Econometrica Supplementary Material

SUPPLEMENT TO "A THEORY OF OPTIMAL INHERITANCE TAXATION"

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BY THOMAS PIKETTY AND EMMANUEL SAEZ

S.1. OMITTED PROOFS FROM THE MAIN TEXT

S.1.1. *Case With Endogenous Factor Prices*

INDIVIDUAL ti SOLVES THE PROBLEM

$$\max_{c_{ti}, b_{t+1i} \geq 0} V^{ti}(c_{ti}, \underline{R}_t b_{t+1i}, l_{ti}) \quad \text{s.t.} \quad c_{ti} + b_{t+1i} = \underline{R}_t b_{ti} + \underline{w}_t v_{ti} l_{ti} + E_t, \tag{S.1}$$

where $\underline{R}_t = R_t(1 - \tau_{Bt})$ and $\underline{w}_t = w_t(1 - \tau_{Lt})$ are the after-tax factor prices, and v_{ti} is ability of individual ti so that her pre-tax wage is $w_t v_{ti}$. The individual FOC is $V_c^{ti} = \underline{R}_{t+1} V_{\underline{b}}^{ti}$ if $b_{t+1i} > 0$.

With budget balance each period and no government debt, total capital in period t is $K_t = b_t$. Total labor is $L_t = \int_i v_{ti} l_{ti}$. Total product is $y_t = F(K_t, L_t)$ with CRS production function. Factor prices are given by $R_t = 1 + F_K$ and $w_t = F_L$, so that $F(K_t, L_t) = (R_t - 1)K_t + w_t L_t$.

The government objective is to choose $(\underline{R}_t, \underline{w}_t)_{t \geq 0}$ to maximize

$$SWF = \sum_{t \geq 0} \Delta^t \int_i \omega_{ti} V^{ti}(\underline{R}_t b_{ti} + \underline{w}_t v_{ti} l_{ti} + E_t - b_{t+1i}, \underline{R}_{t+1} b_{t+1i}, l_{ti}),$$

subject to

$$E_t = (w_t - \underline{w}_t)L_t + (R_t - \underline{R}_t)b_t = b_t + F(b_t, L_t) - \underline{w}_t L_t - \underline{R}_t b_t.$$

R_t and w_t have disappeared from the maximization problem. Considering, as above, a tax reform $(d\underline{R}_t = d\underline{R}, d\underline{w}_t)_{t \geq T}$ with $d\underline{w}_t$ set to meet the period-by-period budget constraint, we have

$$-L_t \, d\underline{w}_t + (w_t - \underline{w}_t) \, dL_t - b_t \, d\underline{R}_t + (R_t - \underline{R}_t) \, db_t = 0,$$

so that

$$b_t \, d\underline{R}_t \left(1 - e_{Bt} \frac{R_t - \underline{R}_t}{\underline{R}_t}\right) = -L_t \, d\underline{w}_t \left(1 - e_{Lt} \frac{w_t - \underline{w}_t}{\underline{w}_t}\right), \tag{S.2}$$

where elasticities e_{Bt} and e_{Lt} are again defined with respect to $\underline{R}_t$ and $\underline{w}_t$ and hence are exactly equivalent to our earlier elasticities with respect to $1 - \tau_{Bt}$ and $1 - \tau_{Lt}$; that is, they are pure supply elasticities keeping the pre-tax price of factors constant. Noting that $\frac{\tau_{Bt}}{1 - \tau_{Bt}} = \frac{R_t - \underline{R}_t}{\underline{R}_t}$ and $\frac{\tau_{Lt}}{1 - \tau_{Lt}} = \frac{w_t - \underline{w}_t}{\underline{w}_t}$, calculations follow those from Appendix A.1, and we obtain the same formula (9).

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In the case with government debt, the government dynamic budget constraint

$$a_{t+1} = R_t a_t + (R_t - \underline{R}_t)b_t + (w_t - \underline{w}_t)L_t - E_t$$

can be rewritten as

$$a_{t+1} = a_t + b_t + F(b_t + a_t, L_t) - \underline{R}_t b_t - \underline{w}_t L_t - E_t.$$

We can consider again the same small reform $(d\underline{R}_t = d\underline{R}, d\underline{w}_t)_{t\geq T}$ with $d\underline{w}_t$ set to meet the period-by-period budget constraint (S.2), so that $da_t = 0$ for all t and the calculations are exactly as in the period-by-period budget balance case. Hence, formula (9) remains valid.

S.1.2. *Case With Economic Growth*

We consider standard labor augmenting economic growth at rate $G > 1$ per generation, that is, individual wage rates w_{ti} grow exogenously at rate G. Obtaining a steady state where all variables grow at rate G per generation requires imposing standard homogeneity assumptions on individual utilities, so that $V^{ti}(c, \underline{b}, l) = \frac{(U^{ti}(c,\underline{b})e^{-h_{ti}(l)})^{1-\gamma}}{1-\gamma}$ with $U^{ti}(c, \underline{b})$ homogeneous of degree 1. In that case, the individual maximization problem can be decomposed into two steps.

First, the individual chooses b_{t+1i} taking resources $y_{ti} = Rb_{ti}(1 - \tau_{Bt}) + w_{ti}l_{ti}(1 - \tau_{Lt}) + E_t$ as given, so that we can define the indirect utility:

$$v^{ti}\big(y_{ti}, R(1 - \tau_{Bt+1})\big) = \max_{b_{t+1i}\geq 0} U^{ti}\big(y_{ti} - b_{t+1i}, Rb_{t+1i}(1 - \tau_{Bt+1})\big).$$

With U^{ti} homogeneous of degree 1, $v^{ti}(y, R(1 - \tau_{Bt+1})) = y \cdot \phi^{ti}(R(1 - \tau_{Bt+1}))$ is *linear* in y.

Second, the individual chooses labor supply to maximize $\log[\phi^{ti}(R(1 - \tau_{Bt+1}))] + \log[Rb_{ti}(1 - \tau_{Bt}) + w_{ti}(1 - \tau_{Lt})l_{ti} + E_t] - h_{ti}(l_{ti})$, leading to the first order condition

$$h'_{ti}(l_{ti}) = \frac{w_{ti}(1 - \tau_{Lt})}{Rb_{ti}(1 - \tau_{Bt}) + w_{ti}(1 - \tau_{Lt})l_{ti} + E_t}.$$

Hence, if tax rates converge and w_{ti}, b_{ti}, E_t, all grow at rate G per generation, labor supply l_{ti} will be stationary, so that an ergodic equilibrium exists (under the standard assumptions).

This implies that utility V^{ti} grows at rate $G^{1-\gamma}$ per generation. As $V_c^{ti}/V^{ti} = (1 - \gamma)/y_{ti}$ and y_{ti} grows at rate G, marginal utility V_c^{ti} grows at rate $G^{-\gamma}$ per generation.[24]

[24]This result remains true in the log-case with $\gamma = 1$.

