SIZE OF ESTABLISHMENT AND THE CAPITAL-OUTPUT RATIO: AN EMPIRICAL INVESTIGATION

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The relationship between the size of the firm and the capital-output ratio (K/O) has been investigated by several authors [3, 6, 8, 10, 12]. The conclusion arrived at in all these studies is that, typically, size of establishment and K/O are positively associated.

Several hypotheses were offered by the different authors as explanations for the observed association. In this paper we seek to examine the explanatory power of the major hypotheses against a new set of data. In addition, we investigate the likely results if output, instead of capital, is used as the measure of size.

THE STATISTICAL APPROACH

Differences in K/O can be studied either over time, or across different production units at a given time. The present analysis takes the form of a cross-sectional study of seventeen two-digit manufacturing industries with states as the observation points.

A cross-sectional study offers several advantages over a time series study for the subject under consideration. First, it reduces—although it does not eliminate—the role of technological change as a factor influencing the ratios. When analyzing variations at a point in time one may assume that the available technological possibilities do not vary among the states although the *chosen* technological process—which may be more or less capital intensive—is likely to vary. Given this assumption one may test the relevance of the various hypotheses which seek to explain the differences in the ratios assuming no differences in the state of technology.[1] The second advantage in the use of cross-sectional data is that all observation points are at the same stage in the business cycle and hence one may assume the same degree of capacity utilization for the different states.

As is well known, K/O ratios may be based on gross capital values, or on net (depreciated) capital values depending upon which of the two series is considered to be a better representation of the current value of the stock of capital. A discussion of the relative merits of the two series falls outside the realm of this paper. In this study gross capital is used.[2]

THE DATA

Output. The output data source is 'adjusted value added' in the Annual Survey of Manufactures for 1957. Value added, it may be recalled, is derived by subtracting the

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[1] The validity of this assumption can be tested in two ways. First, an age of capital variable will be introduced to catch quality differences. The second test is the performance of those explanatory variables which assume no systematic differences in technological possibilities among the states.

[2] The reader who is also interested in the findings based on net capital can obtain them by writing to the author.

It might be mentioned that the net series, regardless of its theoretical merits, may introduce distortions due to the differing depreciation accounting practices used. We find that since World War II there have been two occasions on which depreciation accounting practices were changed. For a discussion and references see [4]. In all events it may be worth noting that the states' ranks based on net and the gross ratios are highly correlated in most industries, as can be seen in the Table on pape 54.

Rank Correlation Coefficients of Capital-Output Ratios Based on Net Capital Figures with Those Based on Gross Capital Figures

Industry	Number of observ.	r_s Net & Gross ratios
Food	40	0687
Textiles	20	0949
Lumber	23	0918
Furniture	22	0713
Printing	18	0943
Paper	30	0928
Chemicals	32	0886
Petroleum	18	0936
Rubber	16	0542
Leather	17	0931
Glass	26	0817
Prim. Metals	29	0887
Fab. Metals	33	0797
Machinery	28	0670
Elect. Machinery	25	0890
Transportation	28	0966
Instruments	14	0935

cost of materials, supplies, containers, fuel, purchased electrical energy, and contract work from the value of shipments of manufacturing. This is adjusted to include value added in merchandising operations minus net changes in inventories. Since the data are collected on an establishment basis, and not by firm, it is considered "... the best value measure available for comparing the relative economic importance of manufacturing among industries and geographical areas" [13].

Capital. The capital stock data are from a special 1958 census report [14]. Only 'depreciable and depletable assets' are reported; this consists of plant, equipment and depletable natural resources. The gross capital figures were reported for Dec. 31, 1957 on the basis of the actual costs of the assets at the time of their purchase. We will refer to this as the reported value of capital.[3]

Manhours Employed. In the Annual Survey of Manufactures one finds information on the number of all employees, number of production workers, and manhours of production workers. One has thus the choice of using a labor-input inclusive or exclusive of nonproduction employees. We chose the former.[4]

Wage Rates. The Annual Survey and the Census Reports report salaries and wages as defined for calculating the Federal Withholding Tax. Consequently, they omit some labor costs such as employer payments for legally required insurance plans and other contractual employer payments.[5] For 1957, however, it was possible to take full account of these labor costs; the Special Report, referred to earlier, has these data. Our labor cost figure is consequently inclusive of supplementary employee costs, and the wage rate is total labor cost divided by the number of manhours.

