WEALTH AND INEQUALITY IN EIGHT CENTURIES OF BRITISH CAPITALISM[1]

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Abstract. This paper constructs annual data for the aggregate wealth-income ratio, *W*-*Y*, saving, *s*, and growth, *g*, for Britain over the period 1210-2013 and tests Piketty's central hypothesis that the *W*-*Y* ratio is governed by the *s*-*g* ratio in the long run. Furthermore, Piketty's model is extended by the share of non-reproducibles in total wealth to explain the *W*-*Y* ratio during different eras of capitalism – pre-industrialization, industrialization and the knowledge economy. It is shown that savings, growth and the share of non-reproducibles in total wealth have been the fundamental drivers of the *W*-*Y* ratio over the past eight centuries.

JEL Classification: E1, E2, O4, P1
Key words: *W*-*Y* ratio, inequality, non-reproducibles versus reproducibles factors of production

1. Introduction

The aggregate wealth-income ratio, *W*-*Y*, has returned to the center-stage of economics as a key macroeconomic variable that enables one to assess income and wealth inequality, the share of income going to capital, the marginal productivity of capital, and the wealth of nations. [2] Furthermore, through labor's income share, it has implications for the entry decision of firms and for the decision of workers to join the labor market (Peretto, 2006). However, very little is known about the path of the *W*-*Y* ratio at medium and long term frequencies, the determinants of the level of the *W*-*Y* ratio or the forces underlying its evolution during various historical epochs. Following the predictions of economic growth theories Piketty and Zucman (2014) (henceforth P&Z) and Piketty (2014) argue that the *W*-*Y* ratio is in steady state driven by the s^n-g ratio, however, very little research, if any, has been undertaken

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[2] The *W*-*Y* ratio impinges directly on income inequality and the share of income going to capital , S^K, following Piketty's (2014) two laws of capitalist economics; namely, $S^K = r \times W/Y = r \times s^n/g$, where s^n is the *net*-of-depreciation saving rate and g is the growth in economy-wide net national income. Provided that the returns to fixed capital are relatively constant, it follows that the *W*-*Y* ratio is strongly positively related to inequality as high income earners chiefly receive their income from assets (Piketty, 2015a). The *W*-*Y* ratio has the advantage over other income inequality measures in that it incorporates capital gains on assets as shown in Section 5. Furthermore, Roine and Waldenström (2012) find that realized capital gains add substantially to income inequality for Sweden, Spain and Finland and that the top 1% income share in total income was on average 40% higher over the period 1990-2008 when realized capital gains were allowed for.

to rigorously examine which variables drive the *W*-*Y* ratio at and outside steady state. To fill this lacuna this research compiles annual data for aggregate wealth, saving and growth for the UK spanning eight centuries covering the Black Death, the pre-industrial era, the British Industrial Revolutions and the recent transition to the knowledge economy. The data are used to explain movements in the *W*-*Y* ratio and to test the central hypothesis of Piketty (2014) and P&Z, extended to allow for the fraction of wealth that is reproducible – tests that are *not* feasible with the data that are currently available.

The paper makes three contributions to the literature. First, it is the first study to create an ultra-long annual dataset on the *W*-*Y* ratio over the period 1210-2013, where wealth is defined as the sum of the market values of agricultural and urban land, fixed capital, precious metals, net foreign assets, intangibles, consumer durables and livestock. The data extends and modifies the 1710-2010 data created by P&Z for Britain. Their data are generated from the point estimates of various authors in the years 1700, 1855, 1865, 1875 and 1885, 1901, and 1920-2010, and are interpolated between the benchmark years using saving rates after 1855. These data are not suitable for regression analysis because they miss out the movements in the *W*-*Y* ratios between the data points and, more importantly, the pre-1920 wealth data are inferred from profits reported in tax returns under the assumption of an approximately time-invariant 'years of purchase' or, in modern language, the price-earnings (*P*-*E*) ratio. The difficulty associated with this method is that the *P*-*E* ratio is neither constant in the short run nor in steady state when non-human factors of production are non-reproducible, as discussed below. In his *Principles of Economics*, Marshall (1890) directly warns against using years of purchase as wealth estimates because they could be "very misleading" (p. 136) and very sensitive to the rate of interest. Furthermore, P&Z's estimates are highly sensitive to the choice of data source in years of large shifts in the *W*-*Y* ratio, underscoring the importance of using consistent data sources. As discussed below, the time-profile of the *W*-*Y* ratio estimated here is quite different from that of P&Z.

Second, it is shown that the s^n-g ratio or $s/(g+\delta)$, needs to be extended to allow for non-reproducible factors of production so that the long-run trend in the *W*-*Y* ratio is governed by $s/(g+\delta-\lambda\phi^{N-R})$, where λ is real capital gains on non-reproducible factors of production, δ is the depreciation rate, s the gross saving rate, and ϕ^{N-R} is the share of non-reproducibles in total wealth. The key to understanding this extension is that real capital gains cannot be made on reproducibles in a competitive environment and capital accumulation needs to come from saving; an assumption underlying the Solow growth model. However, real capital gains can be made on urban and agricultural land because it is non-reproducible – asset appreciations cannot be competed away by investment.