Steady-State Maximization

If the government maximizes steady-state social welfare, we obtain the same equation (6) as in the main text. However, the last term in b_{t+1i} has grown by a factor G relative to b_t, so that, when dividing (6) by $Rb_t\, d\tau_B$, we obtain

$$0 = -\bar{b}^{\text{received}}(1+\hat{e}_B) + \frac{1-e_B\tau_B/(1-\tau_B)}{1-e_L\tau_L/(1-\tau_L)}\bar{y}_L - \frac{G\bar{b}^{\text{left}}}{R(1-\tau_B)},$$

which is the same equation as in the main text except that the term $\bar{b}^{\text{left}}$ is multiplied by a factor G. This will lead to the same optimum formula as (7) except that $\bar{b}^{\text{left}}$ is replaced by $G\bar{b}^{\text{left}}$, or equivalently, R is replaced by R/G, that is,

$$\text{(S.3)} \qquad \tau_B = \frac{1 - \left[1 - \dfrac{e_L\tau_L}{1-\tau_L}\right] \cdot \left[\dfrac{\bar{b}^{\text{received}}}{\bar{y}_L}(1+\hat{e}_B) + \dfrac{G}{R}\dfrac{\bar{b}^{\text{left}}}{\bar{y}_L}\right]}{1 + e_B - \left[1 - \dfrac{e_L\tau_L}{1-\tau_L}\right]\dfrac{\bar{b}^{\text{received}}}{\bar{y}_L}(1+\hat{e}_B)}.$$

Social Discounting Maximization

The government maximizes discounted social welfare:

$$SWF = \sum_{t\geq 0}\Delta^t \int_i \omega_{ti} V^{ti}\big(Rb_{ti}(1-\tau_{Bt}) + w_{ti}l_{ti}(1-\tau_{Lt}) + E_t - b_{t+1i},$$
$$Rb_{t+1i}(1-\tau_{Bt+1}), l_{ti}\big),$$

subject to period-by-period budget balance $E_t = \tau_{Bt}Rb_t + \tau_{Lt}y_{Lt}$. Consider again a reform $d\tau_B$ so that $d\tau_{Bt} = d\tau_B$ for all $t \geq T$ (and $d\tau_{Lt}$ to maintain budget balance and keep E_t constant). We assume that T is large enough that all variables have converged for $t \geq T$:

$$dSWF = \sum_{t\geq T}\Delta^t \int_i \omega_{ti} V_c^{ti} \cdot \big(R\, db_{ti}(1-\tau_B) - Rb_{ti}\, d\tau_B - d\tau_{Lt} y_{Lti}\big)$$
$$+ \sum_{t\geq T-1}\Delta^t \int_i \omega_{ti} V_{\underline{b}}^{ti} \cdot (-d\tau_B R b_{t+1i}).$$

We define elasticities e_{Bt} and e_{Lt} exactly as in equation (A.1) in Appendix A.1. We define $g_{ti} = \omega_{ti}V_c^{ti}/\int_j \omega_{tj}V_c^{tj}$ the normalized social marginal welfare weight on individual ti. Importantly, $\int_j \omega_{tj}V_c^{tj}$ now grows at rate $G^{-\gamma}$ per generation so that $G^{\gamma t}\int_j \omega_{tj}V_c^{tj}$ converges to a steady state.

Using the individual first order condition $V_c^{ti} = R(1-\tau_B)V_{\underline{b}}^{ti}$ when $b_{t+1i} > 0$, along with the budget balance equation (A.1), and dividing by $R \cdot G^{\gamma t} \int_j \omega_{tj} V_c^{tj}$ (constant in steady state), allows us to rewrite the first order condition $dSWF = 0$ as

$$0 = \sum_{t \geq T} \Delta^t G^{-\gamma t} \int_i g_{ti} \left[-b_{ti}(1 + e_{Bti}) + \frac{1 - e_{Bt}\tau_B/(1-\tau_B)}{1 - e_{Lt}\tau_L/(1-\tau_L)} \frac{y_{Lti}}{y_{Lt}} b_t \right]$$
$$- \sum_{t \geq T-1} \Delta^t G^{-\gamma t} \int_i g_{ti} \frac{b_{t+1i}}{R(1-\tau_B)}.$$

As everything has converged for $t \geq T$, dividing by $Rb_t G^{-t}$ (which is constant in steady state) and using definition (4) for $\bar{y}_L$, $\bar{b}^{\text{received}}$, $\bar{b}^{\text{left}}$, and $\hat{e}_{Bt} = \int_i g_{ti} b_{ti} e_{Bti} / \int_i g_{ti} b_{ti}$, the first order condition is rewritten as

$$0 = -\sum_{t \geq T} \Delta^t G^{t-\gamma t} \bar{b}^{\text{received}}(1 + \hat{e}_{Bt}) + \sum_{t \geq T} \Delta^t G^{t-\gamma t} \frac{1 - e_{Bt}\tau_B/(1-\tau_B)}{1 - e_{Lt}\tau_L/(1-\tau_L)} \bar{y}_L$$
$$- \sum_{t \geq T-1} \Delta^t G^{t-\gamma t} \frac{G\bar{b}^{\text{left}}}{R(1-\tau_B)}.$$

There are two differences with the case without growth. First, the G in the numerator of the last term appears because bequests left are from the next period and hence bigger by a factor G (exactly as in the steady-state maximization case presented above). Second, the discount factor Δ is replaced by $\Delta G^{1-\gamma}$ because of growth of all quantities (the G factor) and decrease in average marginal utility (the $G^{-\gamma}$ factor).

We define $e_B = (1 - \Delta G^{1-\gamma}) \sum_{t \geq T} (\Delta G^{1-\gamma})^{t-T} e_{Bt}$, $\hat{e}_B = (1 - \Delta G^{1-\gamma}) \times \sum_{t \geq T} (\Delta G^{1-\gamma})^{t-T} \hat{e}_{Bt}$ as the discounted average of the e_{Bt} and $\hat{e}_{Bt}$. We then define e_L so that

$$\frac{1 - e_B\tau_B/(1-\tau_B)}{1 - e_L\tau_L/(1-\tau_L)}$$
$$= \left(1 - \Delta G^{1-\gamma}\right) \sum_{t \geq T} \left(\Delta G^{1-\gamma}\right)^{t-T} \frac{1 - e_{Bt}\tau_B/(1-\tau_B)}{1 - e_{Lt}\tau_L/(1-\tau_L)}.$$

Using those definitions, we can rewrite the first order condition as

$$0 = -\bar{b}^{\text{received}}(1 + \hat{e}_B) + \frac{1 - e_B\tau_B/(1-\tau_B)}{1 - e_L\tau_L/(1-\tau_L)} \bar{y}_L - \frac{G\bar{b}^{\text{left}}}{R\Delta G^{1-\gamma}(1-\tau_B)},$$

where the $\Delta G^{1-\gamma}$ expression in the denominator of the third term appears because the sum for the third term starts at $T-1$ instead of T. Rearranging this expression leads immediately to formula (9) with Δ being replaced by $\Delta G^{-\gamma}$, that is,

$$\tau_B = \frac{1 - \left[1 - \frac{e_L \tau_L}{1-\tau_L}\right] \cdot \left[\frac{\bar{b}^{\text{received}}}{\bar{y}_L}(1+\hat{e}_B) + \frac{1}{R\Delta G^{-\gamma}} \frac{\bar{b}^{\text{left}}}{\bar{y}_L}\right]}{1 + e_B - \left[1 - \frac{e_L \tau_L}{1-\tau_L}\right] \frac{\bar{b}^{\text{received}}}{\bar{y}_L}(1+\hat{e}_B)}. \tag{S.4}$$

When the Modified Golden Rule holds, we have $R\Delta G^{-\gamma} = 1$, so that formula (10) applies unchanged (all the reasoning with endogenous capital stock applies virtually unchanged). The proof of the Modified Golden Rule with growth can be done exactly as in the case with no growth by considering one small reform $d\underline{w}$ at period T and the same reform (multiplied by $-R$) at period $T+1$. By linearity of small changes, the sum of the two reforms is budget neutral. Hence, it has to be welfare neutral as well. The social welfare effect of the period $T+1$ reform is $-R\Delta G^{-\gamma}$ times the welfare effect of the period T reform because (a) it is $-R$ times bigger, (b) it happens one generation later so is discounted by Δ, (c) it affects generations that have marginal utility $G^{-\gamma}$ times as large.