THE SIZE—K/O HYPOTHESES

Extent of Vertical Integration. One explanation mentioned in several studies [3, 6, 12] was based on the fact that in these studies the volume of *sales* was used as the denominator of the K/O ratio. Thus the observed

[3] For three industries—foods, textiles, machinery—we have, for the sake of comparison, deflated the reported values of capital into 1957 prices. It was found that the resulting pattern of interstate variation changed little in comparison with the reported (undeflated) values. It must be noted, however, that the deflators were not satisfactory since investment estimates by states which were necessary for their construction were available for only the five preceding years. Since capital existing in 1957 had been accumulated over a considerably longer period, a deflator based on only five years may be grossly inadequate.

[4] To do this, the number of nonproduction employees was converted into manhours and added to the number of production manhours employed. The conversion factor was 2,000 hours per worker; based on 250 workdays a year, 8 hours a day. Since most layoffs are among production workers this assumption of full employment does not seem unreasonable. In any event, since the purpose is to compare wage rates among different states rather than determine their absolute magnitude, our figures will distort the findings only to the extent that manhours per non-production employee varied considerably among the states for any given industry.

[5] The required payments are: Old Age and Survivors Insurance, Unemployment Insurance (State and Federal), Workmen's Compensation premiums. Some of the contractual payments are: health, welfare, and life insurance plans.

positive association of size and K/O may be due to the higher degree of vertical integration typical of the larger firm. The larger firm may then exhibit a lower volume of sales per unit of capital even though its value added per unit of capital does not necessarily differ from that of the smaller firm. In the present study, this source of K/O variations is absent since value added data are used in the denominator.

The Output Composition Hypothesis. According to this hypothesis [6, 12] the differences in K/O among firms of different sizes may be due to product specialization *within* the industry studied. Specifically, it is argued that the larger firm produces a different, and more capital intensive product mix, and that this is the cause for the observed association of size and K/O.

The Factor Substitution Hypothesis. Some have suggested([3, 5, p. 61, 10, p. 16, 12, p. 20]) that the observed size $-K/O$ association may result from lower capital costs faced by larger firms. Following this line, it could be argued that larger firms face higher labor-capital price ratios because of lower borrowing charges and/or higher labor prices. This induces them to substitute capital for labor which should be expected to manifest itself in higher K/O.

The Age of Capital. Differences in the average age of capital may produce differences in the observed K/O. This might be the case even if deflated capital figures could be used. It is because the available capital deflators do not remove quality differences.[6] Therefore, if improvements in capital goods take place continuously over time we will expect—*ceteris paribus*—the unit with the lower average age of capital goods to exhibit a lower K/O. Since we do not, in fact, possess deflated capital figures, the age of capital will also be associated with a different influence, namely, with an inflationary effect. The effect is based on the likelihood that the production unit with the lower age of capital has acquired its capital at a higher average price. We thus observe that the two effects associated with the age of capital will have opposing influences on K/O. It is difficult to assess the net effect of 'age of capital' on differences in K/O because there is no way of determining the time pattern and the magnitudes of the technological improvements in capital.[7] Hazarding a "guesstimate" it may be suggested that, extending Solow's estimate [11] of a neutral shift in the production function of a 2¼ per cent through 1957, and assuming an annual rate of inflation in capital goods prices of about 4 per cent (based on the 1947–57 experience), one may expect the net effect of the age of capital on K/O to be positive on balance.[8]

We now turn to the specification of the variables which will be used in evaluating the various hypotheses.

