

# **Public Economics: Tax & Transfer Policies**

*(Master PPD & APE, Paris School of Economics)*

Thomas Piketty

Academic year 2014-2015

## **Lecture 5: Optimal Labor Income Taxation**

(February 3<sup>rd</sup> 2015)

*(check [on line](#) for updated versions)*

- **Main theoretical results about optimal taxation of labor income** (for capital income, see lectures 6-7):
- (1) the social optimum usually involves a U-shaped pattern of marginal tax rates = in order to have high minimum income, one needs to withdraw it relatively fast (but not too fast) = relatively consistent with observed patterns (if we take into account transfers)
- (2) optimal top rates depends positively on income concentration (top income shares) and negatively on labor supply elasticities  
(and positively on bargaining power at the top: very important if we want to understand Roosevelt-type confiscatory tax rates)

- Here I will only present the main results and intuitions. For complete technical details and proofs, see the following papers:
- Mirrlees, J., "An exploration in the theory of optimum income taxation", RES 1971
- Diamond, P., "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Rates", AER 1998 [\[article in pdf format\]](#)
- Saez, "Using Elasticities to Derive Optimal Income Tax Rates", RES 2001 [\[article in pdf format\]](#)
- **Piketty-Saez, "Optimal Labor Income Taxation", 2013, [Handbook of Public Economics, vol. 5](#)**
- Piketty-Saez-Stantcheva, "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities", [AEJ 2013](#) (see also [Slides](#))

# The optimal labor income tax problem

- Mirrlees (1971) : basic labor supply model used to analyze optimal labor income taxes
- Each agent  $i$  is characterized by an exogenous wage rate  $w_i$  (=productivity)
- Labor supply  $l_i$
- Pre-tax labor income  $y_i = w_i l_i$
- Income tax  $t = t(y_i)$
- $t(y_i)$  can be  $>0$  or  $<0$  : if  $<0$ , then this is an income transfer, or negative income tax
- After-tax labor income  $z_i = y_i - t(y_i)$
- Agents choose labor supply  $l_i$  by maximizing  $U(z_i, l_i)$

- Social welfare function  $W = \int W(U(z_i, l_i)) f(y_i) dy_i$  subject to budgetary constraint:  $\int t(y_i) f(y_i) dy_i = 0$  (or  $= G$ , with  $G =$  exogenous public spendings)  
(  $f(y_i)$  = density function for  $y_i$  = partly endogenous, given exogenous distribution of productivities  $w_i$  and endogenous labor supply  $l_i$ )
- If individual productivities  $w_i$  were fully observable, then the first-best efficient tax system would be  $t=t(w_i)$ , i.e. would not depend at all on labor supply behaviour, so that there would be no distortion = lump-sum transfers, fully efficient redistribution
- However if the tax system can only depend on income, i.e.  $t = t(y_i)$ , e.g. because of unobservable productivities  $w_i$  (adverse selection), then we have an equity/efficiency trade-off
- >>> Mirrlees 1971 provides analytical solutions for the second-best efficient tax system in presence of such an adverse selection pb

- But problems with the Mirrlees 1971 formula:
- (i) very complicated and unintuitive formulas, hard to apply empirically
- (ii) only robust conclusion: with finite number of productivity types  $w_1, \dots, w_n$ , then zero marginal rate on the top group = completely off-the-mark
- >>> Diamond (1998), Saez (2001): continuous distribution of types (no upper bound, so that the artificial zero-top-rate result disappears), first-order derivation of the optimal tax formulas, very intuitive and easy-to-calibrate formulas