S.1.3. *Optimal Long-Run τ_B in Dynastic Model With Elastic Labor Supply*

DYNASTIC MODEL LONG-RUN OPTIMUM, PERIOD 0 PERSPECTIVE, AND ELASTIC LABOR SUPPLY:

$$\tau_B = \frac{1 - \frac{\bar{b}^{\text{received}}}{\bar{y}_L}\left[1 - \frac{e_L^{\text{pdv}} \tau_L}{1-\tau_L}\right]}{1 - \frac{\bar{b}^{\text{received}}}{\bar{y}_L}\left[1 - \frac{e_L^{\text{pdv}} \tau_L}{1-\tau_L}\right] + e_B^{\text{pdv}}} \quad \textit{or equivalently} \tag{S.5}$$

$$\tau_B = \frac{1 - \frac{1}{\delta R}\frac{\bar{b}^{\text{left}}}{\bar{y}_L}\left[1 - \frac{e_L^{\text{pdv}} \tau_L}{1-\tau_L}\right]}{1 + e_B^{\text{pdv}}},$$

where e_L^{pdv} *is the elasticity of discounted earnings with respect to* $1-\tau_L$ (*see below for exact definition*), e_B^{pdv} *is defined in* (15), *and* $\bar{b}^{\text{received}} = \frac{E[u_c^{ti} b_{ti}]}{b_t E u_c^{ti}}$, $\bar{b}^{\text{left}} = \frac{E[u_c^{ti} b_{t+1i}]}{b_{t+1} E u_c^{ti}}$, $\bar{y}_L = \frac{E[y_{Lti} u_c^{ti}]}{y_{Lt} E u_c^{ti}}$.

PROOF: We consider the small open economy with exogenous R, period-by-period budget balance, and the utilitarian case (w.l.o.g.). The government chooses $(\tau_{Bt}, \tau_{Lt})_{t\geq 0}$ to maximize

$$EV_0 = \sum_{t\geq 0} \delta^t E u^{ti}\big(Rb_{ti}(1-\tau_{Bt}) + (1-\tau_{Lt})w_{ti}l_{ti} + E_t - b_{t+1i}, l_{ti}\big),$$

subject to period-by-period budget balance $E_t = \tau_{Bt}Rb_t + \tau_{Lt}y_{Lt}$ with E_t given.

Consider again a reform $d\tau_B$ so that $d\tau_{Bt} = d\tau_B$ for all $t \geq T$ (and correspondingly $d\tau_{Lt}$ to maintain budget balance and keeping E_t constant). We assume that T is large enough that all variables have converged for $t \geq T$. Using the envelope conditions for l_{ti} and b_{ti}, we have

$$0 = dEV_0 = -\sum_{t\geq T} \delta^t E\big[u_c^{ti} \cdot Rb_{ti}\big]\, d\tau_{Bt} - \sum_{t\geq 1} \delta^t E\big[u_c^{ti} \cdot y_{Lti}\big]\, d\tau_{Lt}.$$

To rewrite this equation in terms of elasticities of b_t and y_{Lt} with respect to $1-\tau_B$ and $1-\tau_L$, we define again e_{Bt} as the elastic response of b_t to the tax reform $d\tau = (d\tau_{Bt}, d\tau_{Lt})_{t\geq 0}$, so that $\frac{db_t}{b_t} = -e_{Bt}\frac{d\tau_B}{1-\tau_B}$, where db_t is the aggregate bequest response to the full reform $d\tau$. Note that the response of b_t may start before period T due to anticipatory effects described in the text. Such anticipatory effects start before T but are vanishingly small as distance to the reform increases. Therefore, we can assume that anticipatory effects take place only after all variables have converged (as long as T is chosen large enough).

The response builds over generations and eventually converges to the long-run steady-state elasticity e_B. We similarly define the elasticity e_{Lt} so that $\frac{dy_{Lt}}{y_{Lt}} = -e_{Lt}\frac{d\tau_{Lt}}{1-\tau_L}$, where dy_{Lt} is the labor supply response to the full reform $d\tau$. Period-by-period budget balance requires

$$Rb_t\, d\tau_B\left(1 - e_{Bt}\frac{\tau_B}{1-\tau_B}\right) = -d\tau_{Lt}y_{Lt}\left(1 - e_{Lt}\frac{\tau_L}{1-\tau_L}\right) \quad \text{for} \quad t \geq T,$$

$$-Rb_t\, d\tau_B e_{Bt}\frac{\tau_B}{1-\tau_B} = -d\tau_{Lt}y_{Lt}\left(1 - e_{Lt}\frac{\tau_L}{1-\tau_L}\right) \quad \text{for} \quad t < T.$$

The equation for $t < T$ does not have the term $Rb_t\, d\tau_B$ on the left-hand side because the $d\tau_B$ reform starts at T. However, through anticipatory responses, b_t responds before T, requiring an adjustment $d\tau_{Lt}$ to balance the budget (and which triggers a labor supply response). Using those equations (and dividing by $Rb_t\,d\tau_B$, as b_t is constant in the long term), we rewrite $dEV_0 = 0$ as

$$0 = -\sum_{t\geq T}\delta^t E\left[u_c^{ti}\frac{b_{ti}}{b_t}\right] + \sum_{t\geq T}\delta^t E\left[u_c^{ti}\frac{y_{Lti}}{y_{Lt}}\right]\frac{1-\dfrac{e_{Bt}\tau_B}{1-\tau_B}}{1-\dfrac{e_{Lt}\tau_L}{1-\tau_L}}$$

$$-\sum_{t<T}\delta^t E\left[u_c^{ti}\frac{y_{Lti}}{y_{Lt}}\right]\frac{\dfrac{e_{Bt}\tau_B}{1-\tau_B}}{1-\dfrac{e_{Lt}\tau_L}{1-\tau_L}}.$$

With $\bar{b}^{\text{received}} = \frac{E[u_c^{ti}b_{ti}]}{b_t E u_c^{ti}}$, $\bar{y}_L = \frac{E[y_{Lti}u_c^{ti}]}{y_{Lt}E u_c^{ti}}$, we get (as all terms have converged and are identical):

$$0 = -\bar{b}^{\text{received}}\sum_{t\geq T}\delta^t + \bar{y}_L\sum_{t\geq T}\delta^t\frac{1-\dfrac{e_{Bt}\tau_B}{1-\tau_B}}{1-\dfrac{e_{Lt}\tau_L}{1-\tau_L}} - \bar{y}_L\sum_{t<T}\delta^t\frac{\dfrac{e_{Bt}\tau_B}{1-\tau_B}}{1-\dfrac{e_{Lt}\tau_L}{1-\tau_L}}.$$

Define the bequest elasticities as in the main text, $e_B^{\text{pdv}} = e_B^{\text{post}} + e_B^{\text{anticip.}}$ with $e_B^{\text{post}} = (1-\delta)\sum_{t\geq T}\delta^{t-T}e_{Bt}$ and $e_B^{\text{anticip.}} = (1-\delta)\sum_{t<T}\delta^{t-T}e_{Bt}$, and define e_L^{pdv} so that

$$\frac{1-e_B^{\text{pdv}}\tau_B/(1-\tau_B)}{1-e_L^{\text{pdv}}\tau_L/(1-\tau_L)} = (1-\delta)\sum_{t\geq T}\delta^{t-T}\frac{1-e_{Bt}\tau_B/(1-\tau_B)}{1-e_{Lt}\tau_L/(1-\tau_L)} - (1-\delta)\sum_{t<T}\delta^{t-T}\frac{e_{Bt}\tau_B/(1-\tau_B)}{1-e_{Lt}\tau_L/(1-\tau_L)}.$$

