

Public Economics: Tax & Transfer Policies

(Master PPD & APE, Paris School of Economics)

Thomas Piketty

Academic year 2014-2015

Lecture 5: Optimal Labor Income Taxation

(February 3rd 2015)

(check [on line](#) for updated versions)

- **Main theoretical results about optimal taxation of labor income** (for capital income, see lectures 6-7):
- (1) the social optimum usually involves a U-shaped pattern of marginal tax rates = in order to have high minimum income, one needs to withdraw it relatively fast (but not too fast) = relatively consistent with observed patterns (if we take into account transfers)
- (2) optimal top rates depends positively on income concentration (top income shares) and negatively on labor supply elasticities
(and positively on bargaining power at the top: very important if we want to understand Roosevelt-type confiscatory tax rates)

- Here I will only present the main results and intuitions. For complete technical details and proofs, see the following papers:
- Mirrlees, J., "An exploration in the theory of optimum income taxation", RES 1971
- Diamond, P., "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Rates", AER 1998 [\[article in pdf format\]](#)
- Saez, "Using Elasticities to Derive Optimal Income Tax Rates", RES 2001 [\[article in pdf format\]](#)
- **Piketty-Saez, "Optimal Labor Income Taxation", 2013, [Handbook of Public Economics, vol. 5](#)**
- Piketty-Saez-Stantcheva, "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities", [AEJ 2013](#) (see also [Slides](#))

The optimal labor income tax problem

- Mirrlees (1971) : basic labor supply model used to analyze optimal labor income taxes
- Each agent i is characterized by an exogeneous wage rate w_i (=productivity)
- Labor supply l_i
- Pre-tax labor income $y_i = w_i l_i$
- Income tax $t = t(y_i)$
- $t(y_i)$ can be >0 or <0 : if <0 , then this is an income transfer, or negative income tax
- After-tax labor income $z_i = y_i - t(y_i)$
- Agents choose labor supply l_i by maximizing $U(z_i, l_i)$

- Social welfare function $W = \int W(U(z_i, l_i)) f(y_i) dy_i$ subject to budgetary constraint: $\int t(y_i) f(y_i) dy_i = 0$ (or $= G$, with $G =$ exogenous public spendings)
($f(y_i)$ = density function for y_i = partly endogenous, given exogenous distribution of productivities w_i and endogenous labor supply l_i)
- If individual productivities w_i were fully observable, then the first-best efficient tax system would be $t=t(w_i)$, i.e. would not depend at all on labor supply behaviour, so that there would be no distortion = lump-sum transfers, fully efficient redistribution
- However if the tax system can only depend on income, i.e. $t = t(y_i)$, e.g. because of unobservable productivities w_i (adverse selection), then we have an equity/efficiency trade-off
- >>> Mirrlees 1971 provides analytical solutions for the second-best efficient tax system in presence of such an adverse selection pb

- But problems with the Mirrlees 1971 formula:
- (i) very complicated and unintuitive formulas, hard to apply empirically
- (ii) only robust conclusion: with finite number of productivity types w_1, \dots, w_n , then zero marginal rate on the top group = completely off-the-mark
- >>> Diamond (1998), Saez (2001): continuous distribution of types (no upper bound, so that the artificial zero-top-rate result disappears), first-order derivation of the optimal tax formulas, very intuitive and easy-to-calibrate formulas

First-order derivation of linear optimal labor income tax formulas

- Linear tax schemes: $t(y) = ty - t_0$
- I.e. $t =$ constant marginal tax rate
- $t_0 > 0$ = transfer to individuals with zero labor income (RMI/RSA in France)
- Define $e =$ labor supply elasticity
- Definition: if the net-of-tax wage rate $(1-t)w_i$ increases by 1%, labor supply l_i (and therefore labor income $y_i = w_i l_i$, for given w_i) increases by $e\%$
- E.g. if $U(z_i, l_i) = z_i - V(l_i)$ (separable utility, no income effect), with $V(l) = l^{1+\mu}/(1+\mu)$, then $e = 1/\mu$
FO condition: $\text{Max } w_i l_i - V(l_i) \rightarrow l_i = w_i^{1/\mu}$
 $\rightarrow dl_i/l_i = e dw_i/w_i$ with $e = 1/\mu$

- More generally, whatever the labor income generating process $y_i = y(\text{wage rate } w_i, \text{ labor hours } l_i, \text{ effort } e_i, \text{ luck } u_i)$, one can always define **e = generalized labor supply elasticity** = elasticity of labor income with respect to the net-of-tax rate: if the net-of-tax rate $(1-t)$ increases by 1%, observed labor income y increases by $e\%$
- I.e. if $t \rightarrow t+dt$, then $1-t \rightarrow 1-t-dt$, so that $1-t$ declines by $dt/(1-t)\%$; therefore we have: $dy/y = -e dt/(1-t)$
- The generalized elasticity reflects changes in labor hours but also endogenous changes in wage rates: with higher taxes, maybe one will put less effort in education investment, or less effort in trying to get a promotion, etc.

