

Public Economics: Tax & Transfer Policies

(Master PPD & APE, Paris School of Economics)

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Lecture 4: Optimal Taxation of Labor Income

(check [on line](#) for updated versions)

Roadmap of lecture 4

- Main theoretical results
- The optimal labor income tax problem
- Derivation of linear optimal tax formulas
- Derivation of non-linear optimal tax formulas
- Derivation of symptotic optimal marginal rates
- Evidence on U-shaped pattern of marginal rates
- Evidence & theory on top marginal rates

Main theoretical results about optimal taxation of labor income

- (1) the social optimum usually involves a U-shaped pattern of marginal tax rates = in order to have high minimum income, one needs to withdraw it relatively fast (but not too fast) = relatively consistent with observed patterns (if we take into account transfers)
- (2) optimal top rates depends positively on income concentration (top income shares) and negatively on labor supply elasticities

(and positively on bargaining power at the top: very important if we want to understand Roosevelt-type confiscatory tax rates)

(for taxation of capital & capital income, see lectures 5-6)

- Here I will only present the main results and intuitions. For complete technical details and proofs, see the following papers:
- Mirrlees, J., "An exploration in the theory of optimum income taxation", RES 1971
- Diamond, P., "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Rates", AER 1998 [\[article in pdf format\]](#)
- Saez, "Using Elasticities to Derive Optimal Income Tax Rates", RES 2001 [\[article in pdf format\]](#)
- **Piketty-Saez, "Optimal Labor Income Taxation", 2013, [Handbook of Public Economics, vol. 5](#)**
- Piketty-Saez-Stantcheva, "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities", [AEJ 2014](#) (see also [Slides](#))

The optimal labor income tax problem

- Mirrlees (1971) : basic labor supply model used to analyze optimal labor income taxes
- Each agent i is characterized by an exogeneous wage rate w_i (=productivity)
- Labor supply l_i
- Pre-tax labor income $y_i = w_i l_i$
- Income tax $t = t(y_i)$
- $t(y_i)$ can be >0 or <0 : if <0 , then this is an income transfer, or negative income tax
- After-tax labor income $z_i = y_i - t(y_i)$
- Agents choose labor supply l_i by maximizing $U(z_i, l_i)$

- Social welfare function $W = \int W(U(z_i, l_i)) f(y_i) dy_i$ subject to budgetary constraint: $\int t(y_i) f(y_i) dy_i = 0$ (or $= G$, with $G =$ exogenous public spendings)
($f(y_i)$ = density function for $y_i =$ partly endogenous, given exogenous distribution of productivities w_i and endogenous labor supply l_i)
- If individual productivities w_i were fully observable, then the first-best efficient tax system would be $t=t(w_i)$, i.e. would not depend at all on labor supply behaviour, so that there would be no distortion = lump-sum transfers, fully efficient redistribution
- However if the tax system can only depend on income, i.e. $t = t(y_i)$, e.g. because of unobservable productivities w_i (adverse selection), then we have an equity/efficiency trade-off
- >>> Mirrlees 1971 provides analytical solutions for the second-best efficient tax system in presence of such an adverse selection pb

- But problems with the Mirrlees 1971 formula:
- (i) very complicated and unintuitive formulas, hard to apply empirically
- (ii) only robust conclusion: with finite number of productivity types w_1, \dots, w_n , then zero marginal rate on the top group = completely off-the-mark
- >>> Diamond (1998), Saez (2001): continuous distribution of types (no upper bound, so that the artificial zero-top-rate result disappears), first-order derivation of the optimal tax formulas, very intuitive and easy-to-calibrate formulas

- Other limitation of Mirrlees model: pure adverse selection pb, i.e. $y_i = w_i l_i$ with private information on individual productivities/wage rates w_i and labor supply l_i
- In practice, the income generating process also involves effort and luck: income y_i is a stochastic function of w_i , l_i and effort e_i (work intensity, job search intensity, promotion effort). **I.e. moral hazard and not only adverse selection.**
- See e.g. model studied in [lecture 2](#):
 $y_0 =$ low-paid job; $y_1 =$ high-paid job ;
 Probability ($y_i=y_1$) = $\pi_0 + \theta e_i$ if parental income = y_0
 Probability ($y_i=y_1$) = $\pi_1 + \theta e_i$ if parental income = y_1
- **The optimal tax formulas that I will present today work for all cases, i.e. any combination of adverse selection and moral hazard:** all what matters is the elasticity of income y_i with respect to changes in the tax rate, independantly of whether this elasticity comes from l_i , e_i , etc.
= “sufficient statistics” approach

First-order derivation of linear optimal labor income tax formulas

- Linear tax schemes: $t(y) = ty - t_0$
- I.e. $t =$ constant marginal tax rate
- $t_0 > 0 =$ transfer to individuals with zero labor income (RMI/RSA in France)
- Define $e =$ labor supply elasticity
- Definition: if the net-of-tax wage rate $(1-t)w_i$ increases by 1%, labor supply l_i (and therefore labor income $y_i = w_i l_i$, for given w_i) increases by $e\%$
- E.g. if $U(z_i, l_i) = z_i - V(l_i)$ (separable utility, no income effect), with $V(l) = l^{1+\mu}/(1+\mu)$, then $e = 1/\mu$
FO condition: $\text{Max } w_i l_i - V(l_i) \rightarrow l_i = w_i^{1/\mu}$
 $\rightarrow dl_i/l_i = e dw_i/w_i$ with $e = 1/\mu$

- More generally, whatever the labor income generating process $y_i = y(\text{wage rate } w_i, \text{ labor hours } l_i, \text{ effort } e_i, \text{ luck } u_i)$, one can always define **e = generalized labor supply elasticity** = elasticity of labor income with respect to the net-of-tax rate: if the net-of-tax rate $(1-t)$ increases by 1%, observed labor income y increases by $e\%$
- I.e. if $t \rightarrow t+dt$, then $1-t \rightarrow 1-t-dt$, so that $1-t$ declines by $dt/(1-t)\%$; therefore we have: $dy/y = -e dt/(1-t)$
- The generalized elasticity reflects changes in labor hours but also endogenous changes in wage rates: with higher taxes, maybe one will put less effort in education investment, or less effort in trying to get a promotion, etc.

