

A Theory of Optimal Capital Taxation

Thomas Piketty, Paris School of Economics

Emmanuel Saez, UC Berkeley

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ROUGH DRAFT - TO BE COMPLETED - NOT FOR CIRCULATION

Abstract: This paper develops a realistic, tractable normative theory of socially-optimal capital taxation.

1. Introduction

Economists have a problem with capital taxation. According to the profession's most popular theoretical models, socially optimal tax rates on capital should be equal to $\tau_K=0\%$ in the long run - including from the viewpoint of those agents and dynasties who own no capital at all. This is a very strong result indeed, especially since it applies to all forms of capital taxation, either on the capital stock or on the capital income flow, and either at the time of wealth transmission or during one's lifetime. That is, if we were to take this result seriously, then we should be pushing for a 0% inheritance tax rate, a 0% property tax rate, a 0% corporate profit tax rate, a 0% interest and dividend income tax rate, etc., and transfer the resulting revenue loss onto increased tax rates on labor income and/or consumption. Very few economists however seem to endorse such a radical policy agenda. Presumably this reflects a lack of faith in the profession's most popular models. This is in our view one of the most important failures of modern economics.

The objective of this paper is to develop a realistic, tractable normative theory of socially optimal capital taxation. By realistic, we mean a theory providing normative conclusions that are not fully off-the-mark with respect to the real world (i.e. positive and significant capital tax rates). By realistic, we also mean a theory offering such conclusions for reasons that are consistent with the reasons which - we feel - are at play in the real world. By tractable, we mean that one can put numbers into optimal tax formulas and quantify the various trade-offs in a simple and plausible way.

In our view, the two key ingredients for a proper theory of capital taxation are, first, the importance of inheritance, and, next, the imperfection of capital markets. With zero inheritance (i.e. if lifecycle savings account for 100% of aggregate wealth accumulation), and with perfect capital markets (i.e. if agents can costlessly and risklessly transfer resources across periods at a fixed interest rate $1+r$), then we believe that the case for zero capital taxation would indeed be very strong.

This paper makes two main contributions. First, we develop a theory of optimal inheritance taxation. Novelty: two dimensional inequality in the long run (top labor earners and top successors are never exactly the same people), implying a non-

degenerate trade-off between labor taxation and inheritance taxation. Other novelties: proper definition of welfare; finite long run elasticity.

Next, we show that if we introduce capital market imperfections, we can turn our positive optimal inheritance tax results into more general optimal capital tax results. The point is that the ratio (capitalized bequest)/bequest is very large, volatile, unpredictable, and to a large extent uninsurable. Therefore one may want to transfer some of optimal inheritance tax burden on capital taxes paid during one's lifetime >>> the key point is that if the optimal inheritance tax involves 10% of national income in inheritance tax revenues, then one can very well collect 1% in inheritance tax strictly speaking and 9% in lifetime capital taxes (through corporate income tax or interest dividend rent tax or property tax etc.). See Table 1: stylized facts on capital and inheritance taxation EU & US; our theory can rationalize these stylized facts.
(also show stylized facts on top inheritance tax rates: or later on)

2. Relation to existing literature

The main differences with the existing literature are the following. There are two main results in the literature in support of zero capital income taxation.

First, Chamley-Judd show that the optimal capital income tax should be zero in the long-run. This zero long-run result happens for two reasons. First, and as originally emphasized by Judd, the zero rate results happens because social welfare is measured exclusively from the initial period (or dynasty). In that context, a constant tax rate on capital income creates a tax distortion growing exponentially overtime which cannot be optimal. In the context of inheritance taxation where each period is a generation, this strikes us as a particularly bad definition of social welfare, and we will adopt instead a definition of social welfare based on long-run equilibrium steady-state utility. Second, even adopting a long-run steady-state utility perspective, the optimal capital income tax rate is still zero in the standard Chamley-Judd model [see Piketty, 2001]. This is because the supply side elasticity of capital with respect to the net-of-tax return is infinite in the infinite horizon model with constant discount rate. Our theory leaves this key elasticity as a free parameter to be estimated empirically.

Second, Atkinson-Stiglitz show that there should be no need to supplement the optimal non-linear labor income tax with a capital income tax in a life-cycle model. In that model, the only source of life-time income inequality is labor skill and hence there is no reason to redistribute from high savers to low savers (ie tax capital income) conditional on labor earnings. This key assumption of the Atkinson-Stiglitz model breaks down in a model with inheritances where inequality in life-time income comes from both differences in labor income and differences in inheritances received. In that context and conditional on labor earnings, a high level of bequests left is a signal of a high level of inheritances received, which provides a rationale for taxing bequests.¹

¹ Farhi-Werning propose a related analysis but consider a model from the perspective of the first generation of donors who do not start with any inheritance. In this context, bequests should actually be subsidized as they should be untaxed by Aktinson-Stiglitz (ignoring inheritors) and hence should be subsidized when taking into account inheritors. As we shall see, this result is not robust to models where people both receive and leave bequests.

Other theories of positive capital taxation.