SPECIFICATION OF VARIABLES

The Output Composition Variable. The K/O ratios in this study are for two digit industries since no data were available for computing ratios, by states, on a finer industry breakdown. We propose therefore to investigate whether differences among the constituent industries (three digit) are responsible for the size $-K/O$ association of the two digit industries. In order to do this we construct output-mix corrected ratios ($\widehat{K/O}$).

$\widehat{K/O}$ differs from the U.S. ratio only through the three digit industry weights. It is

[6] For a discussion of the bias entering into a time series analysis of K/O ratios due to the deflation procedures used see [1]. For a thorough analysis of the theoretical questions involved in deflating capital goods see [7].

[7] We are assuming neutral technological change. Otherwise one cannot even be certain about the direction of the productivity change influence on K/O. If, for example, we deal with capital-using innovations the age of capital may be positively associated with K/O depending on the relative magnitudes of the labor replacing effect which is capital deepening, and the increasing productivity effect which is capital lowering.

[8] The correlations of K/O and age of capital are indeed found to be positive in most (thirteen of the seventeen) industries. However, their values are quite low and statistically insignificant.

derived as the average of the (U.S.) industry ratios for the constituent three digit industries, weighted by the shares of these minor industries in the state's two digit aggregate.[9, 10]

The Factor Prices Variable. To evaluate the presence of factor substitution as an explanation for the differences in K/O, one must specify the factor prices variable. The price of labor, w, was derived directly from total labor costs (described earlier); the price of capital, on the other hand, will be estimated on the basis of several assumptions. The cost of capital is represented by its annual opportunity cost $\overline{PK}\ (i + 1/n)$, where $\overline{PK}$ is the purchase price of capital, i is the interest rate, and n is the expected life of the capital good. We shall assume that $\overline{PK}$ and n, for any given industry, do not vary among the states. Thus, differences in the cost of capital will be due solely to differences in i. Our factor price ratio will therefore be expressed as w/i.

We proceed to specify the price ratio under two alternative assumptions. (a) i is the same for all states, (b) i varies among the states. The justification for preferring assumption (a) would be the lack of direct data on interest rates by states. If this assumption is adopted, the factor price ratio will vary only because of differences in w.

Under assumption (b) i, as well as w, is allowed to vary among the states. To obtain estimates of the interest rate, we compute the rate of return on K, which, in competitive equilibrium, must equal i. In notation, $O = wL + iK$, and $i = (O - wL)/K$. We can thus obtain estimates of i and specify the factor price ratio for each state. The statistical findings will be reported for each of the two alternative variables, namely, for w and for w/i.

The Size Variable. This was computed as gross capital for the industry in the state divided by the number of establishments in the state. Later we consider the likely effects if (average) output per firm is used as the size measure.

The Age of Capital Variable. As the age index of capital we propose to use the ratio of net to gross capital. The higher it is, the younger is the average age of capital assumed to be. Clearly, for this index to be useful one must assume that for any given industry depreciation accounting practices have not varied among the states. This assumption is questionable (see footnote 2 earlier).

THE STATISTICAL TESTS

The primary hypothesis—that K/O and size are positively correlated—is evaluated in Table I. We see there that the two are positively associated in all cases, and that the association is significant in twelve of the seventeen industries.[11]

Before examining the explanatory value of the hypotheses which were mentioned earlier, it should be noted that a positive association of K/O with size as measured by capital per establishment (K/N) could be spurious under some circumstances, and, given some reasonable assumptions about the presence of measurement errors, it will be biased upwards.

Specifically, the correlation of K/O and

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[9] Let $\hat{K/O}_{ij}$ = expected capital-output ratio of industry i in state j, where i is a two-digit industry.
$K/O_{ik\text{U.S.}}$ = capital-output ratio of industry i_k in the U.S., where k is a three digit constituent of industry i.
S_{ikj} = share of three digit industry k in two digit industry i for state j. Thus, $\sum_k S_{ikj} = 1$.
Then: $\hat{K/O}_{ij} = \sum_k K/O_{ik\text{U.S.}} S_{ikj}$.