# First-order derivation of linear optimal labor income tax formulas

- Linear tax schemes:  $t(y) = ty - t_0$
- I.e.  $t =$  constant marginal tax rate
- $t_0 > 0$  = transfer to individuals with zero labor income (RMI/RSA in France)
- Define  $e =$  labor supply elasticity
- Definition: if the net-of-tax wage rate  $(1-t)w_i$  increases by 1%, labor supply  $l_i$  (and therefore labor income  $y_i = w_i l_i$ , for given  $w_i$ ) increases by  $e\%$
- E.g. if  $U(z_i, l_i) = z_i - V(l_i)$  (separable utility, no income effect), with  $V(l) = l^{1+\mu}/(1+\mu)$ , then  $e = 1/\mu$   
FO condition:  $\text{Max } w_i l_i - V(l_i) \rightarrow l_i = w_i^{1/\mu}$   
 $\rightarrow dl_i/l_i = e dw_i/w_i$  with  $e = 1/\mu$

- More generally, whatever the labor income generating process  $y_i = y(\text{wage rate } w_i, \text{ labor hours } l_i, \text{ effort } e_i, \text{ luck } u_i)$ , one can always define **e = generalized labor supply elasticity** = elasticity of labor income with respect to the net-of-tax rate: if the net-of-tax rate  $(1-t)$  increases by 1%, observed labor income  $y$  increases by  $e\%$
- I.e. if  $t \rightarrow t+dt$ , then  $1-t \rightarrow 1-t-dt$ , so that  $1-t$  declines by  $dt/(1-t)\%$ ; therefore we have:  $dy/y = -e dt/(1-t)$
- The generalized elasticity reflects changes in labor hours but also endogenous changes in wage rates: with higher taxes, maybe one will put less effort in education investment, or less effort in trying to get a promotion, etc.

- Assume that we're looking for the tax rate  $t^*$  maximizing tax revenues  $R = ty$
- Revenue-maximizing tax rate  $t^*$  = top of the Laffer curve
- Revenue-maximizing tax rate  $t^*$  = social optimum if social welfare function  $W$  = Rawlsian (infinitely concave), i.e. in the limit case where the marginal social welfare  $W' = 0$  for all  $U > U_{\min}$ , i.e. social objective = maximizing minimum utility (maxmin) = maximizing transfer  $t_0$
- = useful reference point: by definition, socially optimal tax rates for non-Rawlsian social welfare functions  $W$  will be below revenue-maximizing tax levels

- First-order condition: if the tax rate goes from  $t$  to  $t+dt$ , then tax revenues go from  $R$  to  $R+dR$ , with:

$$dR = y dt + t dy$$

$$\text{with } dy/y = -e dt/(1-t)$$

- I.e.  $dR = y dt - t e y dt/(1-t)$
- $dR = 0$  if and only if  $t/(1-t) = 1/e$
- **I.e.  $t^* = 1/(1+e)$**
- I.e. pure elasticity effect : if the elasticity  $e$  is higher, then the optimal tax  $t^*$  is lower
- I.e. if  $e=1$  then  $t^*=50\%$ , if  $e=0,1$  then  $t^*=91\%$ , etc.
- **= the basic principle of optimal taxation theory: other things equal, don't tax what's elastic**
- Other example: Ramsey formulas on optimal indirect taxation: tax more the commodities with a less elastic demand, and conversely

# First-order derivation of non-linear optimal labor income tax formulas

- General non-linear tax schedule  $t(y)$
- I.e. marginal tax rates  $t'(y)$  can vary with  $y$
- Note  $f(y)$  the density function for labor income, and  $F(y) = \int_{z < y} f(z) dz$  = distribution function ( = fraction of pop with income  $< y$  )
- Assume one wants to increase the marginal tax rate from  $t'$  to  $t' + dt'$  over some income bracket  $[y; y + dy]$ . Then tax revenues go from  $R$  to  $R + dR$ , with:
- $dR = (1 - F(y)) dt' dy - f(y) dy t' e y dt' / (1 - t')$
- $dR = 0$  if and only if  $t'^* / (1 - t'^*) = (1 - F(y)) / y f(y) \quad 1/e$