Models based purely upon K market imperfections: there are papers showing that K market imperfections are sufficient to get $\tau_K > 0$, even in the absence of inheritance. Typically, it is good to tax capital as a way to redistribute from those with no credit constraints (the owners of capital) toward those with credit constraints (non-owners of capital). Chamley (2001) makes this point formally. There are also numerical calibrations, see e.g. Conesa, Kitao and Krueger (2009), who calibrate an optimal tax, OLG model with uninsurable idiosyncratic labor productivity shocks and borrowing constraints, and find $\tau_K = 36\%$ and $\tau_L = 23\%$. The main effect seems to be that capital tax is an indirect way to tax more the old and to tax less the young, so as to alleviate their borrowing constraints. This is an interesting and potentially important effect, but we do not believe this is the most important explanation for $\tau_K > 0$. If the old vs young issue was the main issue, we would find other, more direct ways to address it (e.g. age-varying income taxes; some policies do depend on age).

Models based upon time-inconsistency: zero capital tax results are long run results, but do not hold in the short run. I.e. since it takes a long time before adverse effects on capital accumulation show up, it's tempting for short-sighted governments to have $\tau_K > 0$ so as to reduce τ_L , even though optimal long run τ_K is equal to 0%. We do not believe this is an important part of the explanation as to why we observe positive capital taxes in the real world. First, we show that we do not need time-inconsistency to get $\tau_K > 0$. Next, this short run bias could really go both ways, i.e. it could push towards excessive capital taxation, but it could also push towards excessive labor taxation. In endogenous growth models with both physical capital and human capital accumulation, both capital and labor taxation can potentially have long run adverse effects, and it is unclear which long run effect dominates. This really depends on the fine details of the models (production functions, preferences, and the structure of endogenous productivity growth process) (see e.g. Roubini on optimal taxation in endogenous growth models).

3. A Theory of Optimal Inheritance Taxation

3.1. Notations and definitions

We consider a small open economy facing a world rate of return $r > 0$. We assume the following deterministic, stationary, continuous-time OLG demographic structure.² Everybody becomes adult at age $a = A$, has exactly one kid at age $a = H > A$, and dies at age $D > H$. So everybody inherits at age $a = I = D - H > A$. E.g. if $A = 20$, $H = 30$ and $D = 70$, then $I = 40$. This is a gender free population. Total adult population N_t includes a mass $N_t(a) = 1$ of individuals of age $a \in [A, D]$ and is permanently equal to $D - A$. All adults are assumed to supply inelastically one unit of labor.

We use a standard wealth accumulation model with exogenous growth:

$$Y_t = F(K_t, H_t) = K_t^\alpha H_t^{1-\alpha} = \text{domestic output}$$

K_t = domestic (non-human) capital stock

$$H_t = L_t e^{gt} = \text{domestic human capital} = \text{efficient labor}$$

$$L_t = N_t = D - A = \text{raw labor supply}$$

g = exogenous rate of productivity growth

$$Y_{Lt} = (1 - \alpha)Y_t = \text{aggregate labor income}$$

W_t = aggregate national wealth³

$$Y_{nt} = Y_t + r(W_t - K_t) = \text{national income}$$

$y_t, y_{nt}, y_{Lt}, w_t, k_t$ = per capita variables ($Y_t, Y_{nt}, Y_{Lt}, W_t, K_t$ divided by $N_t = D - A$)

Cohort size $N_t(a) = 1 \rightarrow$ aggregate bequest flow B_t = per decedent inheritance b_t

$$\beta_K = K_t / Y_t = \alpha / r = \text{domestic capital-output ratio}$$

$$\beta_F = (W_t - K_t) / Y_t = \text{foreign wealth-domestic output ratio}$$

$$b_y = B_t / Y_t = (1 + r\beta_F)B_t / Y_{nt} = \text{inheritance flow-domestic income ratio}$$

² In order to obtain meaningful theoretical formulas for inheritance flows (i.e. formulas that can be used with real numbers), we need a dynamic model with a realistic age structure. Models with infinitely lived agents or perpetual youth models will not do, and standard two or three-period OLG models will not do either. Here we follow the continuous-time OLG model introduced by Piketty (2010, sections 5-7).

³ W_t can be \geq or $\leq K_t$, depending on whether the economy has a positive or negative net external asset position. Below we assume away government assets and debt and require budget balance (see below), so that national wealth and private wealth are identical.