[10] It must be noted that the expected ratios are subject to several shortcomings. The weights (shares) used are based on 1958 and not on 1957 data as required. Also, there were changes in classification between the 2 years for which, at times, no proper adjustment could be made. But far more important, the reported composition of output for some states was not always complete.

[11] Unless otherwise stated, reference to statistical significance in the correlation results denotes coefficients different from zero at the 5% level or better. The five industries in which the values are *not* significant are food, printing, rubber, glass, instruments.

K/N will be positive if: (a) capital per establishment and output per establishment were completely independent of each other, or if: (b) output per establishment is constant and so is *planned* capital, except that observed capital is allowed to vary randomly around the planned amount. In such a case there could be no economic reason for a positive association between K/O and K/N since in fact K/O is planned to be constant regardless of K/N. Similarly in case (a) no economic reason could be suggested for any systematic association of K/O and K/N, since, by assumption, O/N and K/N are distributed independently. Yet, for statistical reasons we will observe a positive association in both cases.[12] Clearly, inasmuch as the assumptions underlying these two cases are extremely unreasonable one may consider them theoretical trivia. We *do* know that K/N and O/N are highly correlated, as indeed they should be; and the dispersion of O/N is substantial. A more serious difficulty arises from the effect of measurement errors when K/O and K/N are the variables to be correlated. The problem is that errors in the reported value of capital will influence both K/O and K/N in the same direction thus biasing the reported correlations upwards.[13] We may

TABLE I

CORRELATION COEFFICIENTS, SIZE OF ESTABLISHMENT (K/N) WITH K/O*

Industry	Number of observations	r
Food	40	.089
Textiles	20	.687**
Lumber	23	.817**
Furniture	22	.689**
Paper	30	.802**
Printing	18	.293
Chemicals	32	.659**
Petroleum	18	.711**
Rubber	16	.022
Leather	17	.771**
Glass	26	.350
Primary Met.	29	.494**
Fabricated Met.	33	.794**
Machinery	28	.551**
Elect. Machinery	25	.487*
Transportation	28	.427*
Instruments	14	.095

* Variables are measured in their logarithms. Single and double stars denote significance at the 5% and 1% levels respectively.

[12] Let K, O, and N stand for capital, output and number of establishments, and let:

$X = \log(K/N)$

$Y = \log(O/N)$

$X - Y = \log(K/O)$

n = number of observations.

The correlation of K/O and K/N is then:

$$r_{(X-Y,X)} = \frac{\sigma_X^2 - \sum(X-\bar{X})(Y-\bar{Y})/n}{\sigma_{(X-Y)}\sigma_X}.$$

It is seen that in case (a)—when X and Y are assumed to be independent—the correlation will be positive and equal to:

$$r_{(X-Y,X)} = \frac{\sigma_X}{\sqrt{\sigma_X^2 + \sigma_Y^2}}.$$

Case (b) (output per establishment assumed constant) is characterized by $\sigma Y^2 = 0$. We will therefore find

$$r_{(X-Y,X)} = \frac{\sigma_X^2}{\sigma_X\sqrt{\sigma_X^2 + \sigma_Y^2}} = 1.$$

[13] Let:

$X = \log$ (true) capital $- \log N = \log (K/N)_t$

$X + v = \log$ (reported) capital $- \log N = \log (K/N)_p$

$Y = \log$ output $- \log N = \log (O/N)$

r_t = correlation of K/O and K/N, with true capital

r_p = correlation of K/O and K/N, with reported capital.

Then, assuming v to be distributed independently of the other variables with a zero mean, we obtain:

$$r_p = \frac{\sigma_x^2 - \sum(X-\bar{X})(Y-\bar{Y})/n + \sigma_v^2}{\sigma_{(X+v-Y)}\sigma_{(X+v)}} = \frac{\sigma_x^2 - \sum(X-\bar{X})(Y-\bar{Y})/n + \sigma_v^2}{\sqrt{\sigma^2_{(X-Y)} + \sigma_v^2}\sqrt{\sigma_X^2 + \sigma_v^2}}$$

and,

$$r_t = \frac{\sigma_x^2 - \sum(X-\bar{X})(Y-\bar{Y})/n}{\sigma_{(X-Y)}\sigma_X}.$$

Thus for any given $r_t < 1$, we will find $r_p > r_t$ and the difference will depend upon the value of r_t, and the relation of σ_v^2 to σ_x^2 and σ_y^2.