The introduction of the $\lambda\phi^{N-R}$ term is crucial for understanding the evolution of the *W*-*Y* ratio in pre-industrial and the post-1980 eras in which wealth accumulation and decumulation are dominated by real capital gains and losses on non-reproducible factors of production. During the British pre-industrial epoch farmland was the dominant non-human factor of production and there were no forces ensuring that the W-Y ratio was kept at a level that is consistent with the returns required by investors. In other words, the W-Y ratio was *not* governed by $s/(g + \delta)$ alone because of capital gains and losses on farmland. As Britain industrialized, fixed capital gradually became the dominant non-human factor of production, and since fixed capital is reproducible, capital gains are ruled out. The W-Y ratio starts climbing again after the first oil price shock in 1973/74 as the professional service sector, dominated by ICT, finance and other advanced service industries, have gradually taken over from manufacturing and created a high demand for urban space and, again, increased the share of non-reproducible wealth in total wealth.

Third, the paper tests which factors have driven the level of and the growth in the W-Y ratio over the past eight centuries, focusing on s, g, and ϕ^{N-R} as the fundamental drivers of the W-Y ratio in steady state. Instruments are used to deal with endogeneity and errors-in-variables.

The exercise yields some striking results. First, it is found that the W-Y ratio has been 600% on average in the ultra-long run, while fluctuating substantially around its long-run equilibrium. Second, it is shown that the W-Y ratio fell significantly below its long-run average in the approximate period 1833-1985. It is shown that the W-Y ratio started to decline in the first quarter of the 1800s as Britain industrialized and reproducible factors of production became the dominant factor of production and resulted in a declining ϕ^{N-R}-ratio; i.e. the share in total wealth of reproducible factors of production increased. The post-1980 increase in the W-Y ratio has mostly been driven by an increase in ϕ^{N-R} induced by house prices (well-known) as well as an increase in the value of fixed capital and consumer durables (not well-known). Finally, it is found that the extended P&Z model presented in this paper explains the growth as well as the level of the W-Y ratio well.

From the outset it is important to note that the data are subject to potentially large measurement errors. The measurement errors, however, need not increase much as we go back in time as the economy was back then much simpler and easier to measure; in fact, I spend more time making sense of the 20th century data than the earlier data. The data probably offer a substantial improvement over the existing wealth statistics that have been constructed in pre-WWII period as a result of the painstaking data construction effort by economic historians since the early 1960s.

The next section derives the model for the evolution of the W-Y ratio, while the main elements of the construction of the W-Y ratio are presented in Section 3. Regression analysis is undertaken in Section 4 and the growth in wealth is decomposed into volume and capital gain effects in Section 5 to gain a sense of the sources of wealth accumulation over the past millennium. Section 6 concludes.

2. Long-run determinants of the *W*-*Y* ratio

One of the key hypotheses in Piketty (2014) and P&Z is that the *W*-*Y* ratio is governed by the s^n/g ratio. However, as also acknowledged by Piketty (2014, 2015b), real capital gains on wealth are ruled out in the model. This poses problems for the model because 1) non-reproducible wealth is as important for wealth accumulation and decumulation as saving; and 2) while perpetual real capital gains and losses are generally not possible for reproducibles, they are possible for non- reproducibles, which implies that the reproducible *W*-*Y* ratio cannot increase over time, while non-reproducible wealth can. This section elaborates on these issues and extends Piketty's (2014) second fundamental law of capitalist economics to allow for the distinction between non-reproducible and reproducible wealth.[3]

Consider Piketty's (2014) second fundamental law of capitalist economics, which states that the *W*-*Y* ratio converges to the *s*-*g* ratio in the long run:

$$\left(\frac{W}{Y}\right)^P = \frac{s^n}{g}, \tag{1}$$

where, as mentioned in the Introduction, s^n is the net saving rate; the superscript *P* refers to Piketty's model; and *g* is the growth in total real net national income, NNI, where NNI is GDP minus depreciation of fixed capital plus net foreign income from labor and capital. Time-subscripts are omitted for simplicity. In this growth model population growth lowers per capita income and, therefore, the *W*-*Y* ratio because it dilutes per capita capital stock. Focusing on inequality persistence, Piketty (2014) argues that population growth reduces the *W*-*Y* ratio because it dilutes inheritance.

Eq. (1) is mathematically equivalent to the model governing the *W*-*Y* ratio in the Solow growth model:

$$\left(\frac{W}{Y}\right)^S = \frac{s}{g+\delta}, \tag{2}$$

where δ is the depreciation rate; *s* is the gross saving rate; and the superscript *S* refers to Solow. This equation has the advantage over Eq. (1) in that it is less sensitive to fluctuations in *g*, converges to less extremely high levels in the *W*-*Y* ratio response to low values of *g* and one need not account for endogenous responses in gross savings rates as *g* changes, as pointed out by Krusell and Smith (2015).