Consider the cohort $x=t-l$ (born at time $x=t-l$) receiving average inheritance b_t at time t . The average lifetime resources (capitalized at end-of-life) received by this cohort can be written as the sum of capitalized inheritance and capitalized labor income:

$$\tilde{y}_t = \tilde{b}_t + \tilde{y}_{Lt}$$

$$\text{With: } \tilde{b}_t = b_t e^{rH}$$

Assuming that we are on a steady-state growth path (so that average labor income grows at rate g), the labor term \tilde{y}_{Lt} can be rewritten:

$$\tilde{y}_{Lt} = \int_{A \leq a \leq D} e^{r(D-a)} y_L^x(a) da = \int_{A \leq a \leq D} e^{r(D-a)} y_{Lt} e^{g(a-l)} da$$

$$\text{i.e. } \tilde{y}_{Lt} = \lambda(D-A)y_{Lt} e^{rH} = \lambda Y_{Lt} e^{rH} = \lambda(1-\alpha)Y_t e^{rH}$$

$$\text{With: } \lambda = \frac{e^{(r-g)(l-A)} - e^{-(r-g)(D-l)}}{(r-g)(D-A)}$$

Intuitively, λ corrects for differences between the lifetime profile of labor income flows and the lifetime profile of inheritance flows, i.e. it measures the relative capitalized value of one euro in labor income vs one euro in inheritance. When inheritance happens roughly around mid-life, then λ is typically very close to one.⁴

Denoting $\gamma = \tilde{b}_t / \tilde{y}_{Lt}$, we have: $\gamma = \frac{b_y}{(1-\alpha)\lambda}$.

Now, consider an individual i in cohort x , with lifetime resources $\tilde{y}_{it} = \tilde{b}_{it} + \tilde{y}_{Lit}$:

$$\tilde{y}_{it} = \tilde{b}_{it} + \tilde{y}_{Lit}$$

$$\text{with: } \tilde{b}_{it} = b_{it} e^{rH}$$

$$\tilde{y}_{Lit} = \lambda(D-A)y_{Lti} e^{rH}$$

⁴ Here we assume that average labor income is the same for all age groups ($y_{Lt}(a)=y_{Lt}$). With a non-flat cross-sectional age-income profile, e.g. with a retirement age $R < D$ and a pension system offering a replacement rate $\rho < 1$, the λ formula needs to be modified accordingly. See Piketty (2010).

We take the inequality of labor income \tilde{y}_{Lit} as given. It could come from the inequality of innate productivities, acquired skills or sheer luck, with any structure of shocks and intergenerational persistence.⁵ We are interested in the endogenous inequality of bequests \tilde{b}_{it} and in the steady-state, two-dimensional joint distribution $G(\tilde{b}_{it}, \tilde{y}_{Lit})$.

We assume that each individual i solves the following maximization program:

$$\begin{aligned} & \text{Max } V[U_{Ci}, w_i(D)] \\ \text{s.c.: } & \int_{A \leq a \leq D} e^{r(D-a)} c_i(a) da + w_i(D) \leq \tilde{y}_{ti} = b_{ti}e^{rH} + \tilde{y}_{Lit} \end{aligned}$$

With: $V(U, w) = (1-s_{Bi})\log(U) + s_{Bi}\log(w)$

$U_{Ci} = \left[\int_{A \leq a \leq D} e^{-\theta(a-A)} c_i(a)^{1-\sigma} da \right]^{\frac{1}{1-\sigma}}$ = utility derived from lifetime consumption flows

$w_i(D)$ = end-of-life wealth (= b_{t+Hi} = bequest left to next generation)

This flexible functional form can be interpreted as utility for bequests (individuals care about the bequest they leave to their offspring) or utility for wealth (individuals care directly about the social status or prestige conferred by wealth, or indirectly for the security brought by wealth, i.e. its insurance value). Presumably the exact combination of saving motives varies a lot across individuals, just like other tastes.

One advantage of our log-log formulation for $V(U, w)$ is that this maximization problem can easily be shown to be equivalent to a one-period, two-good program:⁶

$$\begin{aligned} & \text{Max } (1-s_{Bi})\log(\tilde{c}_{ti}) + s_{Bi}\log(w_i(D)) \\ \text{s.c.: } & \tilde{c}_{ti} + w_i(D) \leq \tilde{y}_{ti} = b_{ti}e^{rH} + \tilde{y}_{Lit} \end{aligned}$$

(with: $\tilde{c}_{ti} = \int_{A \leq a \leq D} e^{r(D-a)} c_i(a) da$ = end-of-life capitalized value of consumption flow $c_i(a)$)

So we have a very simple transition equation for bequest along dynasty i :

⁵ For notational simplicity, we assume that each individual i keeps the same relative rank in the labor income distribution during his or her entire lifetime; but this is not necessary for any of the results.

⁶ See appendix for omitted proofs.

$$b_{t+H} = w_i(D) = s_{Bi} [\lambda(D-A)y_{Lti}e^{rH} + b_{ti}e^{rH}]$$

The preference parameter s_{Bi} can naturally be interpreted as the share of lifetime resources that individual i chooses to accumulate in wealth and to pass on to the next generation. We allow this "taste-for-wealth" or "taste-for-bequest" parameter s_{Bi} to vary across individuals.

