{t+1i}) \\
  \text{s.c. } c_{ti} + b_{t+1i} \leq (1 - \tau_B)b_{ti}e^{rH} + (1 - \tau_L)y_{Lt_i}
  \]

  With: $b_{t+1i} =$ end-of-life wealth (wealth loving)  
  $b_{t+1i} = (1 - \tau_B)b_{t+1i}e^{rH} =$ net-of-tax capitalized bequest left  
  (bequest loving)  
  $\tau_B =$ bequest tax rate, $\tau_L =$ labor income tax rate  
  $V_i()$ homogeneous of degree one (to allow for growth)
• **Special case: Cobb-Douglas preferences:**
  \[ V_i(c_{t_i}, b_{t+1i}, b_{t+1i}) = c_{t_i}^{1-s_i} b_{t+1i}^{s_{wi}} b_{t+1i}^{s_{bi}} \] (with \( s_i = s_{wi} + s_{bi} \))
  \[ \rightarrow b_{t+1i} = s_i [(1-\tau_B)z_i b_t e^{rH} + (1-\tau_L)\theta_i y_{Lt}] = s_i y_{ti} \]

• **General preferences:** \( V_i() \) homogenous of degree one:
  \[ \text{Max } V_i() \rightarrow \text{FOC } V_{ci} = V_{wi} + (1-\tau_B)e^{rH} V_{bi} \]
  All choices are linear in total lifetime income \( y_{ti} \)
  \[ \rightarrow b_{t+1i} = s_i y_{ti} \]
  Define \( s_{bi} = s_i (1-\tau_B)e^{rH} V_{bi}/V_{ci} \)
  Same as Cobb-Douglas but \( s_i \) and \( s_{bi} \) now depend on \( 1-\tau_B \)
  (income and substitution effects no longer offset each other)

• We allow for any distribution and any ergodic random process for taste shocks \( s_i \) and productivity shocks \( \theta_i \)
  \[ \text{endogenous dynamics of the joint distribution } \Phi_t(z, \theta) \]
  of normalized inheritance \( z \) and productivity \( \theta \)
• Macro side: open economy with exogenous return \( r \),
domestic output \( Y_t = K_t^\alpha L_t^{1-\alpha} \), with \( L_t = L_0 e^{gH_t} \) and
\( g = \) exogenous productivity growth rate
(inelastic labor supply \( l_{ti} = 1 \), fixed population size = 1)

• Period by period government budget constraint:
\[
\tau_L Y_{Lt} + \tau_B B_t e^{rH} = \tau Y_t
\]
\[
\text{i.e. } \quad \tau_L (1-\alpha) + \tau_B b_{yt} = \tau
\]
With \( \tau = \) exogenous tax revenue requirement (e.g. \( \tau = 30\% \))
\( b_{yt} = e^{rH} B_t / Y_t = \) capitalized inheritance-output ratio

• Government objective:
We take \( \tau \geq 0 \) as given and solve for the optimal tax mix \( \tau_L, \tau_B \)
maximizing steady-state \( \text{SWF} = \int \omega_{z\theta} V_{z\theta} \ d\Psi(z,\theta) \)
with \( \Psi(z,\theta) = \) steady-state distribution of \( z \) and \( \theta \)
\( \omega_{z\theta} = \) social welfare weights
Equivalence between $\tau_B$ and $\tau_K$

- In basic model, tax $\tau_B$ on inheritance is equivalent to tax $\tau_K$ on annual return $r$ to capital as:
  \[
  b_{ti} = (1- \tau_B)b_{ti}e^{rH} = b_{ti}e^{(1-\tau_K)rH}, \text{ i.e. } \tau_K = -\log(1-\tau_B)/rH
  \]

- E.g. with $r=5\%$ and $H=30$, $\tau_B=25\% \leftrightarrow \tau_K=19\%$, $\tau_B=50\% \leftrightarrow \tau_K=46\%$, $\tau_B=75\% \leftrightarrow \tau_K=92\%$

- This equivalence no longer holds with
  (a) tax enforcement constraints, or (b) life-cycle savings, or (c) uninsurable risk in $r=r_{ti}$

  → Optimal mix $\tau_B, \tau_K$ then becomes an interesting question (see below)
• **Special case**: taste and productivity shocks $s_i$ and $\theta_i$ are i.e. across and within periods (no memory)