TABLE II

CORRELATION COEFFICIENTS, SIZE OF ESTABLISHMENT (K/N) WITH SEVERAL VARIABLES*

Industry	Number of observations	Zero order correlations K/N with			
		w/i (1)	w (2)	$\hat{K/O}$ (3)	NK/GK (4)
Textiles	20	.201	−.722**	.501*	.515*
Lumber	23	.827**	.785**	.338	.155
Furniture	22	.605**	−.051	−.013	.152
Paper	30	.532**	.341	.798**	.188
Chemicals	32	.671**	.397*	.626**	−.392*
Petroleum	18	.494*	−.011	.607**	−.314
Leather	17	.580*	.332	.554*	.410
Primary Met.	29	.312	.542**	.607**	.233
Fabricated Met.	33	.436*	.310	.448*	−.032
Machinery	28	.225	.105	.699*	−.023
Elect. Machinery	25	.118	.275	.239	−.276
Transportation	28	.289	.694**	.221	.113

* Variables measured in the logarithms. Single and double stars denote significance at the 5% and 1% levels respectively.

thus conclude that the true correlations of K/O and K/N are lower than the computed ones, although we are unable to gauge the extent of the bias.[14] Bearing this in mind we will now turn to the evaluation of the various economic hypotheses. For this purpose we present (Table II) the simple correlation coefficients of size with each of the explanatory variables; namely, with $\hat{K/O}$ (the output composition variable); with w (factor price ratio when only w is allowed to vary); with w/i (factor price ratio) and with NK/GK (the age of capital index).

Size and $\hat{K/O}$. Recall that according to this hypothesis the variations in the observed K/O are due to output mix differences *within* each two digit industry. We computed therefore the output mix expected ratios ($\hat{K/O}$) and correlated them with the actual ratios (K/O). A look at Col. 3, Table II, shows that in all industries but one—furniture—the association between $\hat{K/O}$ and K/O is positive. The correlation is statistically significant in textiles, paper, chemicals, leather, primary metals, fabricated metals and machinery.

In most of these industries the observed association can be easily related to the broad characteristics of the output mix of the industry.

In the textile industry the observed association is due to the relative specialization of the southeastern states—which display a larger than average establishment size—in the production of coarser fabrics which are typically more capital intensive. The New England and Middle Atlantic states, on the other hand, exhibit smaller size establishments and a relative specialization in the more delicate fabrics which require lower K/O.

In the paper industry the larger size establishments are engaged primarily in the production of basic paper products: pulp, paper and paperboard. The smaller size establishments, on the other hand, specialize in the production of simple paper products, such as paper bags and paper boxes.

In the chemical industry the observed association is due to the fact that larger establishments—primarily located in the South, engage in the production of basic chemicals such as organic and inorganic chemicals and fertilizers. Establishments which produce predominantly secondary chemical products—drugs and medicines, detergents, paints and varnishes—are typically smaller and exhibit lower K/O. A similar distinction between basic and secondary activities explains the association of size

[14] Another test for the size-K/O hypothesis can be made by running a regression of O/N on K/N, when both variables are measured in their logs. If the regression coefficients are smaller than one, it would indicate that K/O increases with K/N. Any bias entering through errors in the common denominator—probably minimal here since the number of establishments is least susceptible to errors—will tend to bias the value of the coefficients upwards. Therefore, requiring that the regression coefficients be less than one is a strong test of the hypothesis that size and K/O are positively associated. When this test is run the coefficients for all industries are found to be smaller than one.

and K/O in the leather industry. The basic activities are tanning and finishing; the secondary activities are the production of footwear, small leather goods and luggage.