- Assume that we're looking for the tax rate t^* maximizing tax revenues $R = ty$
- Revenue-maximizing tax rate t^* = top of the Laffer curve
- Revenue-maximizing tax rate t^* = social optimum if social welfare function $W =$ Rawlsian (infinitely concave), i.e. in the limit case where the marginal social welfare $W' = 0$ for all $U > U_{\min}$, i.e. social objective = maximizing minimum utility (maxmin) = maximizing transfer t_0
- = useful reference point: by definition, socially optimal tax rates for non-Rawlsian social welfare functions W will be below revenue-maximizing tax levels

- First-order condition: if the tax rate goes from t to $t+dt$, then tax revenues go from R to $R+dR$, with:

$$dR = y dt + t dy$$

$$\text{with } dy/y = -e dt/(1-t)$$

- I.e. $dR = y dt - t e y dt/(1-t)$
- $dR = 0$ if and only if $t/(1-t) = 1/e$
- **I.e. $t^* = 1/(1+e)$**
- I.e. pure elasticity effect : if the elasticity e is higher, then the optimal tax t^* is lower
- I.e. if $e=1$ then $t^*=50\%$, if $e=0,1$ then $t^*=91\%$, etc.
- **= the basic principle of optimal taxation theory: other things equal, don't tax what's elastic**
- Other example: Ramsey formulas on optimal indirect taxation: tax more the commodities with a less elastic demand, and conversely

First-order derivation of non-linear optimal labor income tax formulas

- General non-linear tax schedule $t(y)$
- I.e. marginal tax rates $t'(y)$ can vary with y
- Note $f(y)$ the density function for labor income, and $F(y) = \int_{z < y} f(z) dz =$ distribution function (= fraction of pop with income $< y$)
- Assume one wants to increase the marginal tax rate from t' to $t' + dt'$ over some income bracket $[y; y + dy]$. Then tax revenues go from R to $R + dR$, with:
- $dR = (1 - F(y)) dt' dy - f(y) dy t' e y dt' / (1 - t')$
- $dR = 0$ if and only if $t' / (1 - t') = (1 - F(y)) / y f(y) \quad 1/e$

- **Key formula: $t'^*/(1-t'^*) = (1-F(y))/yf(y) \cdot 1/e$**
- I.e. two effects:
- Elasticity effect: higher elasticities e imply lower marginal tax rates t'^*
- Distribution effect: higher $(1-F)/yf$ ratios imply higher marginal rates t'^*
- Intuition : $(1-F)/yf =$ ratio between the mass of people above y (=mass of people paying more tax) and the mass of people right at y (=mass of people hit by adverse incentives effects)
- For low y , the ratio $(1-F)/yf$ is necessarily declining: other things equal, marginal rates should fall
- But for high y , the ratio $(1-F)/yf$ is usually increasing: other things equal, marginal rates should rise
- >>> for constant elasticity profiles, U-shaped pattern of marginal tax rates

Asymptotic optimal marginal rates for top incomes

- With a Pareto distribution $1-F(y) = (k/y)^a$ and $f(y) = ak^a/y^{(1+a)}$, then $(1-F)/yf$ converges towards $1/a$, i.e. t'^* converges towards:
- **$t'^* = 1/(1+ae)$**
- with e = elasticity, a = Pareto coefficient
- Intuition: higher a (i.e. lower coefficient $b = a/(a-1)$, i.e. less fat upper tail = less income concentration) imply lower tax rates, and conversely
- Exemple: if $e=0,5$ and $a=2$, $t'^* = 50\%$
But if $e=0,1$ and $a=2$, $t'^* = 83\%$

- Reminder on key property of Pareto distributions:
ratio average/threshold = constant
- Note $y^*(y)$ the average income of the population above threshold y . Then $y^*(y)$ can be expressed as follows :

- $$y^*(y) = \left[\int_{z>y} z f(z) dz \right] / \left[\int_{z>y} f(z) dz \right]$$

$$= \left[\int_{z>y} dz/z^a \right] / \left[\int_{z>y} dz/z^{(1+a)} \right] = ay/(a-1)$$

- I.e. $y^*(y)/y = b = a/(a-1)$ (and $a = b/(b-1)$)

- In practice : b is usually around 2, but can vary quite a lot
For top incomes

- France 2010s, US 1970s: $b = 1.7-1.8$ ($a=2.2-2.3$)
- France or US 1910s, US 2010s: $b = 2.2-2.5$ ($a=1.7-1.8$)

For top wealth:

- France today: $b = 2.3-2.5$; France 1910s: $b=3-3.5$
- **Higher b coefficients = fatter upper-tail of the distribution = higher concentration of income (or wealth)**

Evidence on U-shaped pattern of marginal rates

- $t^*/(1-t^*) = (1-F(y))/yf(y) \approx 1/e$
- The distribution effect $(1-F(y))/yf(y)$ is typically U-shaped; so if elasticity effect $e=e(y) \approx$ stable over y , then the social optimum involves a U-shaped pattern of marginal tax rates
- Same conclusion with general SWF as long as marginal social welfare weights not too far from Rawlsian social welfare function
- Basic intuition = in order to have high minimum income, one needs to withdraw it relatively fast (but not too fast) = relatively consistent with observed patterns (if we take into account transfers)

- The increasing part of the U-shaped pattern (for upper half incomes) is due to income tax progressivity → rising marginal rates at the top
 - The decreasing part (for bottom half incomes) is due to the withdrawal of income transfers, which also creates high marginal rates
- Observed pattern of marginal rates in France:
U-shaped curve (see [RFE 97 graphs](#), [paper](#))

Figure 1
Taux moyens et taux marginaux (personnes seules)