If we further assume that $E(s_{Bi} | y_{Lti}, b_{ti}) = s_B$ [i.e. tastes for wealth and bequest are iid and independent of labor income and inheritances received], then our individual-level, linear transition equation can easily be aggregated, and we have a unique aggregate steady-state, whatever the structure of shocks on individual labor productivity parameters and taste parameters might be:

Proposition 1. Assuming $E(s_{Bi} | y_{Lti}, b_{ti}) = s_B$, we have a unique aggregate steady-state for the inheritance flow-domestic income ratio b_y :

$$b_y = \frac{s_B \lambda (1 - \alpha) e^{(r-g)H}}{1 - s_B e^{(r-g)H}}$$

Proof: $b_{t+H} = s_{Bi} [\lambda(D-A)y_{Lti}e^{rH} + b_{ti}e^{rH}]$

With $E(s_{Bi} | y_{Lti}, b_{ti}) = s_B$

Then $b_{t+H} = s_B [\lambda Y_{Lt} e^{rH} + b_{t} e^{rH}]$

Dividing by Y_t : $b_{yt+H} = s_B e^{(r-g)H} (\lambda(1-\alpha) + b_{yt})$.

Assuming $s_B e^{(r-g)H} < 1$, this dynamic system has a unique steady-state given by the above equation. **QED.**

3.2. Linear tax systems and steady-state inheritance flows

We now introduce the tax system.

We first assume a linear tax system: $T = \tau_L Y_L + \tau_K Y_K + \tau_C C + \tau_B B = G = \tau Y$
(cross-sectional govt budget balance at any time t with Y domestic income)

E.g. $\tau = 30\%$. If $\alpha = 30\%$ and all taxes on labor, $\tau_L = \tau / (1 - \alpha) = 43\%$.

If broad income tax, $\tau_K = \tau_L = \tau = 30\%$

$V_i = V(U_{Ci}, w_i(D))$ (wealth-lovers or precautionary-savers: don't care about inheritance tax)

$V_i = V(U_{Ci}, (1-\tau_B) w_i(D))$ (bequest-lovers: they care about the after-tax bequest they leave)

Mechanical impact of labor taxes in the case with pure labor taxes $\tau_L = \tau / (1-\alpha)$:

$$\text{With } V = \log, b_y = \frac{s_B \lambda (1 - \tau_L) (1 - \alpha) e^{(r-g)H}}{1 - s_B e^{(r-g)H}} = \frac{s_B \lambda (1 - \tau - \alpha) e^{(r-g)H}}{1 - s_B e^{(r-g)H}}$$

Lifetime budget constraint of a given individual i with all taxes:

$$\int_{A \leq a \leq D} e^{r(D-a)} \frac{c_i(a)}{1 - \tau_c} da + w_i(D) \leq \tilde{y}_{ti} = (1 - \tau_B) b_{ti} e^{rH} + \int_{A \leq a \leq D} e^{r(D-a)} (1 - \tau_L) y_{Li}(a) da$$

(if $\tau_K > 0$, replace r by $(1 - \tau_K)r$)

Note that $\tau_L = \tau_B > 0$ and $\tau_K = 0$ is equivalent to a flat consumption tax $\tau_C = \tau_L = \tau_B$ on all expenditures (consumption and end-of-life wealth). But the point is that in general there's really no reason to favor $\tau_L = \tau_B$, so consumption taxes are not useful in this model (see discussion in section 4).

For the time being: take $\tau_L > 0$, $\tau_B > 0$ (i.e. $\tau_K = \tau_C = 0$; we return to the discussion of $\tau_K > 0$ in section 4).

Mechanical impact of labor and inheritance taxes:

Proposition 2. With $V = \log$ -log, $b_y = \frac{s_B \lambda (1 - \tau_L) (1 - \alpha) e^{(r-g)H}}{1 - (1 - \tau_B) s_B e^{(r-g)H}}$

But: $\tau_B b_y + \tau_L (1 - \alpha) = \tau$

$$\text{So } b_y = \frac{s_B \lambda (1 - \tau - \alpha) e^{(r-g)H}}{1 - s_B (1 + (\lambda - 1) \tau_B) e^{(r-g)H}}$$

Proof: $b_{yt+H} = s_B e^{(r-g)H} ((1 - \tau_L) \lambda (1 - \alpha) + (1 - \tau_B) b_{yt}) = s_B e^{(r-g)H} ((1 - \tau - \alpha) \lambda + (1 + (\lambda - 1) \tau_B) b_{yt})$

I.e. tax mix has no impact on long run inheritance ratio iff $\lambda = 1$

But if $\lambda > 1$ (resp. < 1), then b_y is an increasing (resp. decreasing) function of τ_B

I.e. even in the absence of any behavioral response, the tax mix has an impact on b_y

Intuition: $\lambda > 1$ means that labor resources come relatively earlier in life, so taxing them less increases long run savings and capital accumulation

More generally: with other utility functions $V(U_c, w)$, long run elasticity e of b_y with respect to various taxes.

The important point is that this is a finite elasticity. This differs from dynastic models.

3.3. Optimal linear inheritance tax: basic formulas

First we compute optimal linear inheritance tax rates from the viewpoint of those who receive zero inheritance ("zero receivers"). This corresponds to a Rawlsian objective where welfare weights are concentrated among those who start with no bequests at all. This is a useful first step as focusing on those who receive no bequests is the social welfare criterion most favorable to the taxation of bequests. Even with this objective (and no behavioral responses), 100% taxation of bequests is not desirable because some of those starting with no bequests still enjoy leaving bequests to the next generation and hence are hurt by bequest taxes.