- **Key formula:  $t'^*/(1-t'^*) = (1-F(y))/yf(y) \cdot 1/e$**
- I.e. two effects:
- Elasticity effect: higher elasticities  $e$  imply lower marginal tax rates  $t'^*$
- Distribution effect: higher  $(1-F)/yf$  ratios imply higher marginal rates  $t'^*$
- Intuition :  $(1-F)/yf =$  ratio between the mass of people above  $y$  (=mass of people paying more tax) and the mass of people right at  $y$  (=mass of people hit by adverse incentives effects)
- For low  $y$ , the ratio  $(1-F)/yf$  is necessarily declining: other things equal, marginal rates should fall
- But for high  $y$ , the ratio  $(1-F)/yf$  is usually increasing: other things equal, marginal rates should rise
- >>> for constant elasticity profiles, U-shaped pattern of marginal tax rates

# Asymptotic optimal marginal rates for top incomes

- With a Pareto distribution  $1-F(y) = (k/y)^a$  and  $f(y) = ak^a/y^{(1+a)}$ , then  $(1-F)/yf$  converges towards  $1/a$ , i.e.  $t'^*$  converges towards:
- **$t'^* = 1/(1+ae)$**
- with  $e$  = elasticity,  $a$  = Pareto coefficient
- Intuition: higher  $a$  (i.e. lower coefficient  $b = a/(a-1)$ , i.e. less fat upper tail = less income concentration) imply lower tax rates, and conversely
- Exemple: if  $e=0,5$  and  $a=2$ ,  $t'^* = 50\%$   
But if  $e=0,1$  and  $a=2$ ,  $t'^* = 83\%$

- Reminder on key property of Pareto distributions:  
**ratio average/threshold = constant**
- Note  $y^*(y)$  the average income of the population above threshold  $y$ . Then  $y^*(y)$  can be expressed as follows :

- $$y^*(y) = \left[ \int_{z>y} z f(z) dz \right] / \left[ \int_{z>y} f(z) dz \right]$$

$$= \left[ \int_{z>y} dz/z^a \right] / \left[ \int_{z>y} dz/z^{(1+a)} \right] = ay/(a-1)$$

- I.e.  $y^*(y)/y = b = a/(a-1)$  (and  $a = b/(b-1)$  )

- In practice :  $b$  is usually around 2, but can vary quite a lot  
For top incomes

- France 2010s, US 1970s:  $b = 1.7-1.8$  ( $a=2.2-2.3$ )
- France or US 1910s, US 2010s:  $b = 2.2-2.5$  ( $a=1.7-1.8$ )

For top wealth:

- France today:  $b = 2.3-2.5$ ; France 1910s:  $b=3-3.5$
- **Higher  $b$  coefficients = fatter upper-tail of the distribution = higher concentration of income (or wealth)**

# Evidence on U-shaped pattern of marginal rates

- $t^*/(1-t^*) = (1-F(y))/yf(y) \approx 1/e$
- The distribution effect  $(1-F(y))/yf(y)$  is typically U-shaped; so if elasticity effect  $e=e(y) \approx$  stable over  $y$ , then the social optimum involves a U-shaped pattern of marginal tax rates
- Same conclusion with general SWF as long as marginal social welfare weights not too far from Rawlsian social welfare function
- Basic intuition = in order to have high minimum income, one needs to withdraw it relatively fast (but not too fast) = relatively consistent with observed patterns (if we take into account transfers)

- The increasing part of the U-shaped pattern (for upper half incomes) is due to income tax progressivity → rising marginal rates at the top
  - The decreasing part (for bottom half incomes) is due to the withdrawal of income transfers, which also creates high marginal rates
- Observed pattern of marginal rates in France:  
U-shaped curve (see [RFE 97 graphs](#), [paper](#))