- Assume that we're looking for the tax rate t^* maximizing tax revenues $R = ty$
- Revenue-maximizing tax rate t^* = top of the Laffer curve
- Revenue-maximizing tax rate t^* = social optimum if social welfare function W = Rawlsian (infinitely concave), i.e. in the limit case where the marginal social welfare $W' = 0$ for all $U > U_{\min}$, i.e. social objective = maximizing minimum utility (maxmin) = maximizing transfer t_0
- = useful reference point: by definition, socially optimal tax rates for non-Rawlsian social welfare functions W will be below revenue-maximizing tax levels

- First-order condition: if the tax rate goes from t to $t+dt$, then tax revenues go from R to $R+dR$, with:

$$dR = y dt + t dy$$

$$\text{with } dy/y = -e dt/(1-t)$$

- I.e. $dR = y dt - t e y dt/(1-t)$
- $dR = 0$ if and only if $t/(1-t) = 1/e$
- **I.e. $t^* = 1/(1+e)$**
- I.e. pure elasticity effect : if the elasticity e is higher, then the optimal tax t^* is lower
- I.e. if $e=1$ then $t^*=50\%$, if $e=0,1$ then $t^*=91\%$, etc.
- **= the basic principle of optimal taxation theory: other things equal, don't tax what's elastic**
- Other example: Ramsey formulas on optimal indirect taxation: tax more the commodities with a less elastic demand, and conversely

First-order derivation of non-linear optimal labor income tax formulas

- General non-linear tax schedule $t(y)$
- I.e. marginal tax rates $t'(y)$ can vary with y
- Note $f(y)$ the density function for labor income, and $F(y) = \int_{z < y} f(z) dz =$ distribution function (= fraction of pop with income $< y$)
- Assume one wants to increase the marginal tax rate from t' to $t' + dt'$ over some income bracket $[y; y + dy]$. Then tax revenues go from R to $R + dR$, with:
- $dR = (1 - F(y)) dt' dy - f(y) dy t' e y dt' / (1 - t')$
- $dR = 0$ if and only if $t'^* / (1 - t'^*) = (1 - F(y)) / y f(y) \quad 1/e$

- **Key formula: $t'^*/(1-t'^*) = (1-F(y))/yf(y) \cdot 1/e$**
- I.e. two effects:
- Elasticity effect: higher elasticities e imply lower marginal tax rates t'^*
- Distribution effect: higher $(1-F)/yf$ ratios imply higher marginal rates t'^*
- Intuition : $(1-F)/yf =$ ratio between the mass of people above y (=mass of people paying more tax) and the mass of people right at y (=mass of people hit by adverse incentives effects)
- For low y , the ratio $(1-F)/yf$ is necessarily declining: other things equal, marginal rates should fall
- But for high y , the ratio $(1-F)/yf$ is usually increasing: other things equal, marginal rates should rise
- >>> for constant elasticity profiles, U-shaped pattern of marginal tax rates

Asymptotic optimal marginal rates for top incomes

- With a Pareto distribution $1-F(y) = (k/y)^a$ and $f(y) = ak^a/y^{(1+a)}$, then $(1-F)/yf$ converges towards $1/a$, i.e. t'^* converges towards:
- **$t'^* = 1/(1+ae)$**
- with e = elasticity, a = Pareto coefficient
- Intuition: higher a (i.e. lower coefficient $b = a/(a-1)$, i.e. less fat upper tail = less income concentration) imply lower tax rates, and conversely
- Exemple: if $e=0,5$ and $a=2$, $t'^* = 50\%$
But if $e=0,1$ and $a=2$, $t'^* = 83\%$

- Reminder on key property of Pareto distributions:
ratio average/threshold = constant
- Note $y^*(y)$ the average income of the population above threshold y . Then $y^*(y)$ can be expressed as follows :