Eqs. (1) and (2) are derived under the assumption that *real* market prices of non-human factors of production remain constant over time and, therefore, that real wealth is accumulated through saving

[3] See Peretto and Seater (2014) for a complete theory of the relationship between endogenous growth and non-reproducible factors of production.

and not through real capital gains on assets. This assumption is defensible for reproducible factors of production, but not for capital gains on farmland in the pre-industrial period when agriculture was still the dominant producing sector nor, more recently, for urban land, which has become an important source of wealth in the knowledge economy. In this context it is worth noting that the continuously increasing long run trend in real share prices in many countries is not a reflection of increasing value of units of capital but that some of the earnings of the firm is retained in the company for future investment or share buy-backs. In other words, the capital base of each share increases over time because of retained earnings and share buy-backs that increases the capital claim per share (see, for an analytical exposition, the classical article of Miller and Modigliani, 1961).

Incorporating expected capital gains into the capital accumulation equation yields the following *W*-*Y* ratio:[4]

$$\left(\frac{W}{Y}\right)^* = \beta^* = \frac{s}{g+\delta-\lambda\phi^{N-R}}, \tag{3}$$

where λ is real capital gains on non-reproducible factors of production; $\lambda_t = \Delta \ln P_t^{N-R}$; $P^{N\text{-}R}$ is the real price of non-reproducibles; ϕ^{N-R} is the share of non-reproducible factors in total wealth; and the asterix refers to steady state.

This model is used in the empirical section to explain which structural factors determine the *W*-*Y* path in steady state. The importance of the ϕ^{N-R} term is that unit values of non-reproducible production factors can perpetually increase or decrease over time, while the real unit value of reproducible production factors cannot increase in equilibrium because earnings per unit of capital are kept down by investment of existing and new firms provided that real prices of fixed capital stock are constant. In other words the supply of reproducibles is elastic while the supply of non-reproducibles is inelastic.

The assumption that the real value of fixed capital stock is constant over time may not hold because investment-specific technological progress changes the price of capital goods relative to consumer goods (Greenwood *et al.*, 1997; Karabarbounis and Neiman, 2014). Investment-specific technological progress reduces the real value of machinery and equipment and, as a result, lowers the real value of the existing stock of machinery and equipment. In other words, the real value of the existing fixed capital assets are reduced by creative destruction in the presence of investment-specific

[4] This model is derived as follows. Consider the extended wealth accumulation model:

$$\dot{W} = sY + \lambda W^{N-R} - \delta W,$$

where $W^{N\text{-}R}$ is the value of non-reproducible factors of production (mostly urban and agricultural land). Combining this accumulation model with the total-differential of $w = W/AL$ with respect to time yields Eq. (3) in steady state.

technological progress and the *W-Y* ratio for fixed capital stock decreases in the same way as capital losses on non-reproducibles through the λ-term. This feature is not explicitly accounted for in Eq. (3) through a capital loss term on reproducible capital in the denominator, because the ratio of the investment deflator and the GDP deflator has been relatively constant over the past 250 years for Britain (earlier data are not available). This probably reflects that investment-specific technological progress is not sufficiently accounted for in the construction of the investment deflators and that machinery and equipment capital stock, which is the principal source of investment-specific technological progress, has been a small fraction of total reproducible capital until recently.

To gain better insight into distinction between prices of reproducible and non-reproducible assets consider the real price of *one unit* of a perpetual asset, q^i:

$$q^i = \frac{E^i}{r^i - g^{Ear,i}}, \tag{4}$$

where E^i is real earning per unit of capital for asset i; r^i is the required annual rate of return that is specific to asset i in this particular risk class; and $g^{Ear,i}$ is the expected infinite annual growth in real earnings per unit of asset.

Under perfect competition in the goods market Eq. (4) collapses in steady-state to the following equation for the market price of one unit of reproducible capital, signified by the subscript *R*:

$$q^R = \frac{E^R}{r^R} = \frac{MP^R}{r^R}, \tag{5}$$

where MP^R is the marginal product of reproducible capital. Growth in real earnings per unit of capital in this equation is pinned down to zero by a combination of capital deepening and diminishing returns to capital (see for a general equilibrium exposition, Madsen and Davis, 2006).