$\rightarrow s = E(s_i | \theta_i, z_i)$ → simple aggregate transition equation:

$$b_{t+1|i} = s_i [(1- \tau_B)z_i b_t e^{rH} + (1-\tau_L)\theta_i y_{Lt}]$$

$\rightarrow b_{t+1} = s [(1- \tau_B)b_t e^{rH} + (1-\tau_L)y_{Lt}]$

Steady-state convergence: $b_{t+1} = b_t e^{gH}$

$\rightarrow b_{yt} \rightarrow b_y = \frac{s(1-\tau-\alpha)e^{(r-g)H}}{1-se^{(r-g)H}}$

• $b_y$ increases with $r-g$ (capitalization effect, Piketty QJE’11)

• If $r-g=3\%, \tau=10\%, H=30, \alpha=30\%, s=10\% \rightarrow b_y = 20\%$

• If $r-g=1\%, \tau=30\%, H=30, \alpha=30\%, s=10\% \rightarrow b_y = 6\%$
• **General case:** under adequate ergodicity assumptions for random processes $s_i$ and $\theta_i$:

**Proposition 1** (unique steady-state): for given $\tau_B, \tau_L$, then as $t \to +\infty$, $b_{yt} \to b_y$ and $\Psi_t(z, \theta) \to \Psi(z, \theta)$

• Define: $e_B = \frac{db_y}{d(1-\tau_B)} \frac{1-\tau_B}{b_y}$

• $e_B = \text{elasticity of steady-state bequest flow with respect to net-of-bequest-tax rate } 1-\tau_B$

• With $V_i() = \text{Cobb-Douglas and i.i.d. shocks}$, $e_B = 0$

• For general preferences and shocks, $e_B > 0$ (or $< 0$)

→ we take $e_B$ as a free parameter
• Meritocratic rawlsian optimum, i.e. social optimum from the viewpoint of zero bequest receivers (z=0):

**Proposition 2** (zero-receivers tax optimum)

$$
\tau_B = \frac{1-(1-\alpha-\tau)s_{b0}/b_y}{1+e_B+s_{b0}}
$$

with: $s_{b0} =$ average bequest taste of zero receivers

• $\tau_B$ increases with $b_y$ and decreases with $e_B$ and $s_{b0}$
• If bequest taste $s_{b0}=0$, then $\tau_B = 1/(1+e_B)$
  → standard revenue-maximizing formula
• If $e_B \to +\infty$, then $\tau_B \to 0$: back to Chamley-Judd
• If $e_B=0$, then $\tau_B < 1$ as long as $s_{b0}>0$
• i.e. zero receivers do not want to tax bequests at 100%, because they themselves want to leave bequests
  → trade-off between taxing rich successors from my cohort vs taxing my own children
Example 1: \( \tau = 30\%, \alpha = 30\%, s_{bo} = 10\%, e_B = 0 \)

- If \( b_y = 20\% \), then \( \tau_B = 73\% \) & \( \tau_L = 22\% \)
- If \( b_y = 15\% \), then \( \tau_B = 67\% \) & \( \tau_L = 29\% \)
- If \( b_y = 10\% \), then \( \tau_B = 55\% \) & \( \tau_L = 35\% \)
- If \( b_y = 5\% \), then \( \tau_B = 18\% \) & \( \tau_L = 42\% \)

\( \rightarrow \) with high bequest flow \( b_y \), zero receivers want to tax inherited wealth at a higher rate than labor income (73% vs 22%); with low bequest flow they want the opposite (18% vs 42%)

**Intuition:** with low \( b_y \) (high \( g \)), not much to gain from taxing bequests, and this is bad for my own children

With high \( b_y \) (low \( g \)), it’s the opposite: it’s worth taxing bequests, so as to reduce labor taxation and allow zero receivers to leave a bequest.
Example 2: \( \tau = 30\% \), \( \alpha = 30\% \), \( s_{bo} = 10\% \), \( b_y = 15\% \)

- If \( e_B = 0 \), then \( \tau_B = 67\% \) & \( \tau_L = 29\% \)
- If \( e_B = 0.2 \), then \( \tau_B = 56\% \) & \( \tau_L = 31\% \)
- If \( e_B = 0.5 \), then \( \tau_B = 46\% \) & \( \tau_L = 33\% \)
- If \( e_B = 1 \), then \( \tau_B = 35\% \) & \( \tau_L = 35\% \)

\( \rightarrow \) behavioral responses matter but not hugely as long as the elasticity \( e_B \) is reasonable.

