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# Information Aggregation Through Voting and Vote-Trading

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**Abstract: The price system is widely regarded as an efficient way to use decentralized information, and some authors have argued that a market for vote would make voting more efficient by allowing intensities of preferences to be expressed. We develop a model of vote trading equilibrium to show that when individual preferences are correlated to some privately observed signals the existence of a market for votes can be socially harmful (thereby justifying secret ballots). Although the market is an efficient mechanism to aggregate private information about individual welfare, voting is an efficient mechanism to aggregate private information about collective welfare, and using both mechanisms at the same time when both types of decentralized information are to be aggregated can result into the destruction of the second type of information. This provides some justification for secret ballots and public subsidies to political campaigns.**

# Section 1 : Introduction.

**The market mechanism has been viewed for a long time as an efficient way to aggregate decentralized information[1]: in the Arrow-Debreu model, individual agents only need to know their own preferences and to observe the equilibrium price system, and the resulting allocation cannot be (Pareto-)improved even if a single agent knew everybody's preferences at the same time. The voting mechanism constitutes another commonly used mechanism to allocate ressources[2], whose efficiency properties are usually questionned by economists and other social theorists. Unsurprisingly, several authors[3] have thought of using the market mechanism to mitigate the presumed inefficiency of the voting mechanism: the basic argument is that a market for votes would allow the intensity of preferences to be expressed; that is, a majority may prefer x to y although everybody could be better off if y was implemented and the proponents of y could compensate the proponents of x by buying their votes. However all modern democraties have secret ballots, implying that vote trading contracts are in effect impossible to enforce, and many reasonnable people hold strong views against the idea of a market for votes[4]. This paper attempts to explain why vote trading can indeed be socially**

**[1]The view that the problem of economic organization consists essentially of finding efficient mechanisms to use decentralized information, and in particular that the price system provides us with such a mechanism, dates back at least to Hayek(1945).**

**[2]E.g.,**

**[3]Mostly in the public choice tradition, starting with Buchanan and Tullock(1962).**

**[4]Voting processes within parliaments often don't entail secret ballots, presumably because this is the only way voters can monitor their representative. In effect, this typically leads to massive vote-trading (although lobbying does not usually take the fom of direct vote-trading contracts, the effects are similar; see section 4 for an application to such "quasi-vote-trading"**

**harmful in a world of decentralized information, and by doing so gives some new insights regarding the informational conditions under which voting and the market constitute efficient mechanisms.**

**To understand our basic argument, it may be useful to recall that of Condorcet's Jury Theorem[5]: assume all individuals (say, in a jury) share common values (if the state of nature s=0 they all prefer x to y, if s=1 they all prefer y to x), that they all receive a signal about s, and that the true signal is delivered with probability q>1/2; then nobody wants to be the dictator and everybody wants to delegate the decision-making power to the majority (so as to benefit from information pooling).**

**Now consider the more realistic situation where there is at the same time some decentralized information of the kind described by Condorcet (some voters receive some signals which are of positive value to other voters as well) and some decentralized information about the individual intensity of preferences (different voters have different intensity of preferences). Then the usual benefits of decentralized vote trading (in terms of using efficientely private information about individual preferences by allowing high-valuation agents to vote more by compensating low-valuation agents) can be overwhelmed by the destruction of socially useful signals: typically, some agents can receive a socially-useful signal but still be so uncertain that selling seems privately profitable while some other agents are sufficiently certain about their own preferences that they are ready to buy; in that case a transaction will take place although all uncertain voters (and society as a whole) may be better off if everybody was forced to vote. The key deficiency of the price system is its imperfect informativeness: the same equilibrium**

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**processes).**

**[5]Surprisingly enough, this transparent result does not seem to be as well known as Condorcet's majority cycles story, although both are extremely complementary: voting is an efficient mechanism to aggregate information in a common-values world, but results quickly into troubles (majority cycles) if it is used to aggregate conflicting interests.**