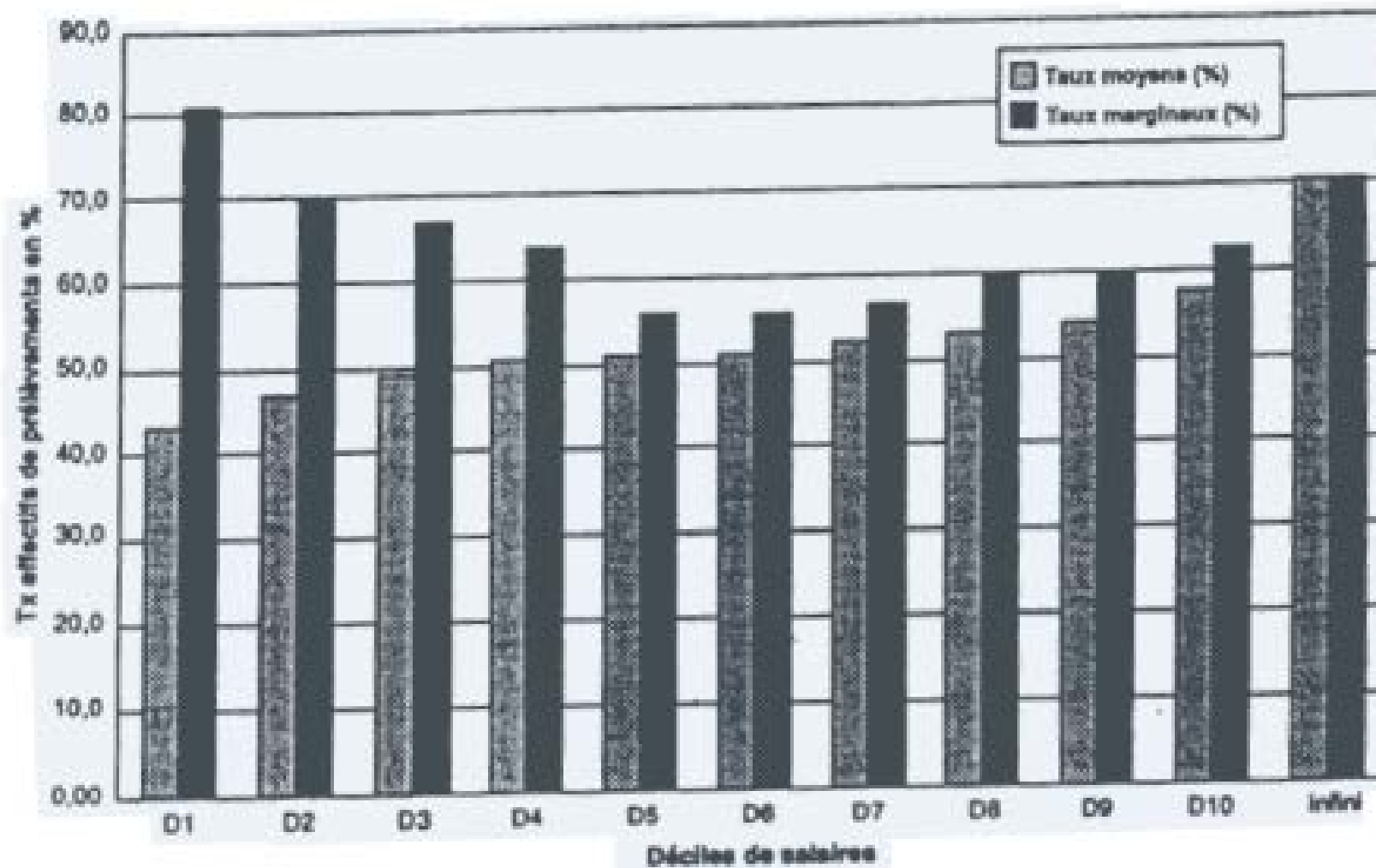
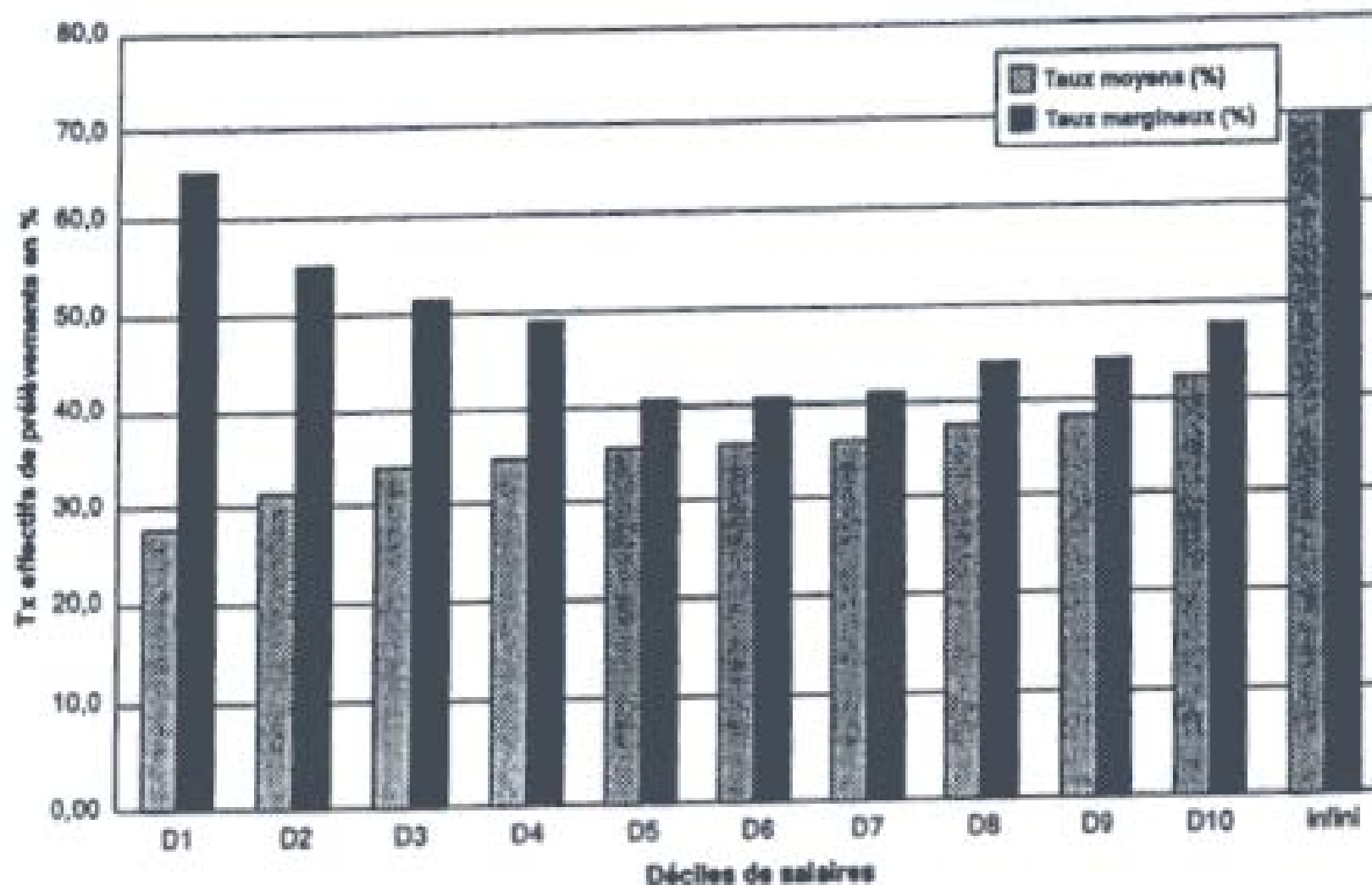


Figure 2
 Taux moyens et taux marginaux (personnes seules) hors cotisations retraites



- Simplified example for France 2013 (see [here](#) for detailed simulations and computer codes for French transfers & taxes)
- If labor income $y=0$, then $t(y)=-t_0$: t_0 = transfer to individuals with zero labor income $\approx 500\text{€}/\text{month}$ for RMI/RSA in France
- If labor income $y=y_{\min}$ =full-time minimum wage, you receive no transfer any more (unless you have children);
net minimum wage $\approx 1100\text{€}/\text{m}$, gross min. wage $\approx 1400\text{€}/\text{m}$,
total labor cost $\approx 1700\text{€}/\text{m}$