Figures 1 and 2 illustrate why capital gains are temporary for reproducibles (fixed capital) and permanent for non-reproducibles (land). Figure 1, which is a phase diagram constructed from a standard optimal control exercise, shows the share market response to an earnings shock. Fixed capital stock, *K*, is on the horizontal axis, Tobin's q is on the vertical axis and the $\dot{K} = 0$ schedule is given by Eq. (5). An increase in earnings per unit of capital shifts the $\dot{q}_0 = 0$ schedule to the right to $\dot{q}_1 = 0$ and results in a short-term capital gain but no long-run capital gain. The increasing earnings per unit of capital lead to a jump in stock prices as signified by Tobin's q (vertical jump to the stable manifold under perfect foresight and to the $\dot{q}_1 = 0$ schedule in the myopic market). Since $q^R > 1$ the jump in Tobin's q results in capital deepening, which in turn, due to diminishing returns to capital, reduces earnings per unit of capital until they are back to their steady state level at $MP^R = \overline{MP}^R$. In other words, the elastic supply of fixed capital has kept the equity price for one unit of capital in real terms down at acquisition costs in steady state and the earnings shock has only had temporary effects on the value of capital.

For non-reproducible capital the story is quite different. There are no factors of production to keep earnings per unit of capital down to required returns to an asset belonging to this particular risk class. Thus the real value of land, q^T, the most prominent non-reproducible asset, is given by:

$$q^T = \frac{E^T(T)}{r^T - g^{Ear,T}}, \tag{6}$$

where T is farmland area. The influence of maintenance costs and the value of prestige and political influence on land prices are excluded from the model (see Data Appendix for a discussion of these factors). It is assumed that the earnings per hectare of farmland at the macro level are a declining function of the farmland area, $E^T(T)' < 0$, because it lowers real prices of food and because earnings declines as more marginal land is cultivated.

Figure 1. Reproducible wealth

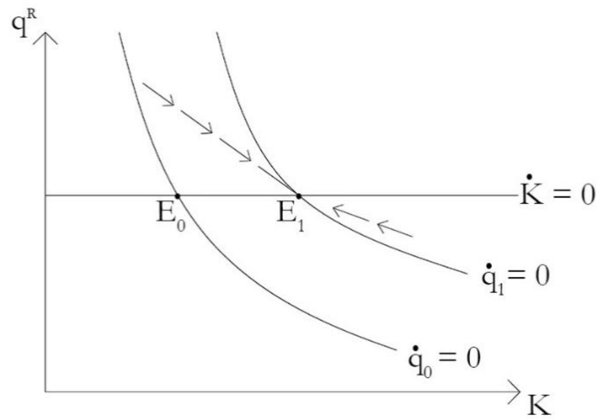
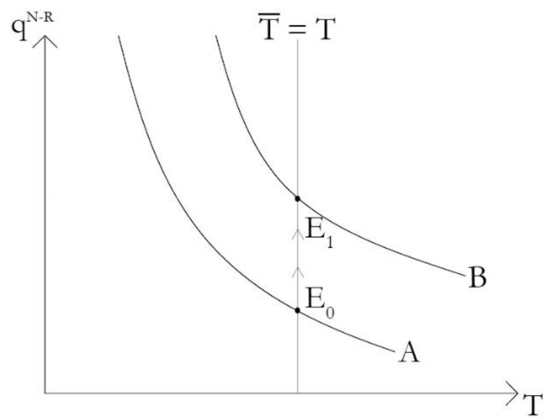


Figure 2. Non-reproducible wealth



Eq. (6) is displayed as the downward sloping line in Figure 2 under the assumption that earnings are always positive for $T \leq \bar{T}$. The supply of land is assumed to be fixed at $T = \bar{T}$. The equilibrium price of land, q^T, is at the point E_0. A positive population shock, say due to a sudden inflow of migrants, increases the demand for food and shifts the A-schedule to the right to the B-schedule and the farmland market jumps up to the point E_1. In this closed economy model, which characterized Britain before the Corn Laws were repealed in 1846, there are no equilibrating factors that could bring land prices down to their initial equilibrium and, as we have seen past 1844, real farmland prices have continued to increase, despite increasing access to imports of foreign food, because the demand for urban land has been increasing and because of productivity advances in agriculture. In other words the price of land is almost entirely determined by demand because the supply of land in inelastic.

From this exposition it follows that the distinction between reproducible and non-reproducible capital is crucial for the dynamics of the W-Y ratio. The Solow model applies only to reproducible capital for which the λW^{N-R}term is zero. When there is an inelastic supply of land, as in pre-industrial Britain or as is the case in urban areas in today's BRITAIN, an increasing population and productivity advances will put upward pressure on urban and agricultural land values and increase the share of non-reproducible wealth in total wealth.

3. Data

Data are constructed over the period 1210-2013, where 1210 signifies a period at which most of the underlying data starts becoming available. Only the main principles behind the data construction are presented here since the data is comprised of more than 200 different raw data series, some of which are directly available and some that are constructed from various books and periodicals. The Data Appendix provides a detailed discussion of data sources, discussion of concepts relating to measurement, validity of the data, and a comparison with previous wealth estimates.