- $$y^*(y) = \left[\int_{z>y} z f(z) dz \right] / \left[\int_{z>y} f(z) dz \right]$$

$$= \left[\int_{z>y} dz/z^a \right] / \left[\int_{z>y} dz/z^{(1+a)} \right] = ay/(a-1)$$

- I.e. $y^*(y)/y = b = a/(a-1)$ (and $a = b/(b-1)$)

- In practice : b is usually around 2, but can vary quite a lot
For top incomes

- France 2010s, US 1970s: $b = 1.7-1.8$ ($a=2.2-2.3$)
- France or US 1910s, US 2010s: $b = 2.2-2.5$ ($a=1.7-1.8$)

For top wealth:

- France today: $b = 2.3-2.5$; France 1910s: $b=3-3.5$
- **Higher b coefficients = fatter upper-tail of the distribution = higher concentration of income (or wealth)**

Evidence on U-shaped pattern of marginal rates

- $t^*/(1-t^*) = (1-F(y))/yf(y) \approx 1/e$
- The distribution effect $(1-F(y))/yf(y)$ is typically U-shaped; so if elasticity effect $e=e(y) \approx$ stable over y , then the social optimum involves a U-shaped pattern of marginal tax rates
- Same conclusion with general SWF as long as marginal social welfare weights not too far from Rawlsian social welfare function
- Basic intuition = in order to have high minimum income, one needs to withdraw it relatively fast (but not too fast) = relatively consistent with observed patterns (if we take into account transfers)

- The increasing part of the U-shaped pattern (for upper half incomes) is due to income tax progressivity → rising marginal rates at the top
 - The decreasing part (for bottom half incomes) is due to the withdrawal of income transfers, which also creates high marginal rates
- Observed pattern of marginal rates in France:
U-shaped curve (see [RFE 97 graphs](#), [paper](#))

- Simplified example for France 2013 (see [here](#) for detailed simulations and computer codes for French transfers & taxes)
- If labor income $y=0$, then $t(y)=-t_0$: t_0 = transfer to individuals with zero labor income $\approx 500\text{€}/\text{month}$ for RMI/RSA in France
- If labor income $y=y_{\min}$ =full-time minimum wage, you receive no transfer any more (unless you have children);
net minimum wage $\approx 1100\text{€}/\text{m}$, gross min. wage $\approx 1400\text{€}/\text{m}$,
total labor cost $\approx 1700\text{€}/\text{m}$

(CSG+employee payroll tax $\approx 20\%$; employer payroll tax $\approx 20\%$)