**price for votes prevails for completely different distribution of signals, so that uncertain voters don't get to learn what they should be fighting for. Therefore it may be socially desirable to forbid vote trading altogether.**

**There exists very few analysis of the efficiency properties of vote trading in decentralized equilibrium. Most of the literature deals with non-equilibirum behavior, pointing out various kinds of equilibrium non-existence, in general for majority-cycles and related reasons or because it is assumed that the distri bution of individual preferences is known with certainty (so that the probabilities of being decisive are certain, which implies that there will always be an excess supply of votes at any positive price and an excess demand at price 0; see Philipson and Snyder(1992) and subsequent references).**

**The basic point that a market for votes cannot operate like a normal market has been made several times, but it is important to realize that the fact a vote-trading contract always involves external effects on othe r agents does not imply per se that allowing vote trading is socially harmful; what this implies is only that the equilibirum will not in general first-best efficient[6]; we explain in section 3 why vote trading, by transferring voting rights to high-valuation agents and compensating low-valuation agents, is very likely to generate positive (though not maximal) Pareto-improvements as long as individual valuations are certain. One needs to invoke uncertain preferences and privately-observed, socially-useful signals to make a robust case against vote trading: even if market power could be regulated[7] there would still exist good reasons for keeping the market out of politics.**

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**[6]See in particular the free-riding argument of Grossman and Hart(1980) and the subsequent literature.**

**[7]For example by supporting the formation of large coalitions of voters with small individual stakes.**

**Our point is also related to Hirshman(1971)'s argument about the limited compatibility between "voice" and "exit": Hirshman argues that the introduction of "exit" (the competitive mechanism) into a situation where "voice" was the primary regulatory mechanism can be socially harmful since it may induce agents who can "voice" most effectively to exit. If one interprets "voice" as a direct information-aggregation mechanism through communication (of which voting is a simple example), then we also suggest that the private regulation of individual weights within a "voice club" can be socially harmful[8].**

**The rest of the paper is organized as follows: section 2 defines a competitive, rational-expectations equilibrium concept for vote trading; section 3 applies it to show why competitive vote trading can result into substantial destruction of socially useful information; section 4 applies these same ideas to the financing of political campaigns; section 5 gives concluding comments.**

## Section 2 : A Model of Equilibrium Vote Trading.

**A finite set of agents I=(1;...;n) has to choose between two possible collective decisions x and y [9]. The preferences of agent i are represented by**

$$U_i = U_i(x,s) + t_i \text{ if x is implemented}$$

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**[8]The main difference with Hirshman's argument, apart from the formal modelling we offer, is that in our model there are prices for votes that could in principle undo this ineficiency by transmitting information (this strenghthens the argument).**

**[9]**

$$U_i = U_i(y,s) + t_i \text{ if } y \text{ is implemented}$$

where $s \in S$ is the state of nature and $t$ is some monetary transfer[10]. We note $d_s=U_i(x,s)-U_i(y,s)$ the utility differential between x and y. For notational simplicity we assume that each agent can be of finitely many different types $d^1,...,d_H$; if i is of type h, it means that $d_s=d^h(s)$. It is common knowledge that types are independentely distributed according to some law $(\pi_h)_{1 \leq h \leq H}$. In addition, each agent i knows his own type $h_i$ and receives a signal $s_i$. It is common knowledge that signals are independentely distributed according to some conditional distribution $\pi(s'/s)$ (the probability of receiving signal s' when the true state of nature is s) and agents have a common prior $\pi(s)$ over states of nature. Say for example that S=(0,1), $\pi(0)=\pi(1)=1/2$ and $\pi(0/0)=\pi(1/1)=q>1/2$.

Thus for each realization of uncertainty $w=(h_i,s_i)_{1 \leq i \leq n} \in W$ involves two different types of private information: private information $h_i$ about individual values and private information $s_i$ about collective (multi-individual) values. We note $\pi(w)$ the ex-ante probability that w occurs, and $\pi(w/s_i)$ the conditional probabilities.