Kopczuk-Slemrod 2001: \( e_B = 0.2 \) (US)

(French experiments with zero-children savers: \( e_B = 0.1-0.2 \))
• Proposition 3 ($\z$%-bequest-receivers optimum):

$$\tau_B = \frac{1-(1-\alpha-\tau) s_{bz}/b_y-(1+e_B+s_{bz})z/\theta_z}{(1+e_B+s_{bz})(1-z/\theta_z)}$$

• If $z$ large, $\tau_B<0$: top successors want bequest subsidies

• But since the distribution of inheritance is highly concentrated (bottom 50% successors receive $\sim5\%$ of aggregate flow), the bottom-50%-receivers optimum turns out to be very close to the zero-receivers optimum

• Perceptions about wealth inequality & mobility matter a lot: if bottom receivers expect to leave large bequests, then they may prefer low bequest tax rates

→ it is critical to estimate the right distributional parameters
• **Proposition 7** (optimum with elastic labor supply):

\[
\tau_B = \frac{1-(1-\alpha-\tau\cdot(1+e_L))s_{b0}/b_y}{1+e_B+s_{b0}\cdot(1+e_L)}
\]

• Race between two elasticities: \(e_B\) vs \(e_L\)
• \(\tau_B\) decreases with \(e_B\) but increases with \(e_L\)

**Example**: \(\tau=30\%\), \(\alpha=30\%\), \(s_{b0}=10\%\), \(b_y=15\%\)
• If \(e_B=0 \& e_L=0\), then \(\tau_B=67\% \& \tau_L=29\%\)
• If \(e_B=0.2 \& e_L=0\), then \(\tau_B=56\% \& \tau_L=31\%\)
• If \(e_B=0.2 \& e_L=0.2\), then \(\tau_B=59\% \& \tau_L=30\%\)
• If \(e_B=0.2 \& e_L=1\), then \(\tau_B=67\% \& \tau_L=29\%\)
Other extensions

• **Optimal non-linear bequest tax**: simple formula for top rate; numerical solutions for full schedule

• **Closed economy**: $F_K = R = e^{rH} - 1 = \text{generational return}$
  $\rightarrow$ optimal tax formulas continue to apply as in open economy with $e_B, e_L$ being the pure supply elasticities

• **Lifecycle saving**: assume agents consume between age $A$ and $D$, and have a kid at age $H$. E.g. $A=20$, $D=80$, $H=30$, so that everybody inherits at age $I=D-H=50$.
  $\rightarrow$ Max $V(U, b, b)$ with $U = \left[ \int_{A \leq a \leq D} e^{-\delta a} c_a^{1-\gamma} \right]^{1/(1-\gamma)}$
  $\rightarrow$ same $b_y$ and $\tau_B$ formulas as before, except for a factor $\lambda$ correcting for when inheritances are received relative to labor income: $\lambda \approx 1$ if inheritance received around mid-life
  (early inheritance: $b_y, \tau_B \uparrow$; late inheritance: $b_y, \tau_B \downarrow$)
Part 2: From inheritance tax to lifetime K tax

• One-period model, perfect K markets: equivalence btw bequest tax and lifetime K tax as $(1 - \tau_B)e^{rH} = e^{(1-\tau_K)rH}$

• Life-cycle savings, perfect K markets: it’s always better to have a big tax $\tau_B$ on bequest, and zero lifetime capital tax $\tau_K$, so as to avoid intertemporal consumption distortion

• However in the real world most people seem to prefer paying a property tax $\tau_K=1\%$ during 30 years rather than a big bequest tax $\tau_B=30\%$

• Total K taxes = 9% GDP, but bequest tax <1% GDP

• In our view, the observed collective choice in favour of lifetime K taxes is a rational consequence of K markets imperfections, not of tax illusion
Simplest imperfection: fuzzy frontier between capital income and labor income flows, can be manipulated by taxpayers (self-employed, top executives, etc.) (= tax enforcement problem)

**Proposition 4:** With fully fuzzy frontier, then $\tau_K = \tau_L$ (capital income tax rate = labor income tax rate), and bequest tax $\tau_B > 0$ is optimal iff bequest flow by sufficiently large

Define $\tau_B = \tau_B + (1 - \tau_B)\tau_K R / (1 + R)$, with $R = e^{r\bar{H}} - 1$.