- Simplified example for France 2013 (see [here](#) for detailed simulations and computer codes for French transfers & taxes)
- If labor income  $y=0$ , then  $t(y)=-t_0$  :  $t_0$ = transfer to individuals with zero labor income  $\approx 500\text{€}/\text{month}$  for RMI/RSA in France
- If labor income  $y=y_{\min}$ =full-time minimum wage, you receive no transfer any more (unless you have children);  
net minimum wage  $\approx 1100\text{€}/\text{m}$ , gross min. wage  $\approx 1400\text{€}/\text{m}$ ,  
total labor cost  $\approx 1700\text{€}/\text{m}$

(CSG+employee payroll tax  $\approx 20\%$ ; employer payroll tax  $\approx 20\%$ )

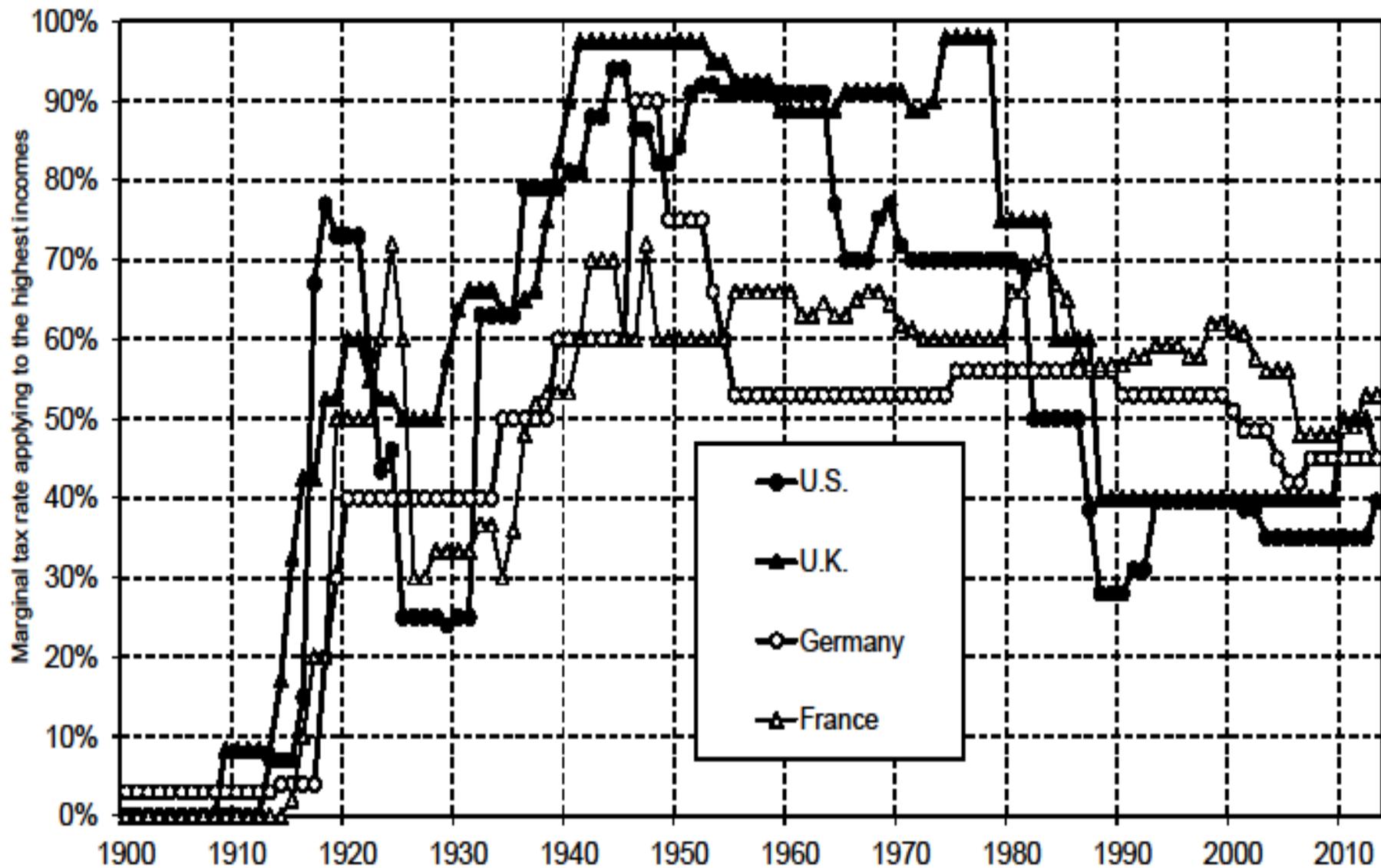
- Note: total labor cost would be  $\approx 2000\text{€}/\text{m}$  at the level of the minimum wage in the absence of low-wage payroll tax cut:  
employer payroll tax  $\approx 20\%$  at  $y_{\min}$   $\rightarrow$  back to  $\approx 40\%$  at  $1,6 \times y_{\min}$