In the primary metals industry the distinction can be drawn along the same lines: the basic steel activities—blast furnaces, steel works, etc.—and their high capital requirements, against the lower capital requirements for rolling, drawing and extruding of metals. In the remaining two industries—fabricated metals and machinery—the host of products cannot be broken down as conveniently into high—and low—capital requiring activities.

Size and w. The coefficients of correlation of size of establishment and the wage rate are presented in Col. 2 of Table II. We note that in nine of the twelve the association is positive and that in five of these the relationship is statistically significant. The five are: textiles, lumber, chemicals, primary metals, and transportation. Of the three instances where the association is negative—textiles, furniture and petroleum, the association is high and significant only in the first. The high negative association in the case of textiles results from the behavior of the southeastern states which are characterized by low wages, high K/O and large-size establishments.

Size and w/i. The correlation is positive in all twelve cases and statistically significant in seven of these (lumber, furniture, paper, chemicals, petroleum, leather, and fabricated metals). We thus note that this factor price variable is superior to w alone: the correlations are higher in nine of the twelve industries.

Size and Capital Age. The age of capital index and size do not show any consistent pattern of association. The association is positive in seven instances and negative in all others, and is never significantly different from zero aside from textiles and chemicals.

Summarizing the correlation findings we note that in six industries (paper, chemicals, petroleum, leather, primary metals, fabricated metals) *both* variables—the factor price ratio (w, or w/i) and the output mix variable—are positively and significantly associated with size; in all remaining industries (furniture excluded) one or the other of the variables is significantly associated with size.

These findings do not appear surprising. That factor price ratios, particularly w/i, are positively associated with size in many industries seems reasonable especially as far as i is concerned. There is considerable evidence showing that interest rates and size of firm are inversely related. Also the fact that larger size establishments are found to be producing a more capital intensive output mix is readily understood when considering that our size measure is the value of capital in use. Thus, an establishment producing a more capital intensive output has, by definition, a higher K/O ratio and, *ceteris paribus*, employs a larger size when measured by capital. But more on this in the next section.

Having seen that the size $-K/O$ positive association may well be due to the operation of other variables, namely, factor price ratios and the output mix, there remains the question whether size and K/O are correlated independently of these two factors. To answer this we use multivariate analysis, where K/O is the dependent variable, and factor price ratios, output mix and size of establishment are the independent variables. We use two regression equations. They are in logarithmic form since these produced the most consistent regression coefficients and the highest coefficients of determination. The two equations are:

$$(1)\quad \log K/O_1 = a_{01} + a_{11} \log w/i + a_{21} \log \hat{K/O} + a_{31} \log K/N$$

$$(2)\quad \log K/O_2 = a_{02} + a_{12} \log w + a_{22} \log \hat{K/O} + a_{32} \log K/N$$

The regression coefficients and their standard errors are presented in Table III.

In Table IV, Column (1) we find the proportion of the variance in K/O explained by