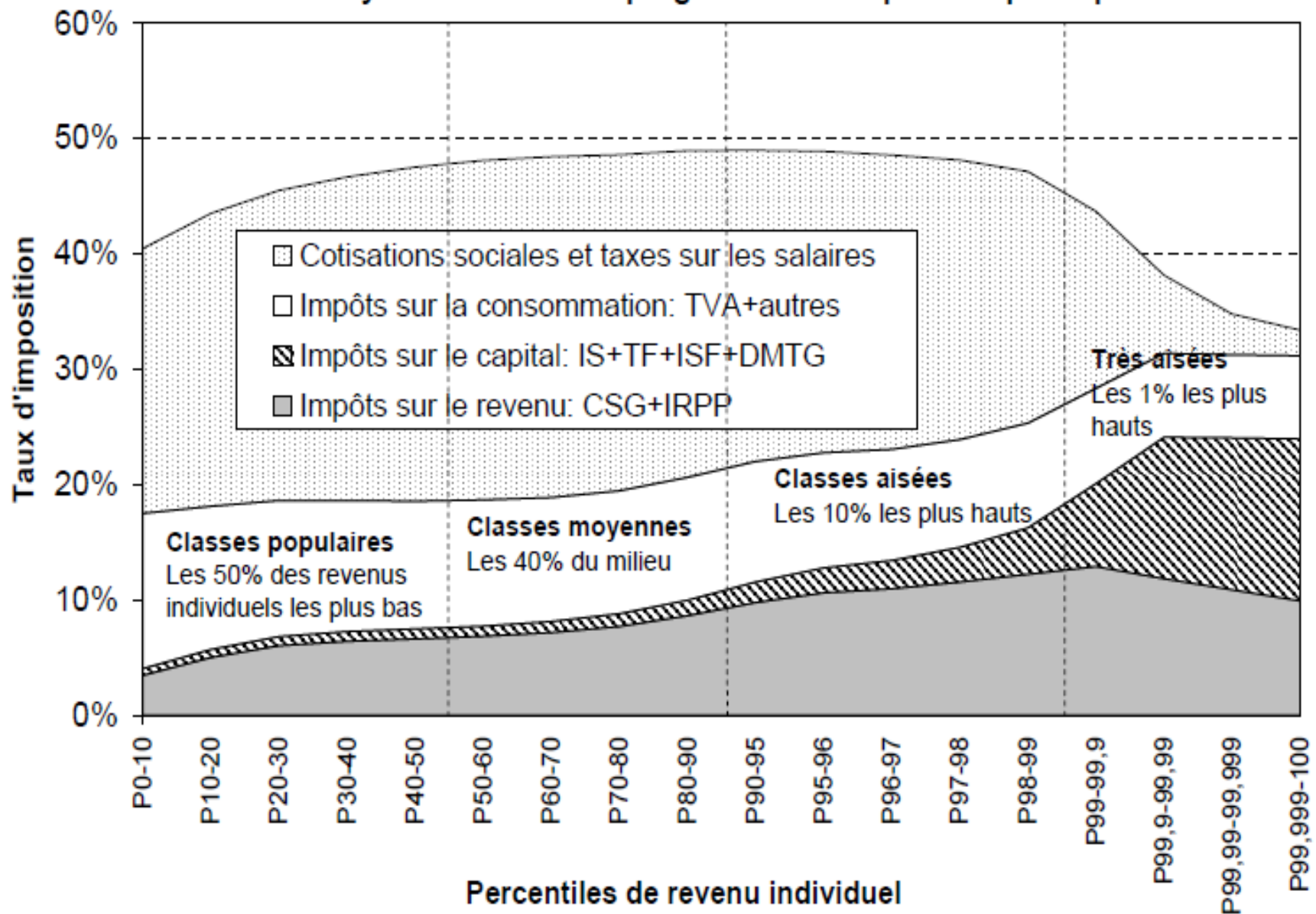
(CSG+employee payroll tax $\approx 20\%$; employer payroll tax $\approx 20\%$)

- Note: total labor cost would be $\approx 2000\text{€}/\text{m}$ at the level of the minimum wage in the absence of low-wage payroll tax cut:
employer payroll tax $\approx 20\%$ at y_{\min} \rightarrow back to $\approx 40\%$ at $1,6 \times y_{\min}$

- As pre-tax income y goes from $y=0$ to $y=1700\text{€}$, after-tax income $y-t(y)$ goes from 500 to 1100€, and $t(y)$ goes from -500 to +600€, i.e. rises by 1100€
 - marginal tax rate associated to the transition between pre-tax incomes 0 and y_{\min} = $\Delta t/\Delta y = 1100/1700 = 65\%$
(if we include VAT & other indirect taxes, the marginal tax rate on minimum wage workers would be closer to 75-80%)
- The pb is that if one wants to reduce this marginal rate (say, by further cuts in low-wage payroll tax), then one has to raise the marginal rate higher up in the distribution (say, btw y_{\min} and $1,6 \times y_{\min}$)
 - complex trade-off, current U-shaped pattern might be not too far from optimal

- Note: the simplified computations above only apply to labor incomes: rising effective tax rates (=progressivity) and U-shaped marginal tax rates
- If one introduces specific tax regimes and tax exemptions for capital incomes (with preferential tax treatment for capital gains, exemptions from many social contributions, etc.), then one gets a different picture: effective tax rates decline at the very top, i.e. inverted-U-shaped pattern of effective tax rates
- See [Landais-Piketty-Saez 2011](#) for detailed computer codes and micro files on French tax systems; see [IPP](#) reports for updates; see following graph for a summary

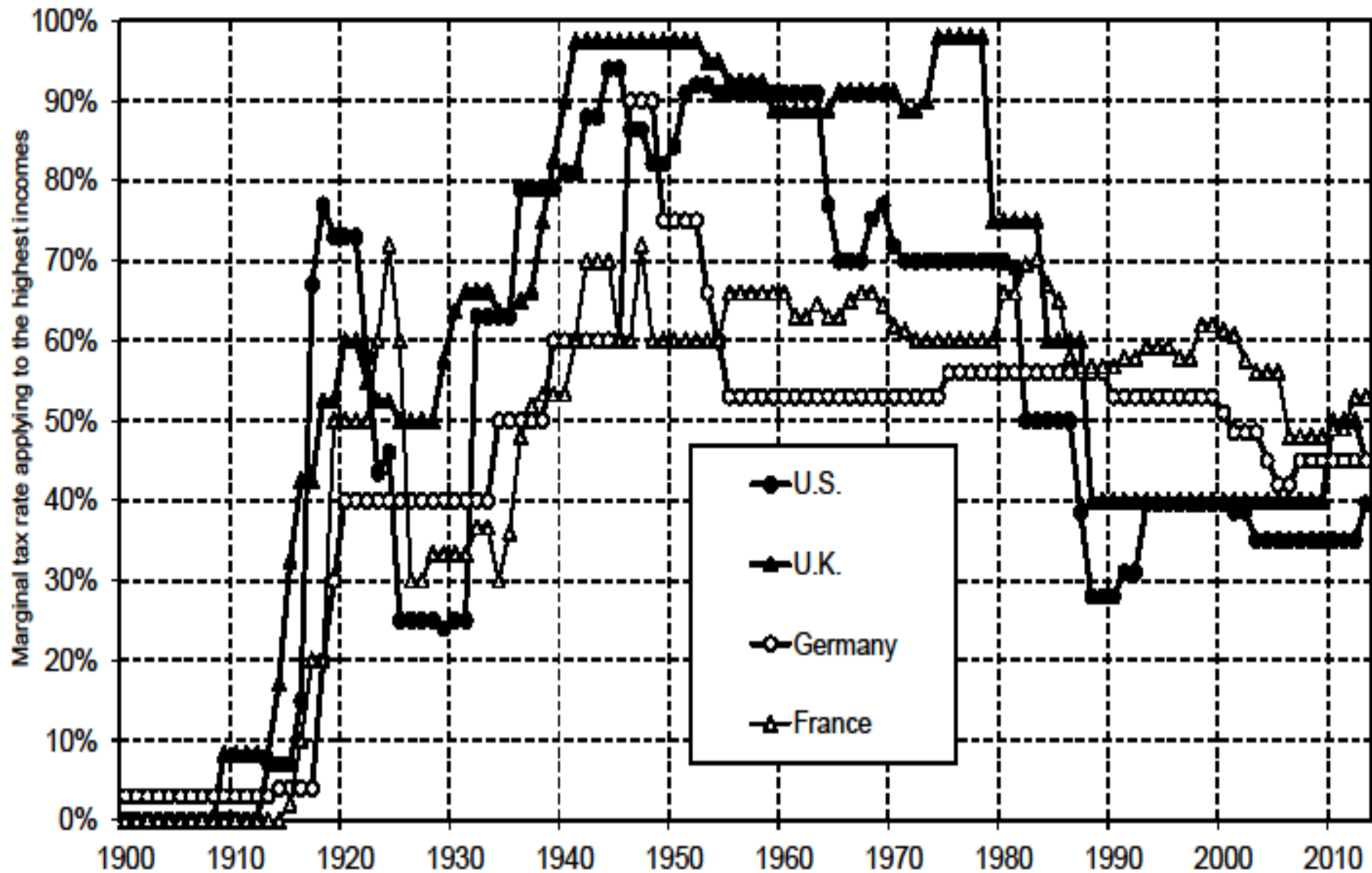
Un système faiblement progressif: décomposition par impôts



