Economics of Inequality
(Master PPD & APE, Paris School of Economics)
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Lecture 4: From capital/income ratios to capital shares
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(check online for updated versions)
Capital-income ratios $\beta$ vs. capital shares $\alpha$

- Capital/income ratio $\beta = K/Y$
- Capital share $\alpha = Y_K/Y$
  
  with $Y_K =$ capital income (=sum of rent, dividends, interest, profits, etc.: i.e. all incomes going to the owners of capital, independently of any labor input)

- I.e. $\beta =$ ratio between capital stock and income flow
- While $\alpha =$ share of capital income in total income flow

- By definition: $\alpha = r \times \beta$
  
  With $r = Y_K/K =$ average real rate of return to capital

- If $\beta=600\%$ and $r=5\%$, then $\alpha = 30\% =$ typical values
• In practice, the average rate of return to capital \( r \) (typically \( r \approx 4-5\% \)) varies a lot across assets and over individuals (more on this in Lecture 6)
• Typically, rental return on housing = 3-4\% (i.e. the rental value of an apartment worth 100 000€ is generally about 3000-4000€/year) (+ capital gain or loss)
• Return on stock market (dividend + k gain) = as much as 6-7\% in the long run
• Return on bank accounts or cash = as little as 1-2\% (but only a small fraction of total wealth)
• Average return across all assets and individuals \( \approx 4-5\% \)
The Cobb-Douglas production function

- Cobb-Douglas production function: \( Y = F(K, L) = K^\alpha L^{1-\alpha} \)
- With perfect competition, wage rate \( v = \) marginal product of labor, rate of return \( r = \) marginal product of capital:
  \[
  r = F_K = \alpha K^{\alpha-1} L^{1-\alpha} \quad \text{and} \quad v = F_L = (1-\alpha) K^{\alpha} L^{-\alpha}
  \]
- Therefore capital income \( Y_K = r K = \alpha Y \)
  & labor income \( Y_L = v L = (1-\alpha) Y \)
- I.e. capital & labor shares are entirely set by technology (say, \( \alpha=30\% \), \( 1-\alpha=70\% \)) and do not depend on quantities \( K, L \)
- Intuition: Cobb-Douglas \( \leftrightarrow \) elasticity of substitution between \( K \) & \( L \) is exactly equal to 1
- I.e. if \( v/r \) rises by 1\%, \( K/L=\alpha/(1-\alpha) \) \( v/r \) also rises by 1\%. So the quantity response exactly offsets the change in prices: if wages \( \uparrow \) by 1\%, then firms use 1\% less labor, so that labor share in total output remains the same as before
The limits of Cobb-Douglas

• Economists like Cobb-Douglas production function, because stable capital shares are approximately stable
• However it is only an approximation: in practice, capital shares $\alpha$ vary in the 20-40% range over time and between countries (or even sometime in the 10-50% range)
• In 19c, capital shares were closer to 40%; in 20c, they were closer to 20-30%; structural rise of human capital (i.e. exponent $\alpha \downarrow$ in Cobb-Douglas production function $Y = K^\alpha L^{1-\alpha}$), or purely temporary phenomenon?
• Over 1970-2010 period, capital shares have increased from 15-25% to 25-30% in rich countries: very difficult to explain with Cobb-Douglas framework
Figure 6.1. The capital-labor split in the United Kingdom, 1770-2010

During the 19th century, capital income (rent, profits, dividends, interest...) absorbed about 40% of national income, vs. 60% for labor income (salaried and non-salaried). Sources and series: see piketty.pse.ens.fr/capital21c.
In the 21st century, capital income (rent, profits, dividends, interest,...) absorbs about 30% of national income, vs. 70% for labor income (salaried and non salaried). Sources and series: see piketty.pse.ens.fr/capital21c.
Figure 6.5. The capital share in rich countries, 1975-2010

Capital income absorbs between 15% and 25% of national income in rich countries in 1970, and between 25% and 30% in 2000-2010. Sources and series: see piketty.pse.ens.fr/capital21c
The CES production function

• CES = a simple way to think about changing capital shares
• CES :  \( Y = F(K,L) = [a \frac{K^{(\sigma-1)/\sigma}}{\sigma} + b \frac{L^{(\sigma-1)/\sigma}}{\sigma}]^{\sigma/(\sigma-1)} \)  
  with \( a, b = \text{constant} \)

\( \sigma = \text{constant elasticity of substitution between K and L} \)

• \( \sigma \rightarrow \infty \): linear production function \( Y = r K + v L \)  
  (infinite substitution: machines can replace workers and vice versa, so that the returns to capital and labor do not fall at all when the quantity of capital or labor rise) (= robot economy)

• \( \sigma \rightarrow 0 \): \( F(K,L)=\min(rK,vL) \) (fixed coefficients) = no substitution possibility: one needs exactly one machine per worker

• \( \sigma \rightarrow 1 \): converges toward Cobb-Douglas; but all intermediate cases are also possible: Cobb-Douglas is just one possibility among many

• Compute the first derivative \( r = F_K \) : the marginal product to capital is given by  
  \[ r = F_K = a \beta^{-1/\sigma} \]  
  (with \( \beta = K/Y \))  
  I.e. \( r \downarrow \) as \( \beta \uparrow \) (more capital makes capital less useful), but the important point is that the speed at which \( r \downarrow \) depends on \( \sigma \)
• With \( r = F_K = a \beta^{-1/\sigma} \), the capital share \( \alpha \) is given by:

\[
\alpha = r \beta = a \beta^{(\sigma-1)/\sigma}
\]