Again, in the case e_{Lt} constant in t, then we have $e_{Lt}\equiv e_L = e_L^{\text{pdv}}$ (e.g., with iso-elastic quasi-linear utility functions of the form $V^{ti}(c,\underline{b},l) = U^{ti}(c - l^{1+1/e_L},\underline{b})$). Using those definitions, we can rewrite the first order condition as

$$0 = -\bar{b}^{\text{received}} + \bar{y}_L\frac{1-e_B^{\text{pdv}}\tau_B/(1-\tau_B)}{1-e_L^{\text{pdv}}\tau_L/(1-\tau_L)}.$$

This can be easily rearranged in the first formula in (S.5). To obtain the second formula in (S.5), we use $\bar{b}^{\text{left}} = \delta R(1-\tau_B)\bar{b}^{\text{received}}$ in the long-run steady state. *Q.E.D.*

S.1.4. *Modified Golden Rule in the Dynastic Model*

We can extend the dynastic model to the case with endogenous factor prices (closed economy) exactly as in our model of Section 3.1. Again, this extension requires to be able to tax both labor income and capital at separate and time varying rates so that the government controls after-tax factor prices $\underline{R}_t$ and $\underline{w}_t$. The optimal τ_B formula carries over to the closed economy case unchanged, and applies both in the period-by-period budget balance case and when the government can use debt.

When the government can use debt optimally, the Modified Golden Rule $\delta R = 1$ holds also in the dynastic model. This can be established exactly in the same way as in our model of Section 3.1. We consider a small reform $d\underline{w}$ at period T and the same reform (multiplied by $-R$) at period $T+1$. By linearity of small changes, the sum of the two reforms is budget neutral. Hence, it has to be welfare neutral as well. The social welfare effect of the period $T+1$ reform is $-R\delta$ times the welfare effect of the period T reform because (a) it is $-R$ times bigger, (b) it happens one generation later so is discounted by δ. This implies that $\delta R = 1$. Aiyagari (2005) obtained the same result but used a government provided public good to establish it. Our proof shows that a public good is not necessary. Any type of reform at periods T versus $T+1$ can prove the result. This shows that the Modified Golden Rule is a robust result of dynamic efficiency.

S.2. RAWLSIAN OPTIMAL FORMULA WITH GENERATIONAL BUDGET

In the case of the Meritocratic Rawlsian optimum where social welfare is concentrated among zero-receivers, it is possible to obtain the long-run optimum tax formula (10) that maximizes discounted social welfare with dynamic efficiency as the solution of the much simpler following static problem. The government maximizes steady-state welfare subject to the alternative "generational" budget balance $\tau_{Bt} b_{t+1} + \tau_{Lt} y_{Lt} = E_t$, so that generation t funds its lump-sum grant E_t with taxes on its labor earnings y_{Lt} and taxes on the bequests it leaves. Bequest taxes are collected at the end of the period.[25] This derivation is useful because it delivers the Meritocratic Rawlsian version of (10) without having to introduce discounting and dynamic efficiency issues.

Formally, assuming everything has converged to the steady state (so that t subscripts can be dropped), the government maximizes

$$(\text{S.6}) \qquad SWF = \max_{\tau_L, \tau_B} \int_i \omega_i V^i\big(w_i l_i (1-\tau_L) + E - b_i, R b_i (1-\tau_B), l_i\big) \quad \text{s.t.}$$

$$\tau_B b + \tau_L y_L = E.$$

Note that bequests received are not included in lifetime resources because ω_i is zero for bequest receivers. We denote by $g_i = \omega_i V_c^i / \int_j \omega_j V_c^j$ the normalized social marginal welfare weight on individual i. g_i measures the social value of

[25]This is equivalent to collecting them on capitalized bequests Rb_{t+1} at the end of next period and discounting those taxes at rate $1/R$, as they accrue one period later.

increasing consumption of individual i by \$1 (relative to increasing everybody's consumption by \$1).

Consider a small reform $d\tau_B > 0$; budget balance with $dE = 0$ requires that $d\tau_L$ is such that

$$b\, d\tau_B\left(1 - e_B\frac{\tau_B}{1-\tau_B}\right) = -d\tau_L y_L\left(1 - e_L\frac{\tau_L}{1-\tau_L}\right), \tag{S.7}$$

where we have used the standard elasticity definitions (3).

Using the fact that b_i and l_i are chosen to maximize individual utility, and applying the envelope theorem, the effect of the reform $d\tau_B, d\tau_L$ on steady-state social welfare is

$$dSWF = \int_i \omega_i V_c^i \cdot (-d\tau_L y_{Lti}) + \omega_i V_{\underline{b}}^i \cdot (-d\tau_B R b_i).$$

At the optimum, $dSWF = 0$. Using the individual first order condition $V_c^i = R(1-\tau_B)V_{\underline{b}}^i$ when $b_i > 0$, expression (S.7) for $d\tau_L$, and the definition of g_i, we have

$$0 = \int_i g_i \cdot \left(\frac{1 - e_B\tau_B/(1-\tau_B)}{1 - e_L\tau_L/(1-\tau_L)}\frac{y_{Li}}{y_L} b\, d\tau_B - d\tau_B\frac{b_i}{1-\tau_B}\right).$$

The first term captures the positive effect of reduced labor income tax and the second term captures the negative effect on bequest leavers.

Let $\bar{y}_L$ and $\bar{b}^{\text{left}}$ be the population averages of $g_i \cdot y_{Li}/y_L$ and $g_i \cdot b_i/b$; we have

$$0 = \frac{1 - e_B\tau_B/(1-\tau_B)}{1 - e_L\tau_L/(1-\tau_L)}\bar{y}_L - \frac{\bar{b}^{\text{left}}}{1-\tau_B},$$

hence the following holds:

MERITOCRATIC RAWLSIAN STEADY STATE WITH GENERATIONAL BUDGET BALANCE: *The optimal tax rate τ_B that maximizes long-run welfare of zero-bequest receivers with period-by-period "generational" budget balance $\tau_{Bt}b_{t+1} + \tau_{Lt}y_{Lt} = E_t$ is given by*

$$\tau_B = \frac{1 - \left[1 - \frac{e_L\tau_L}{1-\tau_L}\right] \cdot \frac{\bar{b}^{\text{left}}}{\bar{y}_L}}{1 + e_B}. \tag{S.8}$$

This formula is consistent with the dynamically efficient formula because it considers the "generational" budget constraint $\tau_{Bt}b_{t+1} + \tau_{Lt}y_{Lt} = E_t$ instead of the cross-sectional budget constraint $\tau_{Bt}Rb_t + \tau_{Lt}y_{Lt} = E_t$. This works for zero-receivers because the welfare trade-off involves solely current labor taxes versus taxes paid on bequests left for the *same* generation t. If the social welfare function puts weight on bequests receivers, this "generational" budget fails to be consistent with the dynamic efficient case because of the welfare term involving bequests received.[26] In contrast, the cross-sectional budget (from the main text) works for the term involving bequests received, but fails for bequests left. Hence, in the general case involving both bequest receivers and bequest leavers in social welfare, two generations are involved and there is no steady-state budget short-cut that can be consistent with the dynamically efficient case. In that case, we need to go back to the analysis presented in the main text.