We assume a fixed revenue requirement τ as a percentage of total domestic product.

Proposition 3 (V=log, zero receivers objective, $\lambda=1$):

The optimal tax mix from the point of view of those receiving no bequests is:

$$\tau_B = \frac{b_y - s_{B0}(1 - \alpha - \tau)}{b_y(1 + s_{B0})} \quad \text{and} \quad \tau_L = \frac{\tau - \tau_B b_y}{1 - \alpha} \quad \text{where } s_{B0} \text{ is the average bequest saving}$$

rate among zero receivers.

Proof: Take a given individual i from cohort $x=t-1$, receiving inheritance $b_{ti}=0$ at time t .

His net-of-labor taxes lifetime resources are $\tilde{y}_{ti} = (1 - \tau_L)\lambda y_{Lit}$ which he divides into life-

time consumption $C = (1 - s_{Bi})\tilde{y}_{ti}$ and gross bequests: $b_{t+Hi} = s_{Bi}\tilde{y}_{ti}$

His utility is:

$$V_i = s_{Bi} \log b_{t+Hi} + (1 - s_{Bi}) \log C = s_{Bi} \log [s_{Bi}(1 - \tau_B)\tilde{y}_{ti}] + (1 - s_{Bi}) \log [(1 - s_{Bi})\tilde{y}_{ti}] =$$

$$s_{Bi} \log [s_{Bi}(1 - \tau_B)(1 - \tau_L)\lambda y_{Lit}] + (1 - s_{Bi}) \log [(1 - s_{Bi})(1 - \tau_L)\lambda y_{Lit}] =$$

$$s_{Bi} \log (1 - \tau_B) + \log (1 - \tau_L) + \text{terms independent of taxes.}$$

(note if wealth-lover rather than bequest-lover, drop $(1 - \tau_B)$ term)

$$\text{We have } 1 - \tau_L = \frac{1 - \alpha - \tau + \tau_B b_y}{1 - \alpha}, \text{ so that,}$$

$$V_i = s_{Bi} \log (1 - \tau_B) + \log (1 - \alpha - \tau + \tau_B b_y) + \text{terms independent of taxes.}$$

Let s_{B0} is the average of the individual s_{Bi} over all those who received no bequests, then the government chooses t_B to maximize:

SWF = $s_{B0} \log(1 - \tau_B) + \log(1 - \alpha - \tau + \tau_B b_y) +$ terms independent of taxes.

When $\lambda=1$, b_y is independent of τ_B [for a given τ] which gives $\tau_B = \frac{b_y - s_{B0}(1 - \alpha - \tau)}{b_y(1 + s_{B0})}$

If $\lambda > 1$ (resp. < 1), positive aggregate effect $\frac{db_y}{d\tau_B} > 0$ (resp. < 0), so τ_B higher (resp. lower)

than this formula: this case is considered in Proposition XX (below).

(Note if wealth-lover, $\tau_B=100\%$ if $\lambda \geq 1$). **QED.**

Example 1 (no revenue requirement): If $\tau=0\%$, $b_y=20\%$, $\alpha=30\%$, $s_{B0}=10\%$, then $\tau_B = 59\%$, i.e. bequests are taxed at $\tau_B=59\%$ in order to finance a labor subsidy $\tau_L = -\tau_B b_y / (1 - \alpha) = -17\%$

The important point is that zero receivers do not want to tax bequests at 100%, because they themselves want to leave bequests. There is a non-trivial trade-off between taxing successors from my cohort vs. my own children.

Example 2 (30% revenue requirement): If $\tau=30\%$ and $b_y=20\%$, then $\tau_B=73\%$ & $\tau_L=22\%$. If $b_y=10\%$, then $\tau_B=55\%$ & $\tau_L=35\%$. If $b_y=5\%$, then $\tau_B=18\%$ & $\tau_L=42\%$.

With high bequest flow b_y , zero receivers want to tax inherited wealth at a higher rate than labor income; with very low bequest flow they want the opposite.

Intuition: with low b_y (high g), not much to gain from taxing bequests, and this is bad for my children; with high g what matters is the future; but with high b_y (low g), it's worth taxing bequest, so as to reduce labor taxation.

3.4. Extensions of the linear formula

Proposition 3 can be extended in various directions. First, we can consider other social welfare functions than Rawlsian. Let's assume that we want to maximize the

expected welfare of those who receive $z\%$ of average inheritance. Then we have the following formula:

Proposition 4 ($V=\log$, $\lambda=1$, and $E(s_B|z)=s_B$ independent of z):

$$\tau_B = \frac{b_y(1-z(1+s_B)) - s_B(1-\alpha)}{b_y(1+s_B)(1-z)} \quad \text{and} \quad \tau_L = \frac{-\tau_B b_y}{1-\alpha}$$

Corollary: There is a level $z^* < 1$ such that $\tau_B > 0$ iff $z < z^*$

Proof: Simple generalization of proposition 3 proof. **QED.**

Example: the median voter does not need to be that rich to favor zero inheritance tax.