- As pre-tax income  $y$  goes from  $y=0$  to  $y=1700\text{€}$ , after-tax income  $y-t(y)$  goes from 500 to 1100€, and  $t(y)$  goes from -500 to +600€, i.e. rises by 1100€
  - marginal tax rate associated to the transition between pre-tax incomes 0 and  $y_{\min}$  =  $\Delta t/\Delta y = 1100/1700 = 65\%$   
(if we include VAT & other indirect taxes, the marginal tax rate on minimum wage workers would be closer to 75-80%)
- The pb is that if one wants to reduce this marginal rate (say, by further cuts in low-wage payroll tax), then one has to raise the marginal rate higher up in the distribution (say, btw  $y_{\min}$  and  $1,6 \times y_{\min}$ )
  - complex trade-off, current U-shaped pattern might be not too far from optimal

# Evidence on top marginal rates

- Observed top marginal rates go from 20-30% to 80-90%
- One possible interpretation = different beliefs about elasticities of labor supply (see [this paper](#) for a learning model: it is difficult to estimate  $e$  with certainty)
- **$t'^* = 1/(1+ae)$**  (with  $e$  = elasticity,  $a$  = Pareto coefficient)
- If  $e=1$  and  $a=2$ ,  $t'^* = 33\%$
- If  $e=0,5$  and  $a=2$ ,  $t'^* = 50\%$
- If  $e=0,1$  and  $a=2$ ,  $t'^* = 83\%$

Figure 14.1. Top income tax rates, 1900-2013



The top marginal tax rate of the income tax (applying to the highest incomes) in the U.S. dropped from 70% in 1980 to 28% in 1988. Sources and series: see [piketty.pse.ens.fr/capital21c](http://piketty.pse.ens.fr/capital21c).

- Empirical evidence: real labor supply elasticities  $\approx 0,2-0,3$  at most (higher elasticities usually come from pure income shifting, i.e. if one can transfer income to a less taxed tax base: in principle, this can be solved by a broader tax base)  
→  $t'^* \approx 60-70\%$  ?
- See P. Diamond & E. Saez, "The Case for a Progressive Tax: From Basic Research to Policy Recommendations", [JEP 2011](#)
- For a survey on empirical estimates of labor supply elasticities, see E. Saez, J. Slemrod and S. Gierz, "The Elasticity of Taxable Income with Respect to Marginal Tax Rates: A Critical Review", NBER 2009 [\[article in pdf format\]](#)

- However the perfect-competition model (labor income = marginal product) may not be sufficient to analyze Roosevelt-type tax rates & the recent surge in US top incomes
- A model with imperfect competition and CEO bargaining power (CEOs can sometime extract some than their marginal product) is more promising
- See Piketty-Saez-Stantcheva, "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities", [AEJ 2014](#) (see also [Slides](#))