S.3. CALIBRATION AND NUMERICAL SIMULATIONS DETAILS

All detailed calibration results, computer codes, and formulas are provided in the Data Appendix file available on line as Supplemental Material. Our main sensitivity checks are reported in Figures S.1–S.6, and are commented in Section 4 of the paper. Figures S.1–S.6 are based on formula (17) using the following benchmark values for the parameters: $e_B = \hat{e}_B = 0.2$, $e_L = 0.2$, $\tau_L = 30\%$, $\nu = 70\%$, $R/G = e^{(r-g)H} = 1.82$ with $r - g = 2\%$ and $H = 30$ years. Optimal

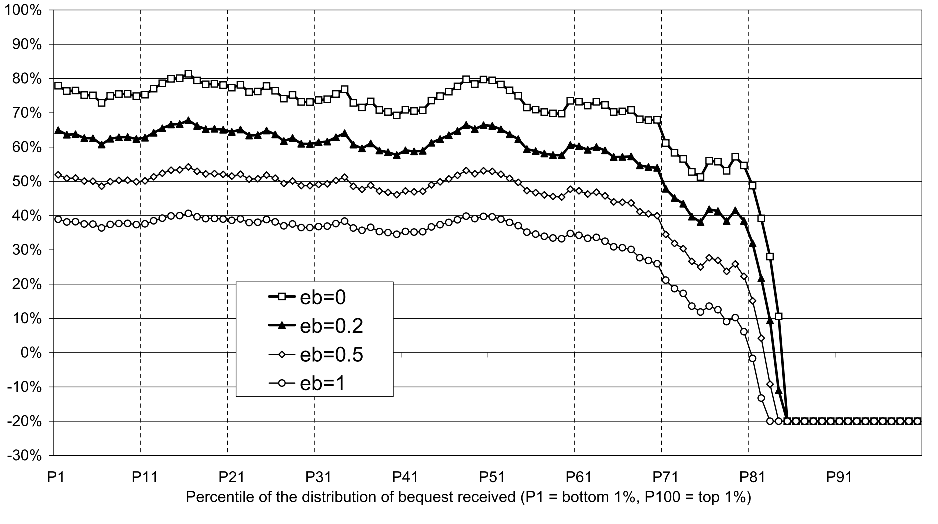

FIGURE S.1.—Optimal linear inheritance tax rates, by percentile of bequest received (France, variants with diff. eb = long-run bequest elasticity).

[26]This term will be blown up by a factor R when using the generational budget. When discounting welfare with discount rate Δ, the blown up factor becomes $R\Delta$, which disappears when the Modified Golden Rule $R\Delta = 1$ holds.

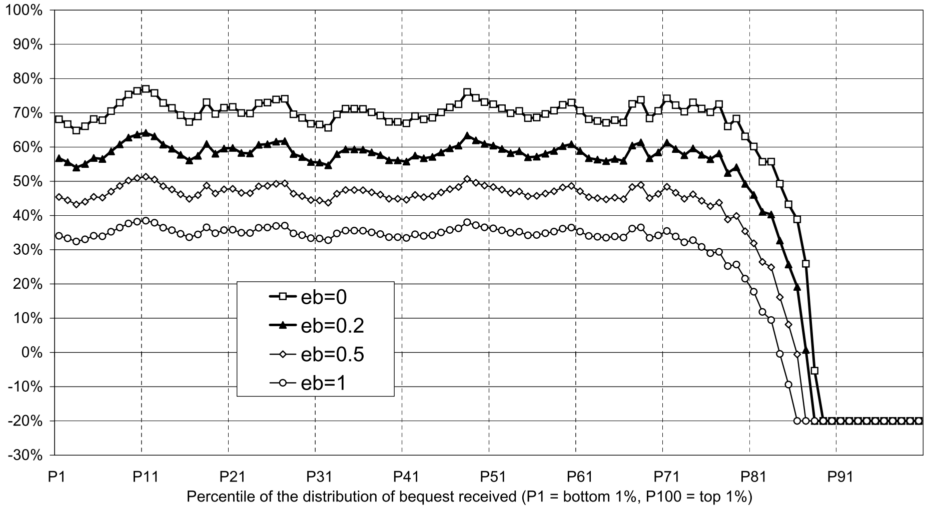

FIGURE S.2.—Optimal linear inheritance tax rates, by percentile of bequest received (U.S., variants with diff. eb = long-run bequest elasticity).

tax rates τ_B are reported for each percentile p of the distribution of bequest received, that is, $\tau_B(p)$ is the optimal τ_B when social welfare weights are fully (and uniformly) concentrated on percentile p of bequest receivers.

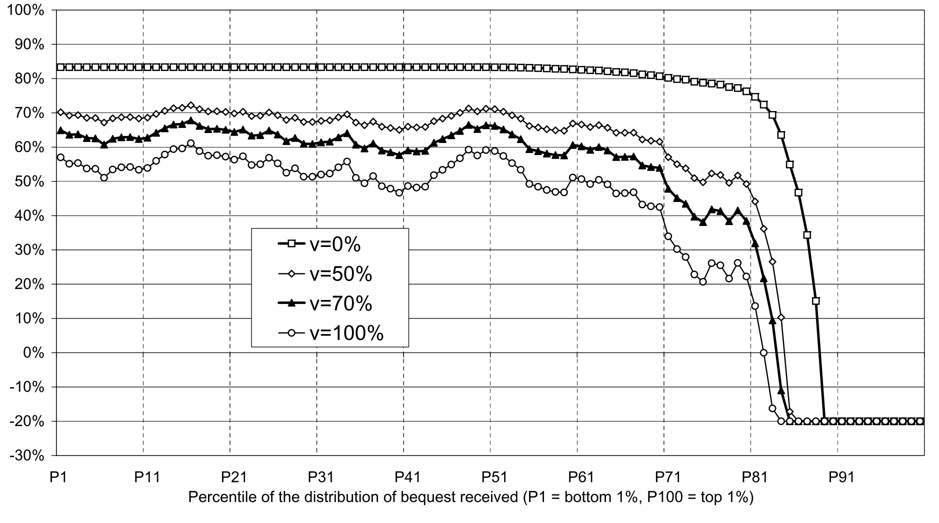

FIGURE S.3.—Optimal linear inheritance tax rates, by percentile of bequest received (France, variants with diff. v = strength of bequest motive).

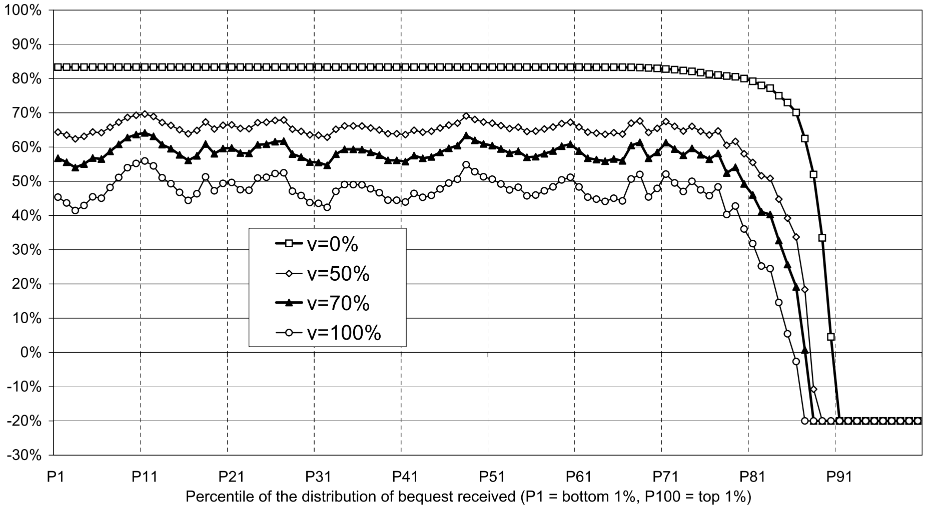

FIGURE S.4.—Optimal linear inheritance tax rates, by percentile of bequest received (U.S., variants with diff. v = strength of bequest motive).