Evidence on top marginal rates

- Observed top marginal rates go from 20-30% to 80-90%
- One possible interpretation = different beliefs about elasticities of labor supply (see [this paper](#) for a learning model: it is difficult to estimate e with certainty)
- **$t'^* = 1/(1+ae)$** (with e = elasticity, a = Pareto coefficient)
- If $e=1$ and $a=2$, $t'^* = 33\%$
- If $e=0,5$ and $a=2$, $t'^* = 50\%$
- If $e=0,1$ and $a=2$, $t'^* = 83\%$

Figure 14.1. Top income tax rates, 1900-2013



The top marginal tax rate of the income tax (applying to the highest incomes) in the U.S. dropped from 70% in 1980 to 28% in 1988. Sources and series: see piketty.pse.ens.fr/capital21c.

- Empirical evidence: real labor supply elasticities $\approx 0,2-0,3$ at most (higher elasticities usually come from pure income shifting, i.e. if one can transfer income to a less taxed tax base: in principle, this can be solved by a broader tax base)
→ $t'^* \approx 60-70\%$?
- See P. Diamond & E. Saez, "The Case for a Progressive Tax: From Basic Research to Policy Recommendations", [JEP 2011](#)
- For a survey on empirical estimates of labor supply elasticities, see E. Saez, J. Slemrod and S. Gierz, "The Elasticity of Taxable Income with Respect to Marginal Tax Rates: A Critical Review", JEL 2010 [\[article in pdf format\]](#)

- However the perfect-competition model (labor income = marginal product) may not be sufficient to analyze Roosevelt-type tax rates & the recent surge in US top incomes
- A model with imperfect competition and CEO bargaining power (CEOs can sometime extract some than their marginal product) is more promising
- See Piketty-Saez-Stantcheva, "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities", [AEJ 2014](#) (see also [Slides](#))

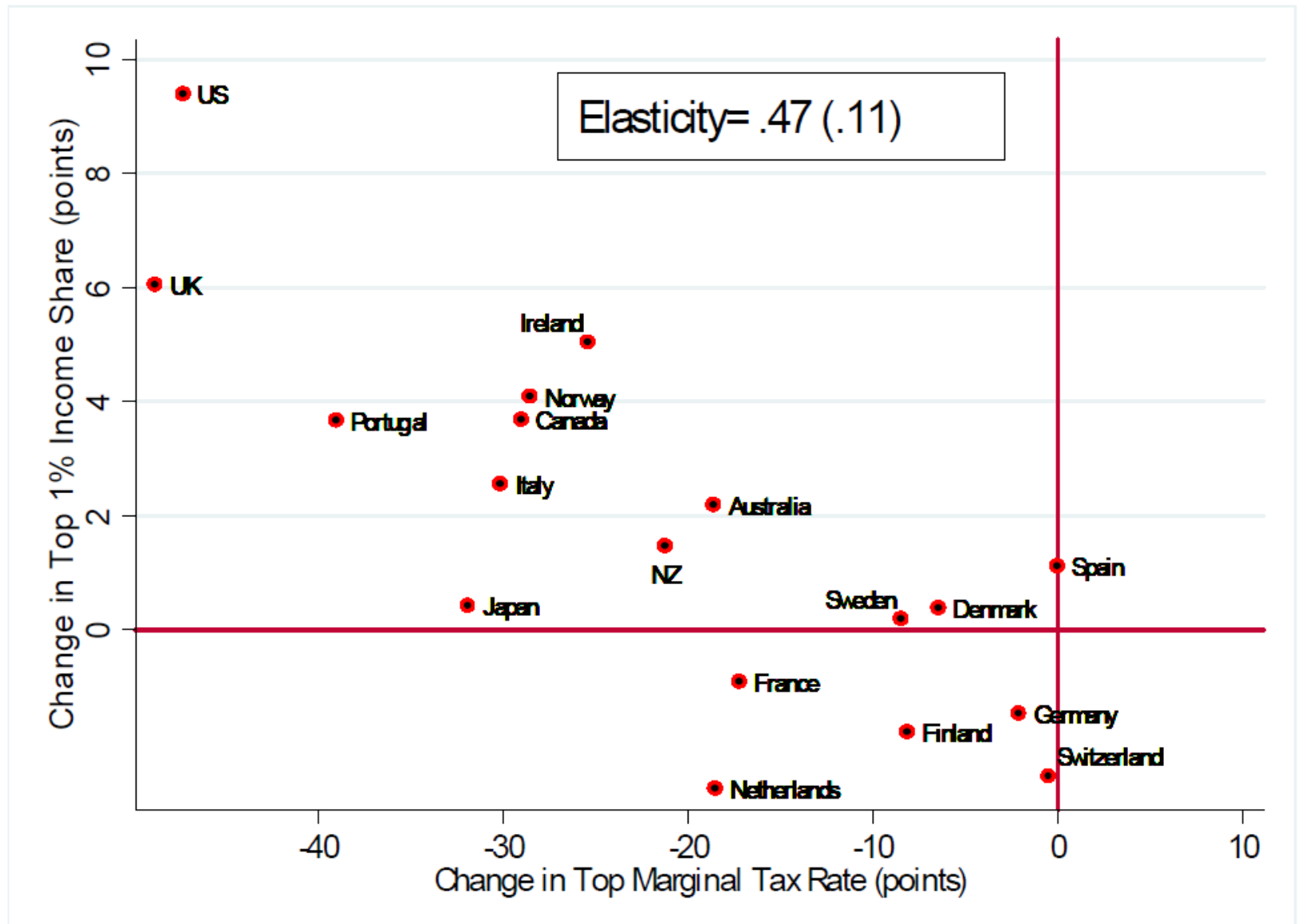
- With imperfect competition and bargaining power, the optimal tax formula becomes more complicated and can justify confiscatory tax rates
 - **Augmented formula: $\tau = (1+tae_2+ae_3)/(1+ae)$**
 - With $e = e_1 + e_2 + e_3$
 = labor supply elasticity e_1 + income shifting elasticity e_2 + bargaining elasticity e_3 (= more intensive bargaining with lower tax rate)
 - **Key point: $\tau \uparrow$ as elasticity $e_3 \uparrow$**
- for a given total elasticity e , the decomposition between the three elasticities e_1, e_2, e_3 is critical