3.1 The W-Y ratio: Computational Method

According to Campion (1939) there are essentially three methods that have been used to compute wealth in the past. The first method scales up inheritance income flow by mortality rates (rarely used); the second creates wealth stocks from profit flows (dominant method until recently and used by P&Z before 1920); and the third method multiplies real wealth stock by market unit prices (currently used by statistical agencies and used here). The guiding principle behind the construction of the wealth data constructed here is that 1) commercial fixed capital is valued by the volume of fixed capital stock multiplied by Tobin's q; 2) values of urban and farmland are valued at market prices; and 3) the value of residential and farm structures, consumer durables, gold and silver and livestock are valued at acquisition costs.

More specifically, national wealth, W^N, and private wealth, W^P, are computed as:

$$W_t^N = q_t P_t^K K_t^{Fix} + P_t^K K_t^{Agr} + P_t^R K_t^{Res} + P_t^K K_t^{Dur} + P_t^{Agr} T_t^{Agr} + P_t^{Urb} T_t^{Urb}$$
$$+ P_t^L S_t^{Live} + P_t^{G\&S} G\&S_t + NFA_t + P_t^I S_t^{Int}, \qquad (7)$$

and

$$W_t^P = W_t^N - W_t^G = W_t^N + Debt_t^G - P_t^K K_t^G\ , \qquad (8)$$

where K^{Fix} is the stock of fixed non-residential and non-agricultural capital; P^K is the price of fixed capital (non-residential investment price deflator); q is Tobin's q; K^{Agr} is agricultural fixed capital stock; K^{Res} is the stock of residential structures; P^{Res} is the price of residential stock (residential investment deflator); K^{Dur} is the stock of durable goods; T^{Agr} is the agricultural land area (arable plus pastoral land); P^{Agr} is the price of agricultural land; T^{Urb} is the size of the urban land; P^{Urb} is the price of urban land; S^{Live} is the livestock; P^L is the price of livestock; $G\&S$ is the quantity of monetary and non-monetary gold and silver; $P^{G\&S}$ is the price of gold and silver; NFA is the value of net foreign assets; S^{Int} is intangible stock; P^I is the price of intellectual knowledge; W^G is government wealth, K^G is government fixed capital; and $Debt^G$ is net government debt.

I have the following comments to Eqs. (7) and (8). First, the wealth coverage is broader than that of P&Z. P&Z omit gold, silver, livestock, intellectual knowledge stock and durables from their estimates. As shown below it is crucial to include gold, silver, and livestock in the wealth estimates in the pre-1900 period where their values totaled between 100% and 200% of NNI, and these items are, after all, wealth.

Second, the value of housing capital is decomposed into structures, K^{Res}, and the underlying land, T^{Urb}, acknowledging that the value of structures is determined by factors that are quite different from that of urban land. The value of structures is determined by acquisition costs because they are reproducible, while urban land prices are determined by the shortage of urban land and agricultural land prices because land is a non-reproducible factor of production. Multiplying the stock of structures by a house price index, as done in most of past wealth estimates, is, therefore, only a good approximation if the value of land in total housing wealth is low. Here the price of structures is assumed to be determined by acquisition costs while the price of urban land is assumed to move proportionally with prices of agricultural land before data on house prices become unavailable in 1895, because farmland prices determine prices of urban land and the urban fringes and, through ripple effects, feed through to the center of towns and cities. The data are spliced with the balance sheet accounts of ONS (Office of National Statistics) after 1985 as detailed in the Data Appendix.

Third, non-residential, non-agricultural fixed capital stock is converted into market values by multiplying the value of fixed capital at acquisition costs (real capital multiplied by the investment deflator) by Tobin's q to account for medium-term deviations of market prices from their acquisition costs. Tobin's q is measured as the deviation of the log of real share prices from its time trend, where stock prices are deflated by the GDP deflator. The trend is removed from stock prices to filter out the increase in stock prices that is driven by accumulated retained earnings per share and share buy-backs. As noted in the previous section, real stock prices increase over time almost only because of retained earnings that are reinvested in the company. If companies do not retain earnings or buy back stocks, real stock prices would collapse to Tobin's *q* (see, for exposition, Madsen and Davis, 2006). Under the assumption that the retention ratio and the stock returns are both relatively constant, the log of real stock prices will fluctuate around a trend and the deviation of the log of stock prices around this trend will reflect the log of Tobin's *q*. Empirically, Barro (1990) finds that the deviation of real stock prices from their trend is a good approximation of Tobin's *q*.

Fourth, it is not clear whether government debt should be included as private wealth as the debt ultimately has to be paid by the taxpayers (Barro, 1974). Following Piketty (2014) and P&Z, government debt is included as a part of private wealth. The distinction is not important for the principal results in the empirical section. Finally, wealth is normalized by NNI as opposed to GDP because it is the income that is ultimately left for compensation to labor and capital.