Many supplementary sensitivity checks are provided in the excel file. One can also use the file to change the parameters and graph the resulting optimal tax rates series, for both linear and two-bracket tax specifications (with thresh-

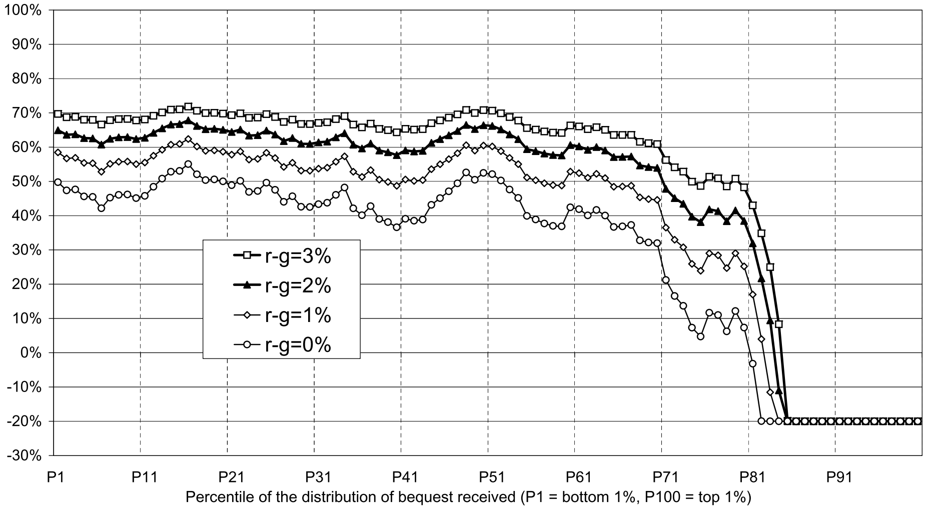

FIGURE S.5.—Optimal linear inheritance tax rates, by percentile of bequest received (France, variants with diff. $r - g$ = capitalization factor).

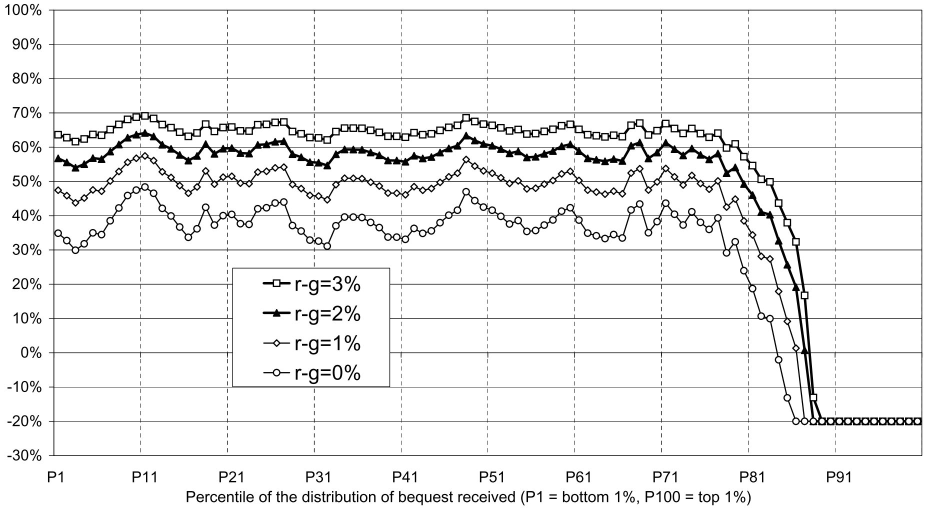

FIGURE S.6.—Optimal linear inheritance tax rates, by percentile of bequest received (U.S., variants with diff. $r - g =$ capitalization factor).

olds at $500,000 or € and $1,000,000 or €). Here we clarify and highlight a number of technical issues and limitations of our calibrations, which should be better addressed in future research.

Reporting Bias. Most importantly, we did not try to correct for reporting biases in either EP 2010 or SCF 2010. This is potentially a serious problem, because respondents in wealth surveys are known to massively underreport bequest and gift receipts. In France, the aggregate annual flow of bequests and gifts reported in household wealth surveys is less than 50% of the aggregate flow found in fiscal data—which is troubling, given that the latter ignores tax exempt assets such as life insurance, and hence is a lower bound for the true economic flow (see Piketty (2011)). When the underreporting rate is the same for all bequest receivers, then the distributional ratios $\bar{b}^{\text{received}}$ and $\bar{b}^{\text{left}}$ are unaffected, and our resulting optimal tax rates are unbiased.

However, there are reasons to believe that reporting rates are not randomly distributed. For instance, it could be that individuals who have gone through a downward sloping wealth trajectory—that is, who inherited $500,000 twenty years ago and only have $100,000—tend to forget to report their inheritance more often than average. On the contrary, it could be that individuals with high current net worth like to present themselves as "self-made" individuals and therefore tend to not report bequests and gifts (even if they represent only part of their current wealth). It could also be that both types of underreporting are present whenever bequest receipts are very large: large inheritors just tend to forget, whatever happens to their wealth trajectory.

Preliminary analysis of the data suggests that this latter bias is indeed what is happening, probably in both countries, and particularly so in the United States: there are too few individuals reporting large bequests and gifts in the retrospective questionnaires (as compared to the number of decedents with large wealth in previous surveys). In both countries, a substantial fraction of the population actually reports no bequest or gift receipt at all. Per se, this is not necessarily problematic: given the large concentration of wealth (bottom 50% receivers usually receive less than 5% of aggregate bequest flow), it is natural that the bottom half reports very little bequest and gift or even not at all. Hence, we randomly attribute bequest received to bottom percentiles so as to obtain a continuous distribution and replicate the actual wealth shares.[27] In France, about 50% of the population aged 70-year-old and over reports positive bequest or gifts (up from about 30% within the 18-to-29-year-old), which is consistent with tax data. In the United States, however, it is only 30% (up from about 10% among the 18-to-29-year-old). This can be partly explained by the higher level of wealth inequality observed in the U.S., but this does not seem to be sufficient. Another possible explanation is the stigma associated to inheritance in U.S. society (where "self-made" values are particularly strong in moral and political discourses). Yet another possible explanation is the fact that the retrospective questionnaire is more detailed in the French wealth survey than in the U.S. survey. In particular, the French survey asks separate questions about bequests and gifts received by each spouse, whereas there is only one question for both spouses in the SCF (so it is possible that the respondent sometime responds solely for himself or herself, although he or she is asked not to do so). In any case, there is a basic inconsistency between the self-reported bequest flow in current wealth survey and the theoretical bequest flow that one could compute by applying mortality rates to parental wealth reported in previous wealth surveys. This is likely to bias downward optimal tax rates (if only a very small percentage of the population reports any positive bequest, then, by construction, zero-receivers make the vast majority of the population and accumulate almost as much as the average, so that $\bar{b}^{\text{left}}$ is close to 100%, which leads to lower τ_B). This should be addressed in future research.