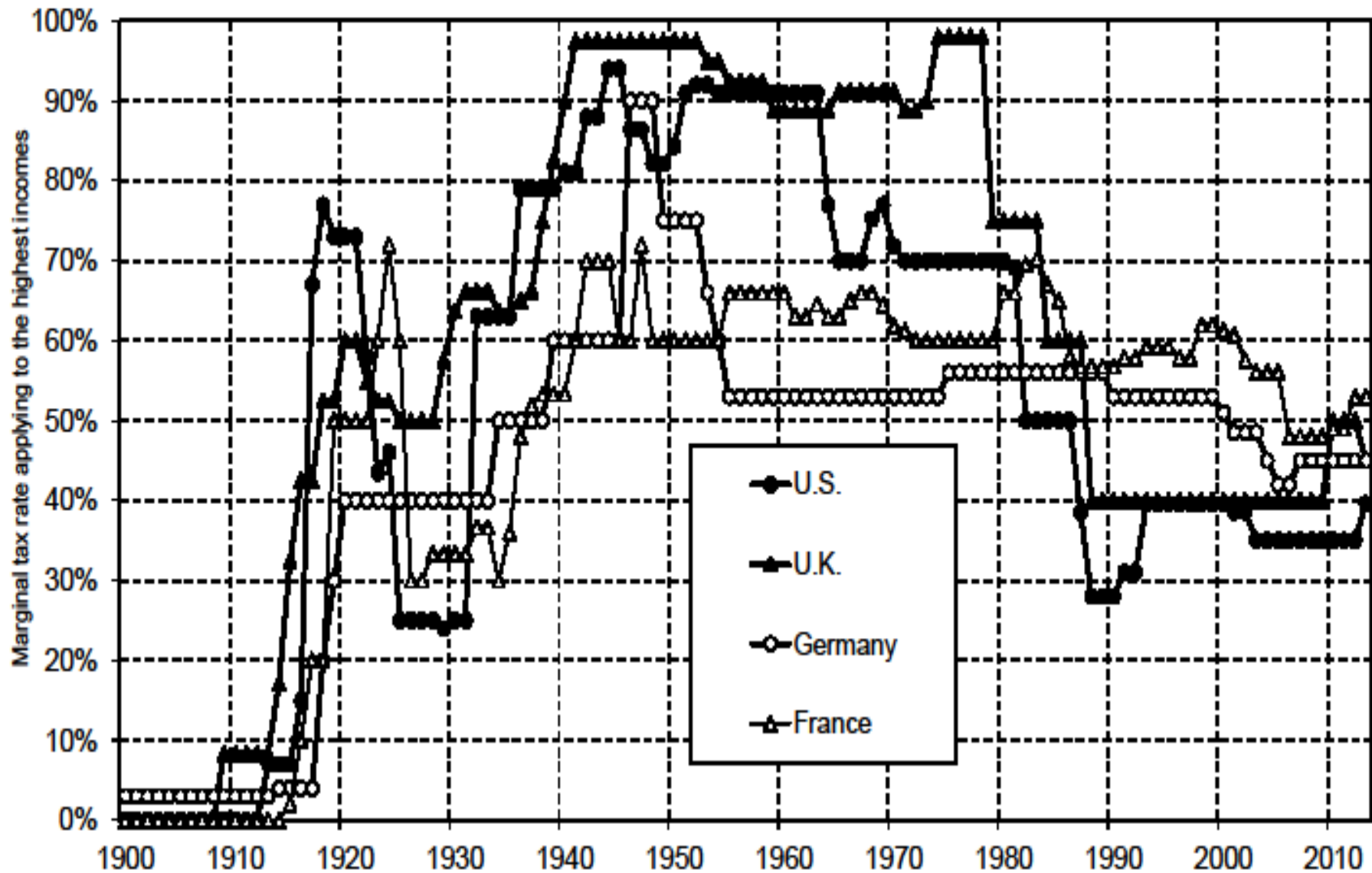
- Note: total labor cost would be $\approx 2000\text{€}/\text{m}$ at the level of the minimum wage in the absence of low-wage payroll tax cut:
employer payroll tax $\approx 20\%$ at y_{\min} \rightarrow back to $\approx 40\%$ at $1,6 \times y_{\min}$

- As pre-tax income y goes from $y=0$ to $y=1700\text{€}$, after-tax income $y-t(y)$ goes from 500 to 1100€, and $t(y)$ goes from -500 to +600€, i.e. rises by 1100€
 - marginal tax rate associated to the transition between pre-tax incomes 0 and y_{\min} = $\Delta t/\Delta y = 1100/1700 = 65\%$
(if we include VAT & other indirect taxes, the marginal tax rate on minimum wage workers would be closer to 75-80%)
- The pb is that if one wants to reduce this marginal rate (say, by further cuts in low-wage payroll tax), then one has to raise the marginal rate higher up in the distribution (say, btw y_{\min} and $1,6 \times y_{\min}$)
 - complex trade-off, current U-shaped pattern might be not too far from optimal

Evidence on top marginal rates

- Observed top marginal rates go from 20-30% to 80-90%
- One possible interpretation = different beliefs about elasticities of labor supply (see [this paper](#) for a learning model: it is difficult to estimate e with certainty)
- **$t'^* = 1/(1+ae)$** (with e = elasticity, a = Pareto coefficient)
- If $e=1$ and $a=2$, $t'^* = 33\%$
- If $e=0,5$ and $a=2$, $t'^* = 50\%$
- If $e=0,1$ and $a=2$, $t'^* = 83\%$

Figure 14.1. Top income tax rates, 1900-2013



The top marginal tax rate of the income tax (applying to the highest incomes) in the U.S. dropped from 70% in 1980 to 28% in 1988. Sources and series: see piketty.pse.ens.fr/capital21c.