If vote trading is forbidden (say, by imposing vote secrety), then x(w)=x iff a majority prefers x to y, i.e. iff

$$\text{size}(i \text{ s.t. } qd^{hi}(s_i)+(1-q)d^{hi}(1-s_i)>0\ ) > n/2$$

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[10] If utility for the private consumption good (money) exhibits decreasing returns and agents are intially endowed with different consumption levels, then poor (resp. rich) agents would tend to sell (resp. buy) votes more than they should, and the effect of vote trading on the quality of decision-making would be lowered instantaneously (redistribution would be involved, but there's no reason to believe that this is minimizing the incentive costs of redistribution over all possible policy tools); although this may be important in practice, we abstract from these issues in order to concentrate on other, deeper reasons for vote-trading inefficiency.

**and conversely for x(w)=y (ties are broken by equal-probability randomization).**

**In general x(w) is not efficient, i.e. one can find some x'(w) which everybody prefers ex ante. The question is whether vote trading is going to result in something more efficient. We must first define what a competitive vote trading equilibrium is. The timing is as follows: first some state (w,s) is realized, then vote trading takes place, then an election takes place where each agent votes as many times as his trading allows him, and eventually the decision obtaining the majority of the votes is implemented.**

**Consider a state w and assume that each agent knows that w is in some subset A of W (in addition to his private information). If an agent i anticipates that agents of type h and signal s will buy D(h,s) votes (with D=-1,0,1,...), then i can compute the probability $\pi_i(z)$ that x wins by z votes (z=...-1,0,1,...) in case he didn't vote, that is, the probability of being decisive by buying D votes; we note $D(h_i,s_i;p;(D(h,s);A)$ this demand function. Then p(w,A) is an equilibrium price if and only there exists some demand functions D(p,h,s) for each type h and signal s such as:**

**(1) $\forall$ h',s' D(h',s';p(w,A);D(h,s);A) = D(h',s')**

**(2) $\Sigma_i$ $D(h_i,s_i) = 0$**

**One can prove the existence of such an equilibrium price p(w,A) for each w,A by using Kakutani's fixed point theorem (first convexify the discrete demands for votes by considering lotteries over possible demand levels D=-1,0,1,..; then the demand correspondance $D(h_i,s_i;p;(D(h,s);A)$ is upper hemi-continuous; the only thing that needs to be taken care of is the way an agent deals with the possible indifferences of other agents; to guarantee existence**

**for finite n one must leave maximum freedom here, in the sense that individual equilibrium demands must be best-replies to some of the best-replies of other agents, which may not coï ncide with the way the Walrasian auctioneer solves individual indifferences; these technical difficulties disappear as n goes to infinity).**

**We must know define what a rational-expectations equilibrium[11] is in this setting. By definition, a r.e. equilibrium is an equilibrium price p(w) for each possible w such that:**

**" w, p(w)=p(w,A) where A=(w' s.t. p(w')=p(w))**

**That is, a r.e. equilibrium is defined as a partition ($A_r$) of W describing exactly the information about w that agents can infer by looking at equilibrium price; i.e. such that:**

**" w,w' Î $A_r$, p(w,$A_r$)=p(w',$A_r$)=p($A_r$)**
**" r,r'  p($A_r$) is different from p($A_{r'}$)**

**It turns out that proving the existence of such a partition ($A_r$) from the existence of an equilibrium price mapping p(w,A) is not easy if one does not impose further restrictions; for exemple it could be that the equilibrium prices associated to different w are very different when agents have no information about w (apart from their private information h,$s_i$), but that they are the same once they have more information, in which case rational inference from prices admits no equilibrium.**

**In any case, it is important to understand that there is no reason in general to expect the**

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**[11] In the sense of Grossman(1981).**