TABLE III

REGRESSION RESULTS, K/O ON SEVERAL VARIABLES*

Industry	Number of observations	Regression 1			Regression 2		
		a_{11}	a_{21}	a_{31}	a_{12}	a_{22}	a_{32}
Textiles	20	0.339	0.630	0.141	−0.398	0.964	0.105
		(0.107)	(0.385)	(0.041)	(0.483)	(0.480)	(0.065)
Lumber	23	0.377	0.307	0.089	0.129	0.484	0.384
		(0.082)	(0.183)	(0.069)	(0.251)	(0.264)	(0.099)
Furniture	22	0.489	2.623	0.102	−0.260	1.390	0.309
		(0.064)	(1.500)	(0.045)	(0.429)	(4.486)	(0.074)
Paper	30	0.584	0.185	0.119	0.872	0.414	0.136
		(0.088)	(0.125)	(0.036)	(0.497)	(0.190)	(0.057)
Chemicals	32	0.610	0.221	0.036	−0.869	0.068	0.368
		(0.101)	(0.201)	(0.066)	(0.340)	(0.289)	(0.087)
Petroleum	18	0.397	−0.116	0.137	0.125	−0.840	0.311
		(0.077)	(0.314)	(0.053)	(0.171)	(0.469)	(0.068)
Leather	17	0.454	0.145	0.220	1.173	0.199	0.380
		(0.080)	(0.161)	(0.063)	(1.138)	(0.500)	(0.114)
Primary Metals	29	0.441	0.557	0.075	0.203	0.693	0.141
		(0.079)	(0.502)	(0.060)	(1.029)	(0.812)	(0.112)
Fabricated Metals	33	0.211	0.248	0.235	−0.618	0.506	0.299
		(0.072)	(0.259)	(0.045)	(0.295)	(0.292)	(0.045)
Machinery	28	0.344	−0.728	0.202	0.467	(0.353	0.221
		(0.072)	(0.668)	(0.063)	(0.539)	(0.909)	(0.086)
Elect. Machinery	25	0.403	0.351	0.142	−0.731	0.960	0.164
		(0.092)	(0.237)	(0.044)	(0.411)	(0.259)	(0.059)
Transportation	28	0.685	0.504	0.080	1.655	0.403	0.333
		(0.075)	(0.162)	(0.050)	(1.447)	(0.364)	(0.149)

* The regression equations are described in the text. Figures in parentheses are standard errrors.

w/i, $\widehat{K/O}$, K/N. Column (3) was computed alike except that w was the factor price variable. In the columns adjacent to each of these we find the coefficients of partial determination for K/N; these values give the proportion of the total variance in K/O which could be ascribed to the effect of K/N alone. Using F ratios to judge the significance of these values, we find that in most industries (nine) there is little likelihood (5% or less) that the additional amount of variance explained by K/N might be due to chance. We conclude therefore that while in the majority of all industries the size $-K/O$ association is due, in part, to the influence of common variations in factor price ratios and/or in the output mix, size is independently associated with K/O in many industries.[15]

At this point it may be worthwhile to mention some additional hypotheses which could not be considered within the present framework, but which might contribute to further understanding of the size $-K/O$ association. First, it is possible that a still finer breakdown of output composition, if it could be done, might improve the performance of the output mix variable. Also there may be quality differences which cannot be easily captured in an output mix variable. Thus, for example, while a smaller firm may be classified into the same industry group as the larger firm, it may be specializing in cus-

[15] Regression (1) yields better results as evidenced by higher R^2 and more consistent regression coefficients. This would agree with our expectation that w/i is the more appropriate factor price variable. However, due to the specification of this variable, its estimated regression coefficients are likely to exhibit an upward bias, if capital is measured with an error term.

tom orders. Finally, using Carter's [2] concept of "imbalance because of expectations of further growth," another possibility arises. It is that, in some industries at some times, larger firms possess excess capacity while small firms typically do not. The argument being that the larger firm is more aware of the expected growth in markets and/or more able, financially, to make allowances for it in its *present* scale of plant.

THE SIZE MEASURE

In this study, as well as in the others [3, 6, 10, 12], the firm's size was measured by the value of its capital. Another approach that could be taken is the use of output per establishment (O/N) as the size measure.[16] In the light of the statistical problems introduced with the use of K/N it may, perhaps, be somewhat consoling to note that the use of O/N would have also introduced a systematic bias, though in another direction. In addition, for economic reasons the relationship of size with K/O could be expected to be different (lower) if O/N were used. We turn now to an elaboration of these points.

Recall that it was shown earlier that the use of K/N biases the observed correlation upwards. It can be similarly shown that when O/N is the size measure, the observed correlation will be biased downwards. This is because errors in the reported values of output bias K/O and O/N in opposite directions.

Even if the variables were not subject to errors of measurement, the correlation of K/O and K/N would still be higher than than of K/O and O/N, provided the two size measures are not perfectly correlated.[17] This means that only in the special case where the two size measures are essentially the same (r of K/N and O/N equals unity) will the two yield identical correlation with K/O.