$\tau_K = \tau_L \rightarrow$ adjust $\tau_B$ down to keep $\tau_B$ the same as before

$\rightarrow$ comprehensive income tax + bequest tax

$= \text{what we observe in many countries}$
Uninsurable uncertainty about future rate of return: what matters is $b_{ti}e_{ri}^H$, not $b_{ti}$; but at the time of setting the bequest tax rate $\tau_B$, nobody knows what the rate of return $1+R_{ti} = e_{ri}^H$ is going to be during the next 30 or 40 years…

(idiosyncratic + aggregate uncertainty)

→ with uninsurable shocks on returns $r_{ti}$, it's more efficient to split the tax burden between one-off transfer taxes and lifetime capital taxes

Exemple: when you inherit a Paris apartment worth 100 000€ in 1972, nobody knows what the total cumulated return will be btw 1972 & 2012; so it’s better to charge a moderate bequest tax and a larger annual tax on property values & flow returns
• Assume rate of return $R_{ti} = \varepsilon_{ti} + \xi e_{ti}$

With: $\varepsilon_{ti} = \text{i.i.d. random shock with mean } R_0$

$e_{ti} = \text{effort put into portfolio management (how much time one spends checking stock prices, looking for new investment opportunities, monitoring one’s financial intermediary, etc.)}$

$c(e_{ti}) = \text{convex effort cost proportional to portfolio size}$

• Define $e_R = \text{elasticity of aggregate rate of return } R \text{ with respect to net-of-capital-income-tax rate } 1-\tau_K$

• If returns mostly random (effort parameter small as compared to random shock), then $e_R \approx 0$

• Conversely if effort matters a lot, then $e_R$ large
• **Proposition 5.** Depending on parameters, optimal capital income tax rate $\tau_K$ can be $>$ or $<$ than optimal labor income tax rate $\tau_L$; if $e_R$ small enough and/or $b_y$ large enough, then $\tau_K > \tau_L$

(=what we observe in UK & US during the 1970s)

**Example :** $\tau=30\%$, $\alpha=30\%$, $s_{bo}=10\%$, $b_y=15\%$, $e_B=e_L=0$

- If $e_R=0$, then $\tau_K=100\%$, $\tau_B=9\%$ & $\tau_L=34\%$
- If $e_R=0.1$, then $\tau_K=78\%$, $\tau_B=35\%$ & $\tau_L=35\%$
- If $e_R=0.3$, then $\tau_K=40\%$, $\tau_B=53\%$ & $\tau_L=36\%$
- If $e_R=0.5$, then $\tau_K=17\%$, $\tau_B=56\%$ & $\tau_L=37\%$
- If $e_R=1$, then $\tau_K=0\%$, $\tau_B=58\%$ & $\tau_L=38\%$
Govt Debt and Capital Accumulation

• So far we imposed period-by-period govt budget constraint: no accumulation of govt debt or assets allowed
• In closed-economy, optimum capital stock should be given by modified Golden rule: $F_K = r^* = \delta + \Gamma g$
  with $\delta = \text{govt discount rate}$, $\Gamma = \text{curvature of SWF}$
• If govt cannot accumulate debt or assets, then capital stock may be too large or too small
• If govt can accumulate debt or assets, then govt can achieve modified Golden rule
• In that case, long run optimal $\tau_B$ is given by a formula similar to previous one (as $\delta \to 0$): capital accumulation is **orthogonal** to redistributive bequest and capital taxation
Conclusion

• (1) Main contribution: simple, tractable formulas for analyzing optimal tax rates on inheritance and capital

• (2) Main idea: economists’ emphasis on $1+r = \text{relative price}$ is excessive (intertemporal consumption distortions exist but are probably second-order)

• (3) The important point about the rate of return to capital $r$ is that
  (a) $r$ is large: $r > g \rightarrow$ tax inheritance, otherwise society is dominated by rentiers
  (b) $r$ is volatile and unpredictable $\rightarrow$ use lifetime $K$ taxes to implement optimal inheritance tax
Extension to optimal consumption tax $\tau_C$

- Consumption tax $\tau_C$ redistributes between agents with different tastes $s_i$ for wealth & bequest, not between agents with different inheritance $z_i$; so $\tau_C$ cannot be a substitute for $\tau_B$
- But $\tau_C$ can be a useful complement for $\tau_B$, $\tau_L$ (Kaldor’55)
- E.g. a positive $\tau_C > 0$ can finance a labor subsidy $\tau_L < 0$: as compared to $\tau_B > 0$, $\tau_L < 0$, this allows to finance redistribution by taxing rentiers who consume a lot more than rentiers who save a lot; given the bequest externality, this is a smart thing to do
  → extended optimal tax formulas for $\tau_B$, $\tau_L$, $\tau_C$
- Extension to optimal wealth tax $\tau_w$ vs $\tau_K$ (2-period model)