Next, we can consider more general utility functions and extend to the case with positive elasticities of bequests flows with respect to inheritance taxes.

Proposition 5 (any V , any λ). $\tau_B = \frac{b_y - s_{B0}(1-\alpha-\tau)}{b_y(1+e+s_{B0})}$ and $\tau_L = \frac{\tau - \tau_B b_y}{1-\alpha}$

With s_{B0} = average effective saving rate of zero receivers and e is the elasticity of bequest flow b_y with respect to $1-\tau_B$.

Example. From French inheritance tax experience, $e=0.1$ or 0.2 seems like an absolute upper bound.

We have seen before, if $b_y=10\%$, $s_{B0}=10\%$, and $e=0$ then $\tau_B=55\%$ & $\tau_L=35\%$. If $e=0.2$, then $\tau_B=46\%$ & $\tau_L=36\%$. If $e=0.5$, then $\tau_B=37.5\%$ & $\tau_L=37.5\%$.

Behavioral responses matter but not hugely as long as elasticity is reasonable.

Note that if $s_{B0} = 0$ (zero receivers never want to leave bequests), we obtain

$\tau_B=1/(1+e)$, the classical revenue maximizing inverse elasticity rule.

Proof: Bequest-lovers: $\text{Max } V=V(U_c, (1-\tau_B)W) \rightarrow V_c = V_w (1-\tau_B)$

(check indirect utility formula: simple degree-1-homogeneity assumption with respect to lifetime capitalized consumption?) SAEZ: yes need $V(\lambda U_c, \lambda W) = \lambda V(U_c, W)$ for balanced growth path.

Assume $d\tau_B > 0$, so as to finance $d\tau_L < 0$

Government budget balance at any time t : $\tau_B b_y + \tau_L(1-\alpha) = \tau$

$$\text{i.e. } -d\tau_L = \frac{b_y d\tau_B}{1-\alpha} \left[1 - \frac{e\tau_B}{1-\tau_B} \right] > 0 \text{ as long as } \tau_B < \frac{1}{1+e}$$

(with $e = -\frac{db_y}{d\tau_B} \frac{1-\tau_B}{b_y}$) (with log utility: $e < 0$ if $\lambda > 1$, $e > 0$ if $\lambda < 1$)

Zero receivers have lifetime resources $\tilde{y}_{ti} = (1-\tau_L)\lambda(1-\alpha)\bar{y}_{Li} Y_t$

So $\frac{d\tilde{y}_{ti}}{\tilde{y}_{ti}} = -\frac{d\tau_L}{1-\tau_L}$ and using $(1-\tau_L)(1-\alpha) = 1-\alpha-\tau+\tau_B b_y$, we have:

$$\frac{d\tilde{y}_{ti}}{\tilde{y}_{ti}} = \frac{b_y d\tau_B}{1-\alpha-\tau+\tau_B b_y} \left[1 - \frac{e\tau_B}{1-\tau_B} \right] > 0 \text{ as long as } \tau_B < \frac{1}{1+e}$$

Max $V = V(U_c, (1-\tau_B)w) \rightarrow w_i(D) = s_{Bi} \tilde{y}_{ti}$ [this is really a definition of s_{Bi}]

$$dV_i = -V_w s_{Bi} \tilde{y}_{ti} d\tau_B + V_c d\tilde{y}_{ti} = -V_w s_{Bi} \tilde{y}_{ti} d\tau_B + V_w (1-\tau_B) d\tilde{y}_{ti}$$

$$dV_i = V_w \tilde{y}_{ti} d\tau_B \left(-s_{Bi} + \frac{b_y(1-(1+e)\tau_B)}{1-\alpha-\tau+\tau_B b_y} \right)$$

$$\rightarrow \tau_{Bi} = \frac{b_y - s_{Bi}(1-\alpha-\tau)}{b_y(1+e+s_{Bi})}$$

(=most-preferred tax rate from the viewpoint of agent i)

Now if we aggregate dV_i over all zero receivers, then we just need to replace s_{Bi} by the average s_{B0} over all zero receivers (we simply need to assume that the welfare weights are orthogonal to the s_{Bi}):

$$\tau_B = \frac{b_y - s_{B0}(1-\alpha-\tau)}{b_y(1+e+s_{B0})}$$

QED.

Note that $\tau_B > 0$ iff $b_y > s_{B0}(1-\alpha-\tau)$

Intuition: assume we start from $\tau_B = 0$, $\tau_L = \tau/(1-\alpha)$. Then $s_{B0}(1-\alpha-\tau) = s_{B0}(1-\alpha)(1-\tau_L)$ is the fraction of income that zero-receivers are going to leave to their children; this measures how much $\tau_B > 0$ is going to hurt them. On the other hand b_y measures how much fiscal resources the bequest tax $\tau_B > 0$ is going to bring, and therefore by how much the labor taxes of zero receivers can be cut.