• I.e. \( \alpha \) is an increasing function of \( \beta \) if and only if \( \sigma > 1 \) (and stable iff \( \sigma = 1 \))

• The important point is that with large changes in the volume of capital \( \beta \), small departures from \( \sigma = 1 \) are enough to explain large changes in \( \alpha \)

• If \( \sigma = 1.5 \), capital share rises from \( \alpha = 28\% \) to \( \alpha = 36\% \) when \( \beta \) rises from \( \beta = 250\% \) to \( \beta = 500\% \)

  = more or less what happened since the 1970s

• In case \( \beta \) reaches \( \beta = 800\% \), \( \alpha \) would reach \( \alpha = 42\% \)

• In case \( \sigma = 1.8 \), \( \alpha \) would be as large as \( \alpha = 53\% \)
Figure 6.5. The capital share in rich countries, 1975-2010

Capital income absorbs between 15% and 25% of national income in rich countries in 1970, and between 25% and 30% in 2000-2010. Sources and series: see piketty.pse.ens.fr/capital21c
Figure 5.3. Private capital in rich countries, 1970-2010

Private capital is worth between 2 and 3.5 years of national income in rich countries in 1970, and between 4 and 7 years of national income in 2010. Sources and series: see piketty.pse.ens.fr/capital21c.
Figure 14: Average return on private wealth 1975-2010
Measurement problems with capital shares

- In many ways, $\beta$ is easier to measure than $\alpha$.
- In principle, capital income = all income flows going to capital owners (independantly of any labor input); labor income = all income flows going to labor earners (independantly of any capital input).
- But in practice, the line is often hard to draw: family firms, self-employed workers, informal financial intermediation costs (=the time spent to manage one’s own portfolio).
- If one measures the capital share $\alpha$ from national accounts (rent+dividend+interest+profits) and compute average return $r=\alpha/\beta$, then the implied $r$ often looks very high for a pure return to capital ownership: it probably includes a non-negligible entrepreneurial labor component, particularly in reconstruction periods with low $\beta$ and high $r$; the pure return might be 20-30% smaller (see estimates).
- Maybe one should use two-sector models $Y=Y_h+Y_b$ (housing + business); return to housing = closer to pure return to capital.
Figure 6.1. The capital-labor split in the United Kingdom, 1770-2010

During the 19th century, capital income (rent, profits, dividends, interest...) absorbed about 40% of national income, vs. 60% for labor income (salaried and non-salaried). Sources and series: see piketty.pse.ens.fr/capital21c.
Figure 6.2. The capital-labor split in France, 1820-2010

In the 21st century, capital income (rent, profits, dividends, interest, ...) absorbs about 30% of national income, vs. 70% for labor income (salaried and non-salaried). Sources and series: see piketty.pse.ens.fr/capital21c.
Figure 6.3. The pure return to capital in the United Kingdom, 1770-2010

The pure rate of return to capital is roughly stable around 4%-5% in the long run.

Sources and series: see piketty.pse.ens.fr/capital21c.
Figure 6.4. The pure rate of return to capital in France, 1820-2010

The observed average rate of return displays larger fluctuations than the pure rate of return during the 20th century.
Sources and series: see piketty.pse.ens.fr/capital21c.
Figure 6.6. The profit share in the value added of corporations in France, 1900-2010

The share of gross profits in gross value added of corporations rose from 25% in 1982 to 33% in 2010; the share of net profits in net value added rose from 12% to 20%. Sources and series: see piketty.pse.ens.fr/capital21c
Figure 6.7. The share of housing rent in national income in France, 1900-2010

The share of housing rent (rental value of dwellings) rose from 2% of national income in 1948 to 10% in 2010.
Sources and series: see piketty.pse.ens.fr/capital21c.
Figure 6.8. The capital share in national income in France, 1900-2010

The share of capital income (net profits and rents) rose from 15% of national income in 1982 to 27% in 2010.

Sources and series: see piketty.pse.ens.fr/capital21c.
Recent work on capital shares


Summing up

• The rate of return to capital $r$ is determined mostly by technology: $r = F_K = \text{marginal product to capital, elasticity of substitution } \sigma$

• The quantity of capital $\beta$ is determined by saving attitudes and by growth ($=\text{fertility + innovation}$): $\beta = s/g$

• The capital share is determined by the product of the two: $\alpha = r \times \beta$

• Anything can happen
• Note: the return to capital \( r = F_K \) is determined not only by technology but also by psychology, i.e. saving attitudes \( s = s(r) \) might vary with the rate of return.

• In models with wealth or bequest in the utility function \( U(c_t, w_{t+1}) \), there is zero saving elasticity with \( U(c, w) = c^{1-s} w^s \), but with more general functional forms one can get any elasticity.

• In pure lifecycle model, the saving rate \( s \) is primarily determined by demographic structure (more time in retirement \( \rightarrow \) higher \( s \)), but it can also vary with the rate of return, in particular if the rate of return becomes very low (say, below 2%) or very high (say, above 6%).
• In the dynastic utility model, the rate of return is entirely set by the rate of time preference (=psychological parameter) and the growth rate:

\[ \text{Max } \sum U(c_t)/(1+\delta)^t, \text{ with } U(c)=c^{1-1/\xi}/(1-1/\xi) \]

→ unique long rate rate of return \( r_t \rightarrow r = \delta + \xi g > g \)

(\( \xi > 1 \) and transversality condition)

This holds both in the representative agent version of model and in the heterogeneous agent version (with insurable shocks); more on this in Lecture 6