**existence of a fully-revealing equilibrium (i.e. an equilibrium such that the associated partition is infinitely precise): in a market for votes, the dimension of the publicly-observable signal (namely, the uni-dimensional price for votes) is naturally lower than the dimension of the relevant uncertainty, so that there is no hope for a general invertibility result; that is, individual agents care about the entire distribution of types and signals, and there's no way the price for votes can transmit all that information (for notational simplicity we assumed finite sets of types and signals, so that W is finite and a fully-revealing equilibrium is logically possible; however, (1) this logical possibility would disappear once these sets are infinite, (2) we will see next section that the basic dimensionality argument against full revealation holds with a finite number of types, (3) a fully-revealing equilibrium would look very strange and would not be particularly interesting: if the distribution w of preferences is known with certainty, then the probabilities of being decisive are certain, and there exists no pure-strategy equilibrium: there is always excess supply when the price is positive and excess demand when the price is 0; only a very unappealing mixed-strategy equilibrium exists).**

## Section 3 : Decentralized Information and Trading Inefficiency.

**We now see why vote trading can destroy socially useful information instead of aggregating it efficientely once there is private information about social values.**

**Assume there are certain and uncertain voters. Certain voters can be of two types h=1,2; voters of type 1 always prefer x to y ($d^1(s)=+b$ " s), and voters of type 2 always prefer y to x ($d^2(s)=-b$ " s). Uncertain voters prefer x when s=0 and y when s=1 ($d^3(0)=+c$ and $d^3(1)=-c$). Certain voters represent (on average) a fraction a of the population, of which 1/2 is of type 1**

**and 1/2 is of type 2, and uncertain voters represent a fraction 1-a (that is, ð(h=1)=ð(h=2)=a/2, ð(h=3)=1-a).**

**As n goes to infinity, the efficient decision is x if s=0 and y if s=1. If vote trading is forbidden, the efficient decision will be implemented with probability 1 as n goes to infinity: as long as q>1/2, the fraction of agents aq reveiving the right signal is larger than the fraction a(1-q) receiving the wrong signal, and the two types of certain voters cancel out.**

**If vote trading is allowed however, only certain voters will vote in equilibrium, so that the socially-useful information about s will be lost. Ex ante, everybody would prefer to commit not to trade votes.**

**Proposition : If q is sufficientely close to 1/2, there exists a r.e. equilibrium such that all uncertain voters sell their vote and all certain voters are indifferent between buying Int(n-m/m) and Int(n-m/m)+1 votes, where m=m(w) is the number of certain voters.**

**Proof (sketch) : To prove that this can be an equilibrium, consider some w and note m=m(w) the number of certain voters. Assume m is the unique m' such that r=Int(n-m'/m')=Int(n-m/m) (what follows can be easily extented to the case where there exists several such m', in which case the equilibrium price does not reveal completely the number of certain voters; one would never have full revealation of m if n was also uncertain). Assume D(h=1,2;s)=r with some proba f and r+1 with some proba 1-f and that D(h=3;s)=-1. This results into decisiveness probabilities ð(z), which are the same for everybody since signals do not influence individual tradings. Let p(m) be the price such that the optimal trade of certain voters is r or r+1 (indifference) (one can show that such a price exists for some f by computing the proba ð(z), as long as m is not too small). Whatever this price may be, if q is sufficientely close to 1/2**

**uncertain voters want to sell their vote: the expected benefit of reversing the decision is arbitrarily small, since the equilibrium price does not reveal anything about others' signals. It follows that there exists a r.e. equilibrium where the price reveals only the number of certain voters (at most), and not the proportion of type 1 and type 2 voters or the distribution signals. CQFD**

**The key reason behind this inefficiency of vote trading is that buyers and sellers do not internalize the social value of private information about collective values. In order to distinguish this externality from the straightforward externality implied by vote trading in general, it may be useful to look at the effects of vote trading in a world where all voters are certain.**