By construction: $b_y = s_B e^{(r-g)H} ((1-\tau_L)\lambda(1-\alpha) + (1-\tau_B)b_y) = s_B e^{(r-g)H} ((1-\tau-\alpha)\lambda + (1+(\lambda-1)\tau_B)b_y)$ measures average inheritance.

Note that the relative labor income of zero-receivers does not matter: this is because inheritance tax revenues are being distributed in a way that is proportional to labor income.

3.4. Optimal non-linear inheritance tax

TO BE COMPLETED

Plan:

- 1) Consider bequest tax only above some threshold b^*
- 2) Consider elastic labor supply as well. This is a difficult case because changes in τ_B will typically affect labor supply in two ways: smaller received inheritances induce receivers to work more (Carnegie effect) but higher tax on bequests discourages labor supply of donors. That means that changes in τ_B have fiscal effects through τ_L as well. This will look like a three good Ramsey model with linear taxes and considerably more complicated (possibly non-tractable) with non-linear taxes. Provide numerical illustration.

4. From Optimal Inheritance Taxation to Optimal Capital Taxation

With perfect capital markets, and full tax instruments, all the tax burden should be paid at the time of inheritance, and there should be no further tax on capital or capital income taxes paid during the course of the inheritors' lifetime: $\tau_B > 0$, but $\tau_K = 0\%$. The intuition is that such lifetime capital taxes impose unnecessary distortion on inter-temporal choices.

Proposition 7. With perfect capital markets, and with full tax instruments (inheritance tax τ_B , labor income tax τ_L , and capital income tax τ_K), then whatever the utility function $V()$ and the social welfare objective, there is no capital income tax in the socially optimal tax structure: $\tau_K = 0\%$.

Note: This will be true under the standard Atkinson-Stiglitz weak separability and homogeneity in sub-utility for consumption if we can use non-linear labor and inheritance taxes. If we have homothetic utility of consumption, then it probably holds with linear taxes on labor and inheritances.

However in the real world inheritance taxes are only a small fraction of total capital taxes: less than 1% of GDP for the former, versus as much as 9%-10% of GDP for the latter (EU 27, 2010; see Table 1) (coming mostly corporate tax + property tax + dividend and interest income tax + sometime annual wealth tax). This is not accidental. This occurs in every single country. There is extensive anecdotal evidence suggesting that inheritance taxes are much more unpopular than lifetime capital taxes. E.g. most people would prefer paying a property tax $\tau_K = 1\%$ during 30 years rather than a big bequest tax $\tau_K = 30\%$.

One possible explanation for this would be tax illusion. However we believe that this mostly reflects the imperfections and incompleteness of real world capital markets and tax instruments.

First, one easy way to generate a positive socially-optimal tax rate on capital income $\tau_K > 0$ in our framework is to assume that the frontier between labor and capital income is fuzzy, i.e. it can be manipulated by taxpayers. This generally will imply that

it is better to have the capital tax rate τ_K not too different from the labor tax rate τ_L . Take the extreme case where the frontier is totally fuzzy (taxpayers can costlessly re-label labor income into capital income, and conversely, and the tax administration has no way to observe this), then clearly this implies that τ_K must exactly be equal to τ_L . Under this assumption, we have the following proposition:

Proposition 8. With perfect capital markets, but with a fuzzy capital-labor frontier, implying a $\tau_K = \tau_L$ constraint (i.e. a comprehensive income tax), then the optimal tax system can be characterized as follows: $\tau_B <$ proposition 3 level, but positive if b_y large enough and/or IES σ low enough.

Note. This corresponds to a tax system with a comprehensive, integrated income tax (treating all forms of capital and labor incomes on a par basis), and an inheritance tax. This optimal tax structure cannot be replicated by a progressive consumption tax, because the latter would treat on a par basis labor income resources and inheritance resources on a par basis, whereas depending upon social welfare objective one might want to tax less labor income resources

Although this capital-labor frontier fuzziness problem is an important issue which needs to be taken into account in the real world,⁷ we do not believe this is the only reason why capital taxes play an important role. For instance, this fuzziness issue does not apply to the property tax example discussed above: it is really difficult - if not impossible - to re-label labor income into rental income or property values. That is, it is often the case that the tax administration is able to properly observe the labor-capital frontier and enforce different labor and capital tax rates, but that the government chooses to have $\tau_K > 0$. One can also find many historical examples where governments actually choose higher rates on capital income than on labor income (i.e. $\tau_K > \tau_L$), for instance in the U.K. and in the U.S. until the 1960s-1970s.⁸

⁷ E.g. dualistic tax systems generate huge manipulations, see reference on Finland etc.