Table 5: Synthesis of Various Scenarios

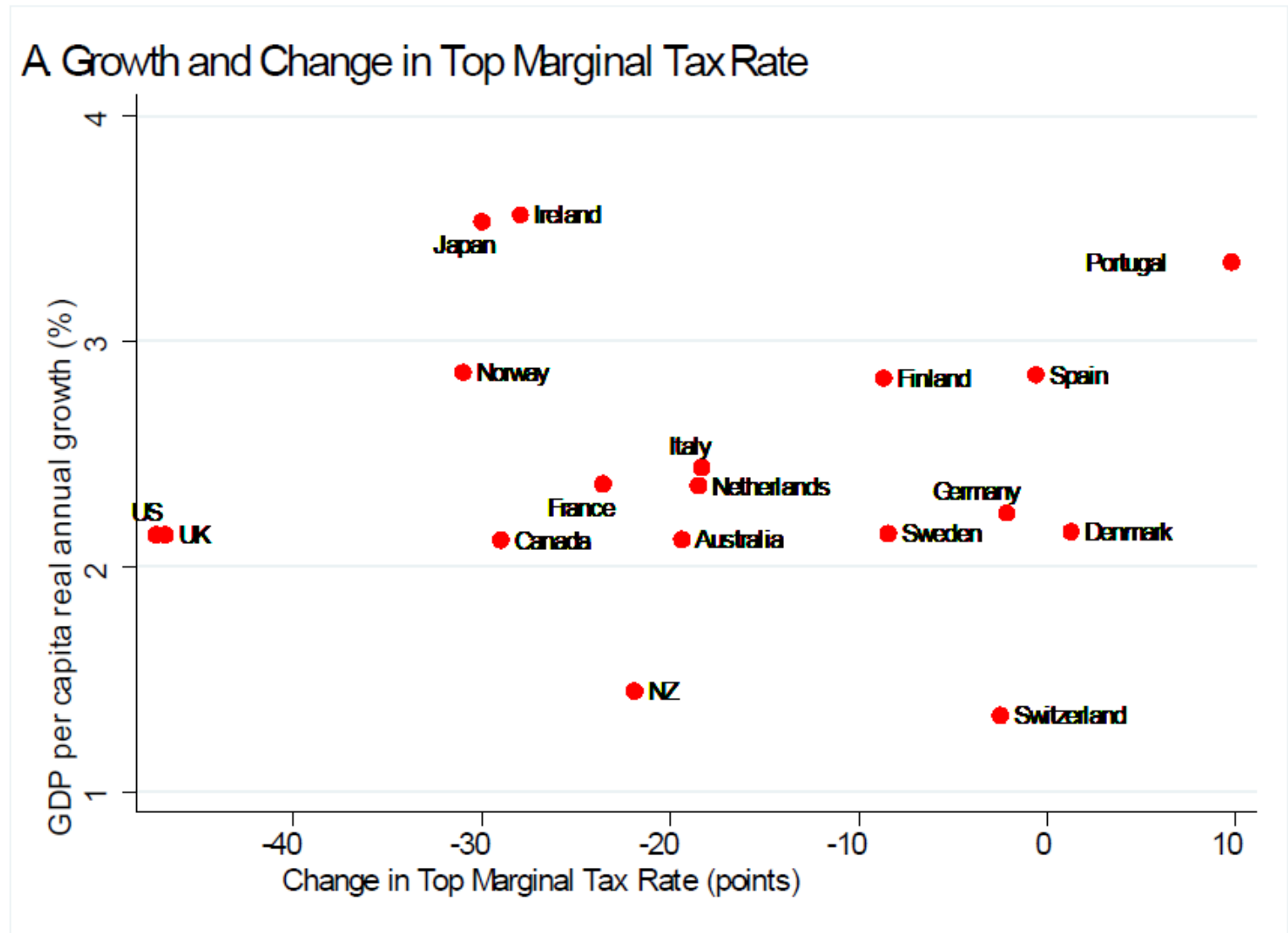
Total elasticity $e = e_1 + e_2 + e_3 =$		0.5								
<p>Scenario 1: Standard supply side tax effects</p> <p>$e_1 = 0.5$</p> <p>$e_2 = 0.0$</p> <p>$e_3 = 0.0$</p>	<p>Scenario 2: Tax avoidance effects</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; text-align: left; padding: 5px;">(a) current narrow tax base</td> <td style="width: 50%; text-align: left; padding: 5px;">(b) after base broadening</td> </tr> <tr> <td style="padding: 5px;">$e_1 = 0.2$</td> <td style="padding: 5px;">$e_1 = 0.2$</td> </tr> <tr> <td style="padding: 5px;">$e_2 = 0.3$</td> <td style="padding: 5px;">$e_2 = 0.1$</td> </tr> <tr> <td style="padding: 5px;">$e_3 = 0.0$</td> <td style="padding: 5px;">$e_3 = 0.0$</td> </tr> </table>	(a) current narrow tax base	(b) after base broadening	$e_1 = 0.2$	$e_1 = 0.2$	$e_2 = 0.3$	$e_2 = 0.1$	$e_3 = 0.0$	$e_3 = 0.0$	<p>Scenario 3: Compensation bargaining effects</p> <p>$e_1 = 0.2$</p> <p>$e_2 = 0.0$</p> <p>$e_3 = 0.3$</p>
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Optimal top tax rate $\tau^* = (1 + ae_2 + ae_3)/(1 + ae)$										
Pareto coefficient $a =$		1.5								
Alternative tax rate $t =$		20%								
<p>Scenario 1</p> <p>$\tau^* = 57\%$</p>	<p>Scenario 2</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; text-align: left; padding: 5px;">(a) $e_2=0.3$</td> <td style="width: 50%; text-align: left; padding: 5px;">(b) $e_2=0.1$</td> </tr> <tr> <td style="padding: 5px;">$\tau^* = 62\%$</td> <td style="padding: 5px;">$\tau^* = 71\%$</td> </tr> </table>	(a) $e_2=0.3$	(b) $e_2=0.1$	$\tau^* = 62\%$	$\tau^* = 71\%$	<p>Scenario 3</p> <p>$\tau^* = 83\%$</p>				
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This table presents optimal top tax rates in the case where the overall elasticity of reported taxable income is $e=0.5$ in three scenarios depending on how this total elasticity breaks down into the standard labor supply elasticity (e_1), the tax avoidance elasticity (e_2), the compensation bargaining elasticity (e_3). In scenario 1, the only elasticity is e_1 . In scenario 2, both e_1 and e_2 are present, income shifted away from the regular tax is assumed to be taxed at rate $t=20\%$. Scenario 2a considers the case of the current narrow base with avoidance opportunities and scenario 2b considers the case where the base is first broadened so that e_2 falls to 0.1 (and hence e falls to 0.3). In scenario 3, both e_1 and e_3 are present. In all cases, top tax rates are set to maximize tax revenue raised from top bracket earners.