In order to visualize the economic reasons which will tend to produce a lower correlation of K/O and O/N, assume that the production function for each industry is homogeneous of the first degree.[18] In such a case

TABLE IV

CORRELATION RESULTS, K/O AND OTHER VARIABLES*

Industry	Number of Observations	$R^2_{1.245}$ (1)	$r^2_{15.24}$ (2)	$R^2_{1.345}$ (3)	$r^2_{15.34}$ (4)
Textiles	20	0.765**	0.419**	0.633*	0.139
Lumber	23	0.869**	0.079	0.730**	0.439**
Furniture	22	0.883**	0.215*	0.518**	0.490**
Paper	30	0.882**	0.298**	0.718**	0.178*
Chemicals	32	0.761**	0.011	0.558**	0.010
Petroleum	18	0.867**	0.313*	0.633**	0.364*
Leather	17	0.923**	0.483**	0.753**	0.458**
Primary Metals	29	0.671**	0.059	0.266*	0.059
Fabricated Metals	33	0.723**	0.479**	0.689**	0.600**
Machinery	28	0.641**	0.297**	0.328*	0.214*
Elect. Machinery	25	0.735**	0.327**	0.561**	0.270*
Transportation	28	0.836**	0.095*	0.304*	0.171*

* Single and double stars represent statistical significance at the 5% and the 1% levels respectively.

Variable 1 is log K/O

" 2 " log w/i

" 3 " log w

" 4 " log K/O

" 5 " log K/N.

$$r_{(X-Y,X)} = \frac{\sigma_X^2 - \sum (X-\bar{X})(Y-\bar{Y})/n}{\sigma_{(X-Y)}\sigma_X},$$

and

$$r_{(X-Y,Y)} = \frac{\sum (X-\bar{X})(Y-\bar{Y})/n - \sigma_Y^2}{\sigma_{(X-Y)}\sigma_X}.$$

After some manipulations one obtains:

$$r_{(X-Y,X)} - r_{(X-Y,Y)} = (1 - r_{XY})\left(\frac{\sigma_X + \sigma_Y}{\sigma_{(X-Y)}}\right).$$

Thus, when $r_{XY} = 1$, $r_{(X-Y,X)} - r_{(X-Y,Y)} = 0$.

[16] For a comprehensive statistical discussion of a similar problem in relation to labor productivity see [9].

[17] Adhering to our previous notation where

$$X = \log (K/N)$$
$$Y = \log (O/N)$$
$$X\text{-}Y = \log (K/O)$$
$$n = \text{number of observations}$$

then:

[18] The same reasoning could be used with a production function homogeneous of a degree smaller than one.

K/O is solely dependent on K/L (the capital-labor ratio). With such a function a change in K/L alone—output being held constant—will cause a change (in the same direction) in K/O *and* in the size of the firm, as measured by its capital. On the other hand, for a positive association of O/N and K/O the requirement is more severe: there must be a change both in K/L *and* O/N. Thus a pure change in K/L is not sufficient to generate the last relationship. An extreme example for this is the situation where all observation points fall on the same iso-product curve though varying with respect to their chosen K/L. In this instance the correlation of K/O and K/N is perfect, and that of K/O and O/N is zero.

The same effect can of course be induced through differences in the output mix. Since the output size measure takes no account of output mix differences, a state producing a more capital intensive output mix will not register as having a larger size when value of output is used, unless it also produces on a larger scale of output. However, using the capital measure it will always appear to have a larger size.

SUMMARY

A positive association of size and K/O was found to be present in twelve of seventeen industries studied. There is reason to believe, however, that the observed values are somewhat biased upwards.

Of the various hypotheses suggested in other studies, the degree of vertical integration in the establishment could not play a role in this study because we dealt here with value added rather than with sales value. The age of capital was not found, almost generally, to be of explanatory value. But this may be due to the nature of the variable which is a catch-all for several, some of them contradictory, influences. The hypotheses which received support are the factor substitution and the output mix hypotheses. Nevertheless even after correcting for the influence of these two factors, size was found to exercise some independent influence on K/O in many, though not in all, industries.

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