⁸ E.g. in the U.S., the top marginal tax rate was 70% for capital income and 50% for labor income until 1981. $\tau_K > \tau_L$ was actually the norm in most income tax systems when the latter were instituted in the early 20th century (generally around 1910-1920). At that time income tax systems typically involved a progressive surtax on all forms of labor and capital income (including imputed rent), and a set of schedular taxes taxing wage income less heavily than interest, dividend, rent or business profit. It is only in the postwar period that this tax equilibrium was gradually changed (following the large shocks incurred by capital during the 1930s-1940s). Since the 1990s-2000s, it has become more common to

In our view, the main reason why $\tau_K > 0$ is the existence of capital market imperfections. Two kinds of K market imperfections:

(1) Liquidity pbs: paying $\tau_B = 30\%$ might require successors to sell the property (borrowing constraints + indivisibility pb). Empirically, this seems to be an important reason why people dislike inheritance taxes (« death taxes ») much more than property taxes & other flow taxes

(2) Uninsurable uncertainty about future rate of return on inherited wealth: what matters is $b_{ti} e^{rH}$, not b_{ti} ; but at the time of setting the bequest tax rate τ_B , nobody has any idea about the future rate of return during the next 30 years... (idiosyncratic + aggregate uncertainty). With uninsurable uncertainty on r , it's more efficient to split the tax burden between one-off transfer taxes and flow capital taxes paid during entire lifetime.

Proposition 9. In case the intertemporal elasticity of substitution is small, then one can replicate almost exactly the τ_B optimum with $\tau_K > 0$. If liquidity pb and/or uninsurable uncertainty on future r are substantial, then it's optimal to do so.

Notes:

(i) historically, $\tau_K > 0$ probably played a major role to prevent inheritance flows from fully returning to 19c levels (see Piketty (2010))

(ii) Proposition 9 related to the view according to which if $\tau_B = 0$ (say, impossible to tax inheritance), then optimal $\tau_K > 0$ (Pestieau); related, but different: in practice, it is possible to tax inheritance, it's even easier than to tax k income; the pb is the k market imperfection

have $\tau_K < \tau_L$. But this mostly reflects a rising concern for international tax competition and tax evasion and the persistent lack of tax coordination (the view is that it is easier to reallocate one's financial portfolio abroad than one's labor income, and that it is harder to apply the residence principle of taxation for capital income; or at least this is a view that became very influential in a number of small open economies, typically in Nordic countries), rather than fundamental economic beliefs that τ_K should be smaller than τ_L for deep economic reasons.

Economists' emphasis on $1+r$ =relative price is inappropriate, or at least excessive.

Other points to be mentioned: of course the optimal balance between inheritance tax and lifetime k income tax clearly depends on behavior, and can vary over time and across societies. E.g. if successors eat up their inheritance entirely and immediately, then lifetime k taxation will only hurt lifecycle savers, which would be both unfair and inefficient. Using $\tau_K > 0$ to implement the social optimum is relatively rough, and it would be better to imagine more creative tax instruments.

The true universal optimum should involve a tax break for self made wealth (and derived capital income); i.e. we should tax only capital income related to $\min(\text{current wealth, capitalized wealth})$. Note that this would still be quite different from progressive consumption taxes. The problem with progressive consumption taxes is that one may not want to treat similarly top labor earners and top successors (there's no reason to put the same weights in the social welfare function, e.g. the latter can be "meritocratic" and put more weight on top labor earners - or even all weight, as in our Rawlsian optimum, in which case progressive consumption taxes are wholly inadequate).

TO BE COMPLETED

5. Concluding comments

TO BE COMPLETED

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Appendix: omitted proofs

Proof of Proposition 1 (section 3.1). The reason why our dynamic, continuous time maximization program is equivalent to a one-period, two-good maximization program is the following. First, with $U_{Ci} = [\int_{A \leq a \leq D} e^{-\theta(a-A)} c_i(a)^{1-\sigma} da]^{\frac{1}{1-\sigma}}$, standard first-order conditions imply that individual i will choose a consumption path $c^x(a) = c^x(A) e^{g_c(a-A)}$ growing at rate $g_c = (r-\theta)/\sigma$ during his or her lifetime. Note that $(1-\sigma)g_c - \theta = g_c - r$. The utility value U_{Ci} of this consumption flow is given by:

$$U_{Ci} = [\int_{A \leq a \leq D} e^{-\theta(a-A)} c_i(a)^{1-\sigma} da]^{\frac{1}{1-\sigma}} = c_i(A) \left(\frac{1 - e^{-(r-g_c)(D-A)}}{r - g_c} \right)^{\frac{1}{1-\sigma}}$$

We note \tilde{c}_{ti} the end-of-life capitalized value of individual i consumption flow:

$$\tilde{c}_{ti} = \int_{A \leq a \leq D} e^{r(D-a)} c_i(a) da = c_i(A) e^{r(D-A)} \frac{1 - e^{-(r-g_c)(D-A)}}{r - g_c}$$

It follows that that the maximization program of individual i is equivalent to:

$$\text{Max } (1-s_{Bi}) \log(\tilde{c}_{ti}) + s_{Bi} \log(w_i(D))$$

$$\text{s.c.: } \tilde{c}_{ti} + w_i(D) \leq \tilde{y}_{ti} = b_{ti} e^{rH} + \tilde{y}_{Lti}$$

QED.