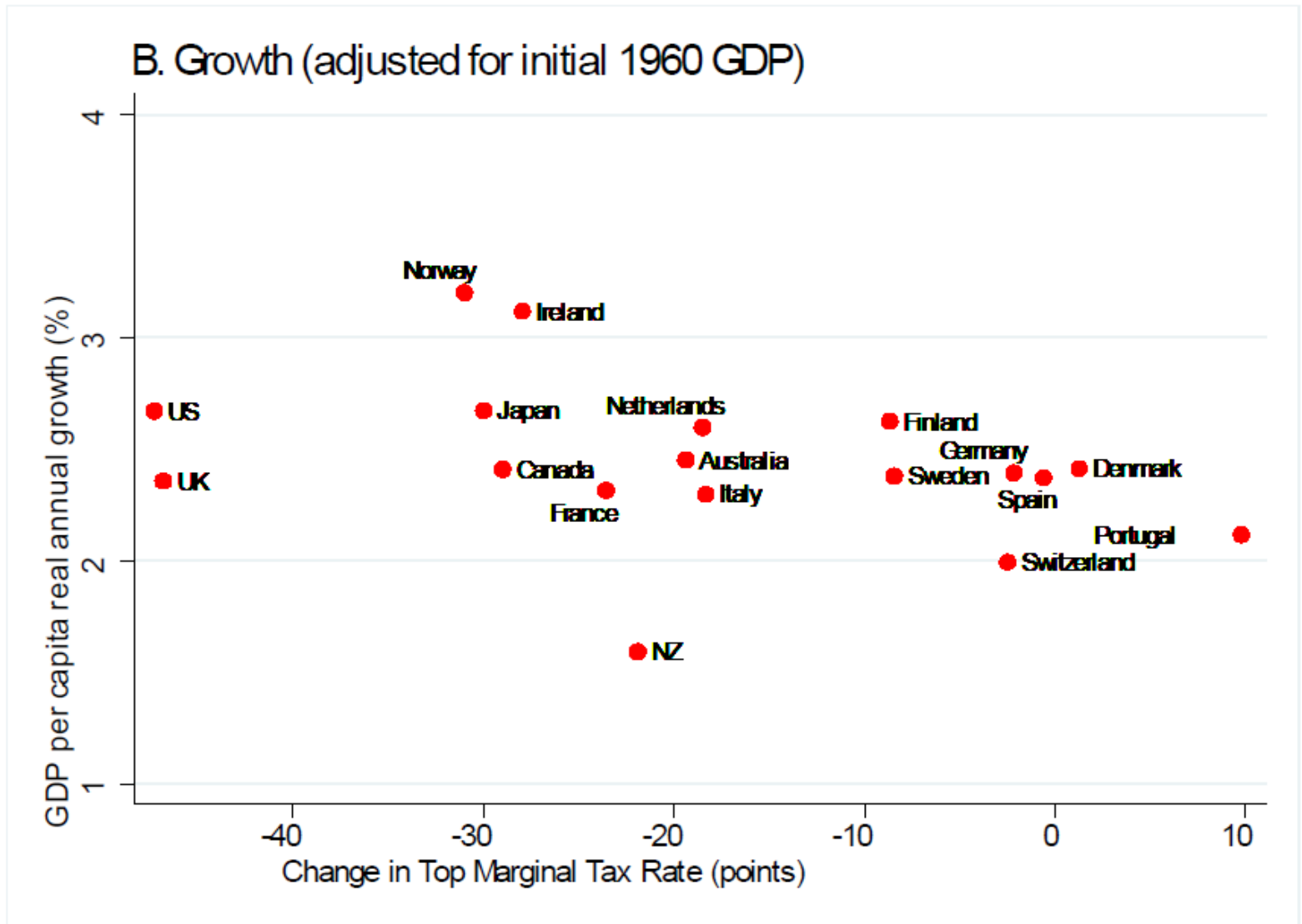
Top 1% share and top tax rates 1960-2009