3.2 W-Y data

The stock of knowledge and fixed capital are mostly based on the perpetual inventory method. Fixed capital stock, K^{Fix}, is constructed from Feinstein's (1988) national account estimates from 1760, while it is constructed as the sum of capital stock constructed for industry, ships, and agriculture before 1760. The pre-1760 fixed capital stock in agriculture, K^{Agr}, the all dominating production sector in medieval Britain, is estimated as the sum of the value of working animals (oxen and horses), the change in the weighted number of non-sheep livestock (pigs, cows, oxen and horses) and crop land under the assumption that the barn/stables/byre capacity is proportional to the size of each animal cohort and crop land (for hay storage) and spliced with Feinstein's (1988) estimates in 1760.

Investment in land improvements and enclosure is included in the saving estimates below but not in the wealth estimates, because these investments are likely to have been capitalized in agricultural land values. Livestock wealth is computed as the sum of the number of horses, oxen, cattle, pigs and sheep multiplied by their respective prices. The value of the stock of ships prior to 1760 (data are available from Feinstein, 1988, after that date) is computed as the estimated total tonnage of ships multiplied by the weighted average of unit labor costs of skilled and unskilled building workers and timber prices. Data on the estimated total tonnage of ships prior to 1760 is collected from several sources: see the Data Appendix for details. Note that canal and turnpike capital is not accounted for before 1760 because their value was negligible back then (Deane, 1961).

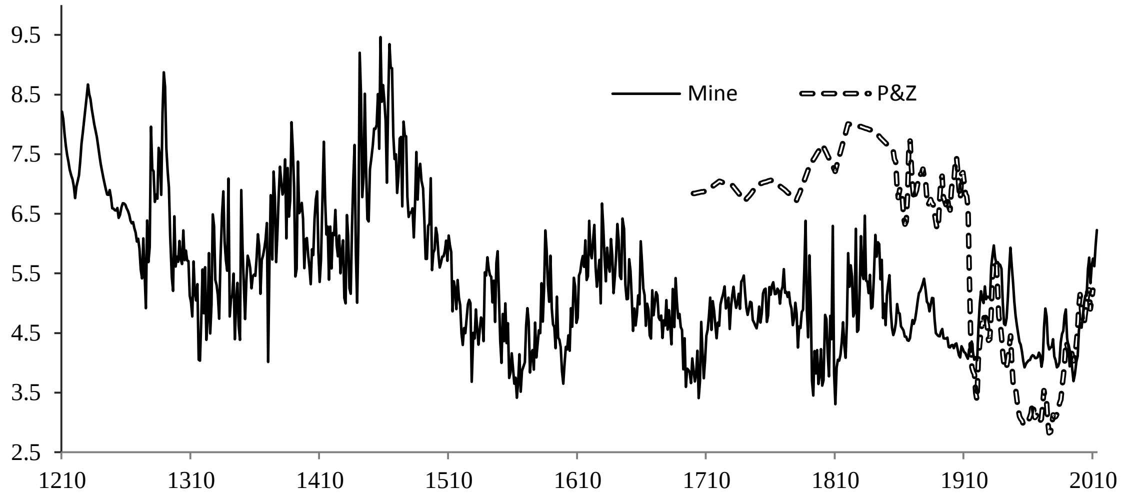
The value of intangible capital is available from ONS (Office of National Statistics) over the period 2008-2013 and is backdated using stock of patents (1557-2008) and the stock of great inventions (1210-1557); both computed by the perpetual inventory method. The data are reflated using the unweighted geometric average of wages and the GDP deflator following Madsen (2008). Consumer durables are estimated by applying the perpetual inventory method to the consumption of durables, which in turn is estimated as the share of durables in total consumption multiplied by consumption up to 1900, when the data on consumption of durables become available from national accounts estimates. Consumption is estimated residually as $C = Y^N - G - S$ before 1900, where C is consumption expenditure, Y^N is nominal GDP, G is government consumption, and S is gross saving. The estimation method for saving is detailed below in Section 3.4.

The value of farmland is computed as the per hectare price of land multiplied by farmland hectares. The urban land area is estimated to be proportional to urban population multiplied by a constant (no official estimates are available and previous estimates are all guestimates). The hectare price of farmland is directly available while the value of urban land computed residually as detailed in the Data Appendix, where the value of urban land is assumed to follow real house prices after 1895 and farmland prices before then.

3.3 The evolution of the W-Y ratio

Aggregated and disaggregated *W-Y* ratios are displayed in Figures 3-9. The *W-Y* estimates of P&Z are also included in Figure 3, but will first be discussed in Section 3.4. A remarkable feature of the aggregate *W-Y* ratio in Figure 3 is that the *W-Y* ratio has fluctuated around a relatively constant mean of 600% in the ultra-long run, and there is a tendency for the *W-Y* ratio to revert towards a slow moving steady state given by Eq. (3) as shown below (see, Peretto, 2015, for theory of the long run *W-Y* ratio in agricultural as well as industrial economies). Another remarkable feature of the graph is that the *W-Y* ratio fluctuates wildly at medium-term frequencies of one or two centuries' duration. The medium-term cycles, which peak in the approximate years 1210, 1457, 1629, 1833, and, most recently, in 2013, are only temporary because the *W-Y* ratio has been outside its steady state during these periods. The three first peaks in the *W-Y* ratio (1210, 1457 and 1629) were driven up by increasing values of agricultural capital stock, farmland and livestock, whereas the last two peaks (1833 and 2013) have been propelled chiefly by increasing values of non-agricultural fixed capital stock (inclusive of dwellings) and consumer durables.