- Empirical evidence: real labor supply elasticities $\approx 0,2-0,3$ at most (higher elasticities usually come from pure income shifting, i.e. if one can transfer income to a less taxed tax base: in principle, this can be solved by a broader tax base)
→ $t'^* \approx 60-70\%$?
- See P. Diamond & E. Saez, "The Case for a Progressive Tax: From Basic Research to Policy Recommendations", [JEP 2011](#)
- For a survey on empirical estimates of labor supply elasticities, see E. Saez, J. Slemrod and S. Gierz, "The Elasticity of Taxable Income with Respect to Marginal Tax Rates: A Critical Review", NBER 2009 [\[article in pdf format\]](#)

- However the perfect-competition model (labor income = marginal product) may not be sufficient to analyze Roosevelt-type tax rates & the recent surge in US top incomes
- A model with imperfect competition and CEO bargaining power (CEOs can sometime extract some than their marginal product) is more promising
- See Piketty-Saez-Stantcheva, "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities", [AEJ 2014](#) (see also [Slides](#))

- With imperfect competition and bargaining power, the optimal tax formula becomes more complicated and can justify confiscatory tax rates
 - **Augmented formula: $\tau = (1+tae_2+ae_3)/(1+ae)$**
 - With $e = e_1 + e_2 + e_3$
 = labor supply elasticity e_1 + income shifting elasticity e_2 + bargaining elasticity e_3 (= more intensive bargaining with lower tax rate)
 - **Key point: $\tau \uparrow$ as elasticity $e_3 \uparrow$**
- for a given total elasticity e , the decomposition between the three elasticities e_1, e_2, e_3 is critical

Table 5: Synthesis of Various Scenarios

Total elasticity $e = e_1 + e_2 + e_3 =$		0.5								
<p>Scenario 1: Standard supply side tax effects</p> <p>$e_1 = 0.5$</p> <p>$e_2 = 0.0$</p> <p>$e_3 = 0.0$</p>	<p>Scenario 2: Tax avoidance effects</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; text-align: left; padding: 5px;">(a) current narrow tax base</td> <td style="width: 50%; text-align: left; padding: 5px;">(b) after base broadening</td> </tr> <tr> <td style="padding: 5px;">$e_1 = 0.2$</td> <td style="padding: 5px;">$e_1 = 0.2$</td> </tr> <tr> <td style="padding: 5px;">$e_2 = 0.3$</td> <td style="padding: 5px;">$e_2 = 0.1$</td> </tr> <tr> <td style="padding: 5px;">$e_3 = 0.0$</td> <td style="padding: 5px;">$e_3 = 0.0$</td> </tr> </table>	(a) current narrow tax base	(b) after base broadening	$e_1 = 0.2$	$e_1 = 0.2$	$e_2 = 0.3$	$e_2 = 0.1$	$e_3 = 0.0$	$e_3 = 0.0$	<p>Scenario 3: Compensation bargaining effects</p> <p>$e_1 = 0.2$</p> <p>$e_2 = 0.0$</p> <p>$e_3 = 0.3$</p>
(a) current narrow tax base	(b) after base broadening									
$e_1 = 0.2$	$e_1 = 0.2$									
$e_2 = 0.3$	$e_2 = 0.1$									
$e_3 = 0.0$	$e_3 = 0.0$									
Optimal top tax rate $\tau^* = (1 + ae_2 + ae_3)/(1 + ae)$										
Pareto coefficient $a =$		1.5								
Alternative tax rate $t =$		20%								
<p>Scenario 1</p> <p>$\tau^* = 57\%$</p>	<p>Scenario 2</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; padding: 5px;">(a) $e_2=0.3$</td> <td style="width: 50%; padding: 5px;">(b) $e_2=0.1$</td> </tr> <tr> <td style="padding: 5px;">$\tau^* = 62\%$</td> <td style="padding: 5px;">$\tau^* = 71\%$</td> </tr> </table>	(a) $e_2=0.3$	(b) $e_2=0.1$	$\tau^* = 62\%$	$\tau^* = 71\%$	<p>Scenario 3</p> <p>$\tau^* = 83\%$</p>				
(a) $e_2=0.3$	(b) $e_2=0.1$									
$\tau^* = 62\%$	$\tau^* = 71\%$									

This table presents optimal top tax rates in the case where the overall elasticity of reported taxable income is $e=0.5$ in three scenarios depending on how this total elasticity breaks down into the standard labor supply elasticity (e_1), the tax avoidance elasticity (e_2), the compensation bargaining elasticity (e_3). In scenario 1, the only elasticity is e_1 . In scenario 2, both e_1 and e_2 are present, income shifted away from the regular tax is assumed to be taxed at rate $t=20\%$. Scenario 2a considers the case of the current narrow base with avoidance opportunities and scenario 2b considers the case where the base is first broadened so that e_2 falls to 0.1 (and hence e falls to 0.3). In scenario 3, both e_1 and e_3 are present. In all cases, top tax rates are set to maximize tax revenue raised from top bracket earners.