We stress that some of the differences that we obtain between France and the United States (in particular, the fact that $\bar{b}^{\text{left}}$ within the bottom 50% receivers is as large as 70%–80% in the U.S., vs. 60%–70% in France; see excel file) might well reflect such reporting biases, rather than true differences in wealth mobility and hence socially optimal tax rates. The calibration results presented in this paper should be viewed as exploratory: they provide illustrative orders of magnitudes for key parameters and optimal tax rates, but should not be used to make fine policy recommendations or comparisons between countries.

[27] We used a uniform law with upper bound equal to bottom reported bequests; we tried several specifications, and this made little difference to the resulting estimates. See excel file.

In order to illuminate the crucial role played by wealth inequality and mobility, and the importance of using the right data sources to estimate these distributional parameters, we provide in the Supplemental appendix file detailed estimates using the micro files of estate tax returns collected by Piketty, Postel-Vinay, and Rosenthal (2011) in the Paris archives over the 1872–1937 period. This is an interesting time period to look at, since it was characterized by large inheritance flows and extreme wealth concentration (with over 90% of aggregate inheritance received by top 10% successors). In addition, these are highly reliable, exhaustive administrative data covering wealth over two generations (something that is usually difficult to do), which do not suffer from the same self-reporting biases as the contemporary survey data. We find that $\bar{b}^{\text{left}}$ is as low as 20%–30% for the bottom 80% receivers (maybe with a slight rise over the period). This would imply very high optimal inheritance tax rates—typically above 80% for the benchmark values parameters used here.[28] This would also suggest that wealth mobility has increased quite spectacularly between Paris 1872–1937 and either France 2010 or the United States 2010 (which would make sense, given the decline in both the aggregate level of inheritance flows and the concentration of inherited wealth). However, given the data sources biases for the recent period, it is difficult to make a precise comparison. It would be valuable to use similar administrative data for the recent period. We leave this to future research.

Individual Bequest Motives and Rates of Return. It would be valuable to introduce individual specific estimates for the strength of bequest motive ν (using available questionnaires) and for capitalization factors (here we applied the same annual real rate of return to all bequests and gifts; this seems to have rather limited impact on optimal tax rates, however; see excel file).

Utilitarian Optimum. It would be interesting to use our estimates to compute the full social optimum implied by various social welfare functions, in particular the utilitarian optimum. In effect, this would amount to computing a weighted average of the optimal tax rates depicted in Figure 1, with weights given by the marginal social value of extra income for the different percentiles of the distribution of bequest received. The exact result will depend on the curvature γ, but it is pretty obvious that, for any reasonably large curvature (putting sufficiently more weights on bottom deciles), the utilitarian optimum will be very close to the bottom 70% receivers' most preferred tax rate. A more complicated issue is to decide whether one should use the same curvature within each percentile of the distribution of bequest received. In effect, our calibrations ignore redistribution issues between individuals in the same percentile of bequest received, but with different labor incomes. The full social welfare optimum should also introduce this dimension of redistribution.

[28]Note also that it is possible that the $\bar{y}_L$ effect pushes in the same direction: in a rentier society where the very rich do not work, then $\bar{y}_L$ can be larger than 100% for the poor and the middle class. Unfortunately, we do not observe labor earnings in estate tax returns, so we cannot really say.

Effect of τ_B on Distributional Parameters. It would be valuable to introduce more structure into our calibrations. In our baseline estimates, we simply compute the optimal tax rates by plugging observed distributional ratios into the optimal tax formula. However, in practice, distributional ratios should respond to change in tax rates, thereby implying that our baseline estimates are biased upward. In particular, one needs to put a minimum structure so that $\bar{b}^{\text{left}}$ depends on τ_B. In the case $\tau_B = 100\%$, $\bar{b}^{\text{left}} = \bar{y}_L$ is natural (as zero-receivers are no longer disadvantaged). The simplest way to proceed is to consider that we estimate $\bar{b}^{\text{left}}$ at the current rate τ_B^{current}, and then assume that $\bar{b}^{\text{left}}(\tau_B)$ is linear in τ_B (as obtained in the linear savings model; see Piketty and Saez (2012)): $\bar{b}^{\text{left}}(\tau_B) = [\bar{b}^{\text{left}}(\tau_B^{\text{current}})(1 - \tau_B) + (\tau_B - \tau_B^{\text{current}})\bar{y}_L]/[1 - \tau_B^{\text{current}}]$.

The main difficulty with this approach is that one needs to specify the current tax system, which in practice is highly nonlinear, and relies much more on the annual taxation of the flow of capital income and corporate profits (and on annual property or wealth taxes) and on inheritance taxes. Taking all forms of capital taxes together, the average effective capital tax rate is about 30%–40% in both France and the United States. Preliminary estimates using this simplified view of the current tax system lead to the conclusion that the extra effects implied by the linear structure would not be very large—as long as the optimal tax rate is not too different from the current one. For instance, if we take $\tau_B^{\text{current}} = 40\%$, and if we start from a situation where $\tau_B = 60\%$ (which is approximately the optimal linear inheritance tax rate for bottom 70% receivers in both France and the U.S.; see Figure 1), then the new corrected optimal tax rate would be reduced to $\tau_B \simeq 55\%$. We leave more sophisticated calibrations—in particular taking into account the nonlinear structure of the tax system—to future research.

Optimal τ_B by Cohort. Another limitation of our calibrations is that we compute optimal tax rates from the viewpoint of a single cohort, namely individuals over 70 years old in 2010. This corresponds to the cohorts born in the 1920s–1930s, who received bequests from their parents mainly in the 1970s–1980s, and who are about to leave bequests to their children in the 2010s–2020s. The problem is that we are not in a steady state. In France, the aggregate annual flow of bequest was slightly over 5% of national income in the 1970s, and has gradually increased in recent decades, up to about 15% of national income in the 2010s (Piketty (2011)); in the United States, the trend is going in the same direction, though probably with a lower slope.[29] In other words, we have computed optimal tax rates from the viewpoint of cohorts who, at the aggre-

[29]The series by Piketty and Zucman (2013) showed that the aggregate wealth-income ratio has increased significantly in the U.S. since the 1970s, but less strongly than in Europe. The U.S. also has larger demographic growth (younger population and lower mortality rates) and larger non-transmissible, annuitized wealth (pension funds), both further moderating the rise in the aggregate bequest flow.

gate level, have received less bequests than what they will leave—which biases downward optimal rates.

Formula Using Aggregate Bequest Flow. In Piketty and Saez (2012), we showed that the optimal tax formula can be re-expressed in terms of the aggregate bequest flow $b_y = B/Y$, and we presented calibrations illustrating the fact that, for a given structure of preferences and shocks, the optimal tax rate is a steeply increasing function of b_y. The intuition is the following: with a low b_y, there is not much gain from taxing high bequest receivers from my own cohort, and in addition, low and high bequest receivers accumulate wealth levels that are not too far apart. In future research, it would be valuable to combine the micro calibrations emphasized here and the macro calibrations presented in the working paper so as to compute cohort-varying, out-of-steady-state optimal tax rates. It is likely that the optimal tax rates from the viewpoint of more recent cohorts will be significantly larger than those for older cohorts.