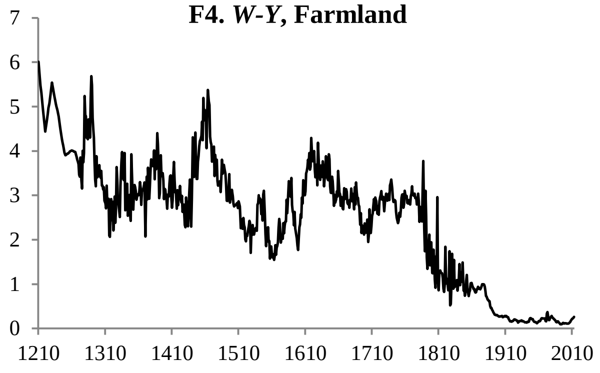
F3. W-Y Total, Private



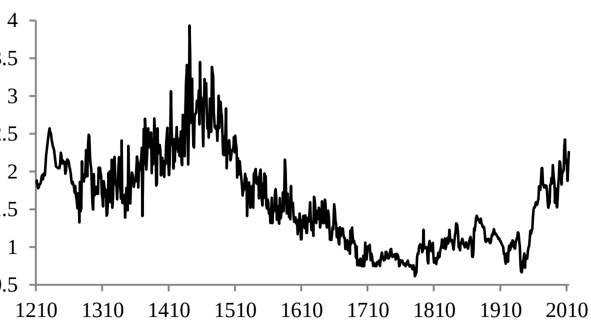
Notes. The data of P&Z are annual after 1855 and before then linearly interpolated from decennial to annual values.

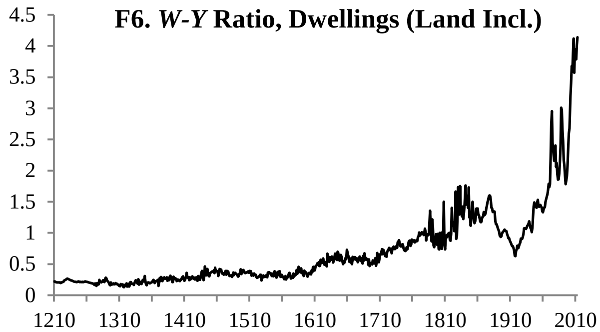
Two remarkable features of Figure 3 is the marked increase in the *W-Y* ratio from the mid-1300s to the mid-1400s and the U-shaped *W-Y* ratio over the past two centuries, only interrupted by increasing war debt absorbed by the private sector during the period 1915-1945. The increase over the period from the mid-1300s to the mid-1400s was partly due to the Plague-induced reduced income – a trend that was first reversed in the second half of the 1400s as the population started to recover from its low. The decline in the *W-Y* ratio from the beginning of the 1800s up to WWI was predominantly the result of declining non-reproducible wealth in total wealth during industrialization (see Figure 10), while the recent increase in the *W-Y* ratio has been driven predominantly by increasing real prices of houses and increasing stock of consumer durables.

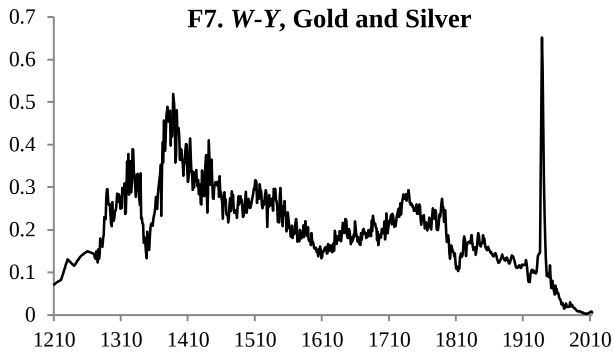
F4. *W-Y*, Farmland

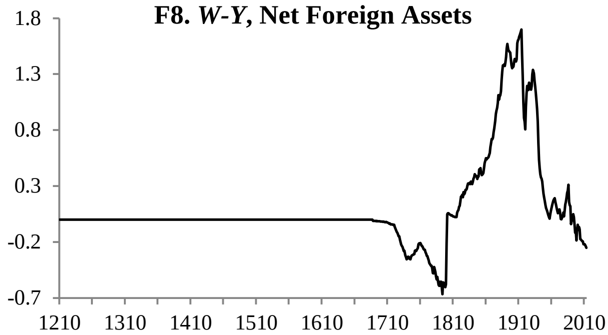


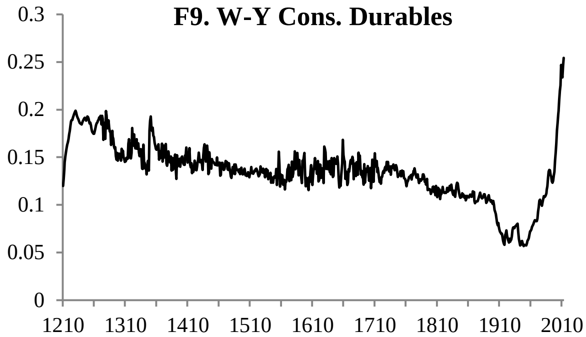
F5. *W-Y*, Non-Res. Fixed Cap. + Livestock