- With imperfect competition and bargaining power, the optimal tax formula becomes more complicated and can justify confiscatory tax rates
  - **Augmented formula:  $\tau = (1+tae_2+ae_3)/(1+ae)$**
  - With  $e = e_1 + e_2 + e_3$   
 = labor supply elasticity  $e_1$  + income shifting elasticity  $e_2$  + bargaining elasticity  $e_3$  (= more intensive bargaining with lower tax rate)
  - **Key point:  $\tau \uparrow$  as elasticity  $e_3 \uparrow$**
- for a given total elasticity  $e$ , the decomposition between the three elasticities  $e_1, e_2, e_3$  is critical

**Table 5: Synthesis of Various Scenarios**

Total elasticity $e = e_1 + e_2 + e_3 =$		0.5								
<p><b>Scenario 1: Standard supply side tax effects</b></p> <p><math>e_1 = 0.5</math></p> <p><math>e_2 = 0.0</math></p> <p><math>e_3 = 0.0</math></p>	<p><b>Scenario 2: Tax avoidance effects</b></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; text-align: left; padding-right: 10px;">(a) current narrow tax base</td> <td style="width: 50%; text-align: left;">(b) after base broadening</td> </tr> <tr> <td><math>e_1 = 0.2</math></td> <td><math>e_1 = 0.2</math></td> </tr> <tr> <td><math>e_2 = 0.3</math></td> <td><math>e_2 = 0.1</math></td> </tr> <tr> <td><math>e_3 = 0.0</math></td> <td><math>e_3 = 0.0</math></td> </tr> </table>	(a) current narrow tax base	(b) after base broadening	$e_1 = 0.2$	$e_1 = 0.2$	$e_2 = 0.3$	$e_2 = 0.1$	$e_3 = 0.0$	$e_3 = 0.0$	<p><b>Scenario 3: Compensation bargaining effects</b></p> <p><math>e_1 = 0.2</math></p> <p><math>e_2 = 0.0</math></p> <p><math>e_3 = 0.3</math></p>
(a) current narrow tax base	(b) after base broadening									
$e_1 = 0.2$	$e_1 = 0.2$									
$e_2 = 0.3$	$e_2 = 0.1$									
$e_3 = 0.0$	$e_3 = 0.0$									
Optimal top tax rate $\tau^* = (1 + ae_2 + ae_3)/(1 + ae)$										
Pareto coefficient $a =$		1.5								
Alternative tax rate $t =$		20%								
<p><b>Scenario 1</b></p> <p><math>\tau^* = 57\%</math></p>	<p><b>Scenario 2</b></p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; text-align: left;">(a) <math>e_2=0.3</math></td> <td style="width: 50%; text-align: left;">(b) <math>e_2=0.1</math></td> </tr> <tr> <td><math>\tau^* = 62\%</math></td> <td><math>\tau^* = 71\%</math></td> </tr> </table>	(a) $e_2=0.3$	(b) $e_2=0.1$	$\tau^* = 62\%$	$\tau^* = 71\%$	<p><b>Scenario 3</b></p> <p><math>\tau^* = 83\%</math></p>				
(a) $e_2=0.3$	(b) $e_2=0.1$									
$\tau^* = 62\%$	$\tau^* = 71\%$									

This table presents optimal top tax rates in the case where the overall elasticity of reported taxable income is  $e=0.5$  in three scenarios depending on how this total elasticity breaks down into the standard labor supply elasticity ( $e_1$ ), the tax avoidance elasticity ( $e_2$ ), the compensation bargaining elasticity ( $e_3$ ). In scenario 1, the only elasticity is  $e_1$ . In scenario 2, both  $e_1$  and  $e_2$  are present, income shifted away from the regular tax is assumed to be taxed at rate  $t=20\%$ . Scenario 2a considers the case of the current narrow base with avoidance opportunities and scenario 2b considers the case where the base is first broadened so that  $e_2$  falls to 0.1 (and hence  $e$  falls to 0.3). In scenario 3, both  $e_1$  and  $e_3$  are present. In all cases, top tax rates are set to maximize tax revenue raised from top bracket earners.