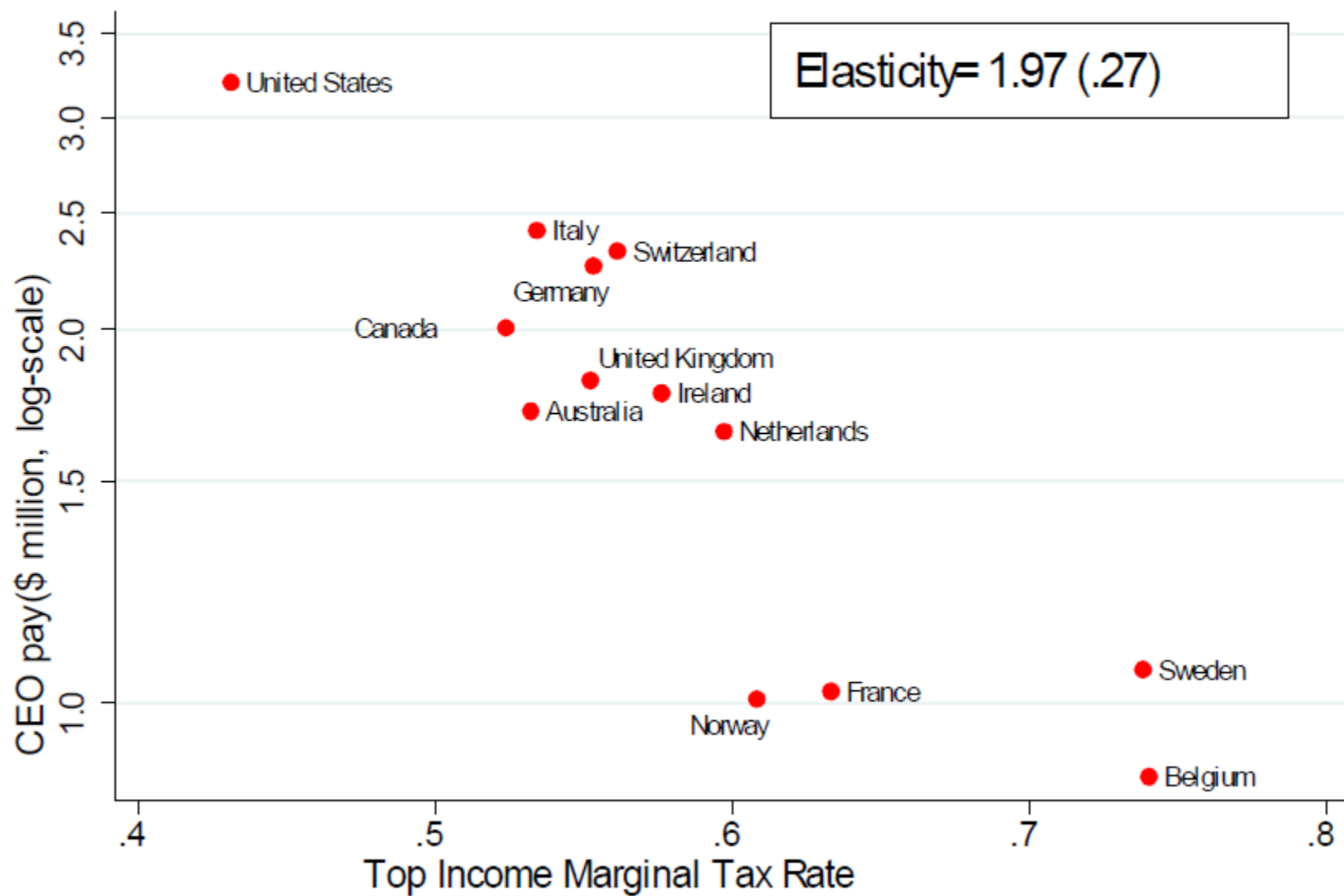
Top tax rates and average growth 1960-2009



Top tax rates and average growth 1960-2009



A Average CEO compensation



B. Average CEO compensation with controls

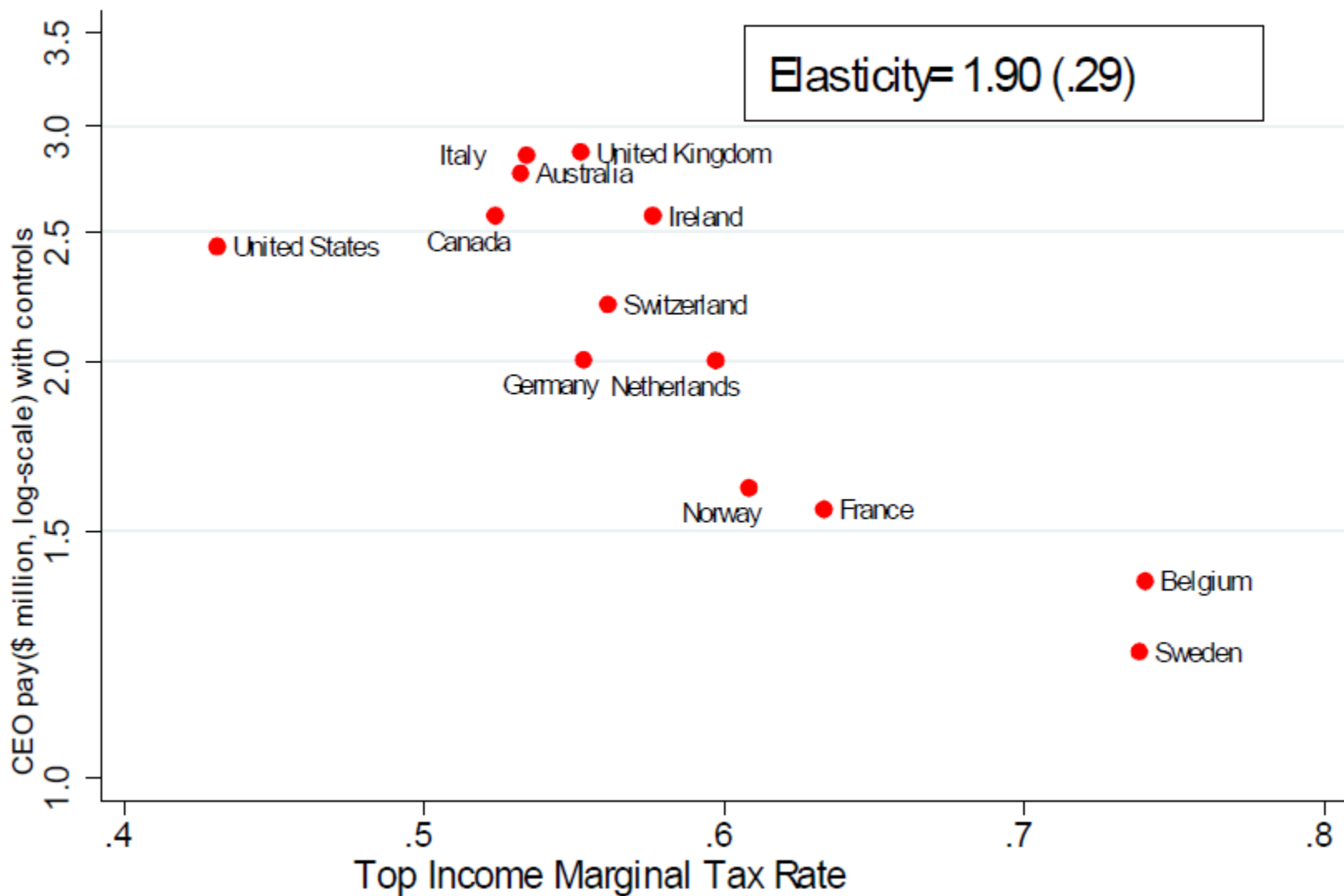


Table 4: International CEO Pay Evidence

Outcome (LHS variable)	Log(CEO pay)	Log(CEO pay)	Log(CEO pay)	Log(CEO pay)	Log(CEO salary)	Log(CEO bonus and equity pay)
	(1)	(2)	(3)	(4)	(5)	(6)
Explanatory variables (RHS variables)						
log(1-Top MTR)	1.97*** (0.27)	1.90*** (0.286)	1.92*** (0.336)	1.90*** (0.328)	0.35* (0.189)	4.68*** (0.782)
Governance index			-0.10*** (0.020)	-0.19*** (0.038)	-0.02 (0.072)	-0.26 (0.201)
log(1-Top MTR)*Governance index				-0.13** (0.057)	0.06 (0.089)	-0.03 (0.281)
Firm and CEO controls	no	yes	yes	yes	yes	yes
Number of observations	2,959	2,844	2,711	2,711	2,691	2,711

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Total elasticity $e = e_1 + e_2 + e_3 =$		0.5								
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