**Assume all types are certain, in the sense that " h $d^h(s)$ does not depend on s. Then one can easily show that in any vote trading equilibrium there exists d<0<d' such that voters of type $d^h$<d or >d' are buyers and voters of type $d^h$ between d and d' are sellers (this simply comes from the fact that in equilibrium all voters face the same decisiveness probabilities ð(z), and that for a given probability of being decisive it is privately more valuable to vote if one values more intensively the decision). In other words, in a world with certain preferences vote trading will transfer the rights to vote to those citizens who value the particular decision at stake the most intensively, and the other voters are compensated for that. From an ex-ante viewpoint this is socially beneficial if the distribution of types ($d^h$,ð(h)) is such that the probability that the decision preferred by the majority of voters with absolute valuation higher than d be the efficient decision increases with d, which is the case of many natural distributions (one can also find some distributions such that this is not true).**

**In sum, although vote trading per se always involves an externality (when I buy a vote, with**

**positive proba I have an effect on everybody's welfare), this is not sufficient to conclude that vote trading is socially harmful: as long as individual preferences are certain, vote trading is likely to generate a Pareto-improvement (at least from an ex ante viewpoint).**

**Note also that the coexistence of certain and uncertain voters is responsible for messing up everything. That is, assume a=0, i.e. that we are in a world of common values and no conflicting interests. Then even if vote trading was allowed, nobody would like to trade: every voter realizes that a voter with another view simply reflects another signal and doesn't want to destroy it by buying his vote. Thus the reason why vote trading is socially harmful is because the world is somewhere in between a world of common, uncertain values (Condorcet's Jury Theorem) and a world of certain, conflicting individual interests.**

**Another point is worth discussing. One may think that the inefficiency of vote trading would vanish if some polls could reveal the distribution of signals to the voters prior to the voting stage. This is wrong in general: polls would simply add a cheap-talk stage to the process, and various voters have an incentive to manipulate the information that gets revealed through cheap talk. For example, certain voters have nothing to learn from this cheap talk, and they don't want uncertain voters to be able to coordinate; most likely, there will several cheap-talk equilibria, most of which are not informative. Polls and other pre-voting communication devices may however improve matters substantially; this is left for further research.**

# Section 4 : Application to the Financing of Political Campaigns.

**Even if direct vote-trading is made illegal (for exemple through secret ballots), the market is**

**not entirely kept out of politics: different interests spend different ammount of money to convey their views and convince others during political campaigns. Whatever the model of political contributions, the logic we developed above for vote-trading will apply: agents with more certain interests will spend more money, not allowing agents with uncertain interests to aggregate their information through undistorted voting.**

**(...)**

**This may explain why in many countries private contributions to political parties are severely limited and campaign spendings are partly financed out of public money. Again, a pure "market-power" argument would do quite do the job: even if small agents are well represented by coalitions, there are still good reasons to limit the role of private money in politics. The reason why most people view Washington lobbying as harmful is not primarily because they believe some well-informed interests are less well represented that others, but because they believe that the "politics of special interests" kills democratic debate. This is exactly what our theory is describing: well-informed interests compete to attracte votes, thereby destroying the benefits of democratic voting for more uncertain voters.**

## Section 5 : Concluding Comments.

**The main conclusion that we draw from this paper is that voting and the market are efficient mechanisms to aggregate decentralized information in different contexts, and that trying to use both at the same time may be socially harmful. Whenever private information about collective welfare is to be aggregated, the market can result into substantial destruction of socially-useful information; and conversely the voting mechanism is ill-suited to aggregate**

**private information about private welfare.**

**Note that nothing precludes us from interpreting the possible decisions x and y to be taken as the identity of different candidates for some decision-making function, and private information as information about the quality of various candidates' information to take decisions. Different distributions of information would naturally lead to different socially-optimal degrees of direct and indirect democratic voting.**

**This raises the question of constitutional choice: what mechanism should be used in order to decide whether the voting mechanism and/or the market mechanism should be used? Since we have assumed that everybody had the same information about the future distributions of private information, in our model everybody agrees ex ante on whether vote trading should be allowed, and therefore there is no aggregation issue. In a more general situation where this information differs across individuals, we would be in the case of private information about collective welfare. Therefore it seems that democratic voting and not the market should be allowed to determine the boundaries between voting and the market.**