S.4. OPTIMAL NONLINEAR INHERITANCE TAXATION

Our formulas can be extended to the case with nonlinear bequest taxation when the nonlinear bequest tax takes the following simple but realistic form. Bequests below a threshold b_t^* are exempt and the portion of bequests above the threshold b_t^* is taxed at the constant marginal tax rate τ_{Bt}. In effect, the tax on b_{ti} is $\tau_{Bt}(b_{ti} - b_t^*)^+$. Actual bequest tax systems often do take such a form. Considering multiple brackets with different rates is unfortunately intractable, as we explain below. We consider only the basic model of Section 2.2 and the Meritocratic Rawlsian criterion (the formulas can be extended to other models as well). We consider the case with "generational" budget balance so as to be consistent with dynamic efficiency (as is possible when considering the zero-receivers optimum as discussed in Section S.2).

Let us denote by $B_{ti} = (b_{ti} - b_t^*)^+$ taxable bequests of individual ti and by $B_t = \int_i B_{ti}$ aggregate taxable bequests. The individual maximization problem is

$$\max_{c_{ti}, b_{t+1i} \geq 0} V^{ti}\big(c_{ti}, R\big[b_{t+1i} - \tau_{Bt+1}\big(b_{t+1i} - b_{t+1}^*\big)^+\big], l_{ti}\big) \quad \text{s.t.}$$

$$c_{ti} + b_{t+1i} = R[b_{ti} - \tau_{Bt} B_{ti}] + w_{ti} l_{ti} (1 - \tau_{Lt}) + E_t.$$

The individual first order condition for bequests left is $V_c^{ti} = R(1 - \tau_{Bt+1}) V_{\underline{b}}^{ti}$ if $B_{t+1i} > 0$ and $V_c^{ti} = R V_{\underline{b}}^{ti}$ if $0 < b_{t+1i} < b_{t+1}^*$. Importantly, $B_{t+1i} V_c^{ti} = R(1 - \tau_{Bt+1}) B_{t+1i} V_{\underline{b}}^{ti}$ is always true.

We take b^* as given and constant with t in the steady state. The government solves

$$SWF = \max_{\tau_L, \tau_B} \int_i \omega_{ti} V^{ti}\big(R(b_{ti} - \tau_B B_{ti}) + w_{ti} l_{ti} (1 - \tau_L) + E_t - b_{t+1i}, R(b_{t+1i} - \tau_B B_{t+1i}), l_{ti}\big), \tag{S.9}$$

with E given and τ_L and τ_B linked to meet the "generational" budget constraint, $E = \tau_B B_{t+1} + \tau_L y_{Lt}$. The aggregate variable B_{t+1} is a function of $1 - \tau_B$ (assuming that τ_L adjusts), and y_{Lt} is a function of $1 - \tau_L$ (assuming that τ_B adjusts). Formally, we can define the corresponding long-run elasticities as

$$e_B = \frac{1-\tau_B}{B_t} \frac{dB_t}{d(1-\tau_B)}\bigg|_E \quad \text{and} \quad e_L = \frac{1-\tau_L}{y_{Lt}} \frac{dy_{Lt}}{d(1-\tau_L)}\bigg|_E .$$

Consider a small reform $d\tau_B > 0$; budget balance with $dE = 0$ requires that $d\tau_L$ is such that

$$B_{t+1}\, d\tau_B \left(1 - e_B \frac{\tau_B}{1-\tau_B}\right) = -d\tau_L y_{Lt} \left(1 - e_L \frac{\tau_L}{1-\tau_L}\right).$$

Using the fact that b_{t+1i} and l_{ti} are chosen to maximize individual utility and applying the envelope theorem, and the fact that $R(b_{ti} - \tau_B B_{ti}) \equiv 0$ for zero-receivers, the effect of $d\tau_B, d\tau_L$ is

$$dSWF = \int_i \omega_{ti} V_c^{ti} \cdot (-d\tau_L y_{Lti}) + \omega_{ti} V_{\underline{b}}^{ti} \cdot (-d\tau_B R B_{t+1i}).$$

At the optimum, $dSWF = 0$. Using the individual first order condition $V_c^{ti} B_{t+1i} = R(1-\tau_B) B_{t+1i} V_{\underline{b}}^{ti}$, and the expression above for $d\tau_L$, and the definition of g_{ti}, we have

$$0 = \int_i g_{ti} \cdot \left[\frac{1 - \dfrac{e_B \tau_B}{1-\tau_B}}{1 - \dfrac{e_L \tau_L}{1-\tau_L}} \frac{y_{Lti}}{y_{Lt}} B_{t+1}\, d\tau_B - \frac{d\tau_B B_{t+1i}}{1-\tau_B} \right].$$

Let $\bar{y}_L$, $\bar{B}^{\text{left}}$ be the population averages of $g_{ti} \cdot y_{Lti}/y_{Lt}$, $g_{ti} \cdot B_{t+1i}/B_{t+1}$. Dividing by $B_{t+1}\, d\tau_B$, the first order condition is rewritten as

$$0 = \frac{1 - e_B \tau_B/(1-\tau_B)}{1 - e_L \tau_L/(1-\tau_L)} \bar{y}_L - \frac{\bar{B}^{\text{left}}}{1-\tau_B}.$$

Finally, as in optimal top labor income taxation (Saez (2001)), we can define the elasticity e_b of top bequests (i.e., the full bequests among taxable bequests) with respect to $1 - \tau_B$. It is related to elasticity of aggregate taxable bequests e_B through the Pareto parameter a of the bequests distribution through the simple equation $e_B = a \cdot e_b$ with $a = b^m(b^*)/[b^m(b^*) - b^*]$, where $b^m(b^*)$ is the average bequest among bequests above the taxable threshold b^*. To see this, note that, for taxable bequests, $b_{ti} - b^* = B_{ti}$, so that $b_{ti} \frac{db_{ti}}{b_{ti}} = (b_{ti} - b^*) \frac{dB_{ti}}{B_{ti}}$, and hence $b_{ti} e_{bti} = (b_{ti} - b^*) e_{Bti}$ at the individual level. Aggregating across all taxable bequests, we get $b^m(b^*) e_b = (b^m(b^*) - b^*) e_B$, that is, $a \cdot e_b = e_B$. Hence, we can state the following:

NONLINEAR TOP RATE STEADY-STATE MERITOCRATIC RAWLSIAN OPTIMUM: *The optimal tax rate* τ_B *above threshold* b^* *that maximizes long-run steady-state social welfare of zero-receivers with "generational" budget balance is given by*

$$\tau_B = \frac{1 - \left[1 - \frac{e_L \tau_L}{1-\tau_L}\right] \cdot \frac{\bar{B}^{\text{left}}}{\bar{y}_L}}{1 + e_B} = \frac{1 - \left[1 - \frac{e_L \tau_L}{1-\tau_L}\right] \cdot \frac{\bar{B}^{\text{left}}}{\bar{y}_L}}{1 + a \cdot e_b}, \tag{S.10}$$

where $\bar{B}^{\text{left}}$ *and* $\bar{y}_L$ *are the average taxable bequests and average labor income among zero-receivers (relative to population wide averages),* e_B *is the elasticity of aggregate taxable bequests, a is the Pareto parameter of the bequest distribution, and* e_b *is the elasticity of full bequests (among taxable bequests).*

ADDITIONAL REFERENCES

PIKETTY, T., AND G. ZUCMAN (2013): "Capital Is Back: Wealth–Income Ratios in Rich Countries, 1870–2010," Working Paper, PSE. [16]

SAEZ, E. (2001): "Using Elasticities to Derive Optimal Tax Rates," *Review of Economic Studies*, 68, 205–229. [18]

Paris School of Economics, 48 Boulevard Jourdan, 75014 Paris, France; piketty@pse.ens.fr

and

Dept. of Economics, University of California at Berkeley, 530 Evans Hall, Berkeley, CA 94720, U.S.A., and NBER; saez@econ.berkeley.edu.

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