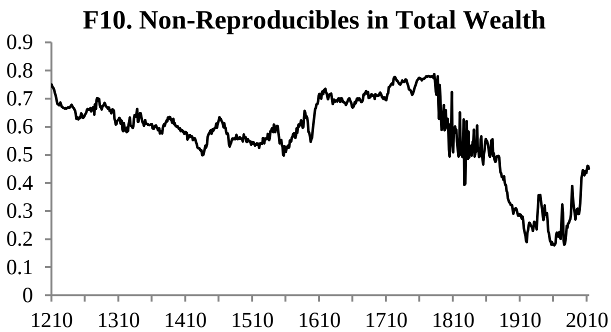


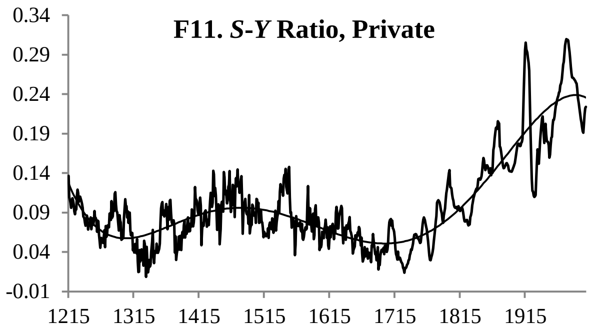












Notes. Non-residential fixed capital includes livestock, farm buildings, industrial buildings and structures and government fixed capital. Housing wealth includes urban land and structures. The saving rate in F11 is estimated as a 9-year centered moving average of annual private saving rates and the trend line is a 6th order polonomial.

Figures 4-9 display the evolution of the most important disaggregated *W-Y* ratios. Consider first the farmland *W-Y* ratio, which fluctuated around 300% up to the end of the Napoleonic War in 1814 (Figure 4). The sharp decline starting in the first quarter of the 1800s signals the end of agriculture as the driving force of economic progress. This was not trigged by an agricultural depression but was almost entirely driven by industrialization that, through Engel's law, reduced the share of agricultural income and employment. The increasing population growth during the 1800s did not push land prices up through higher food prices because the increasing demand for food was met through imports. The second major decline in the agricultural *W-Y* ratio occurred during the Great Agricultural Depression 1873-1896, which contributed to the more than 50% reduction in the farmland *W-Y* ratio over the period 1866-1900 (see, e.g., Fletcher, 1961). The farmland *W-Y* ratio never recovered from the agricultural depression as industrialization gained further momentum and cheap overseas imports kept agricultural prices low.

The *W-Y* ratio for fixed non-residential capital, displayed in Figure 5, fluctuates around 200% up until the late medieval period, showing that fixed capital was a significant factor of production in pre-industrial Britain (noting that land improvement and enclosure capital are *not* included in the wealth estimates because, as noted above, they are capitalized in land values). The marked increase in the fixed non-residential capital *W-Y* ratio over the approximate period 1350-1450 is driven by an increase in livestock, barns, and stalls. The reduced population following the Black Death is likely to have induced farmers to switch from labor intensive arable cultivation to less labor intensive pasture (Clark, 2001). Land cultivation was not only highly labor intensive during the harvest but also during the periods of plowing, weeding, soil manuring etc. In other words, provided labor and crop land were complements while labor and pastoral land were substitutes, the Plague-induced increase in real wages gave landowners an incentive to substitute animal production for corn production. This scenario is consistent with the time-profile of real wages: farmworkers experienced a two-fold increase in real wages over the period 1347-1350 and a 33% reduction over the period 1350-1352 (using another dataset, Clark, 2001, finds that real wages of agricultural workers did not increase). This process was compounded by reduced lending rates that reduced the costs of investing in livestock and farm buildings (Madsen, 2016). The subsequent increase in population and per capita income reduced the capital-income ratio up to around 1700. The sharp increase since 1942 is mostly a result of capital deepening and a two-fold increase in Tobin's *q*.

The dwellings *W-Y* ratio in Figure 6, computed as the sum of the value of urban land and residential structures, shows a growing trend over the period 1210-1875; driven mainly by increasing urbanization and hence, increasing urban land values. Despite continuing urbanization, the ratio is driven down by declining land prices over the period 1875-1920; predominantly induced by ripple effects of declining prices of farmland and the high inflation rates during and immediately after WWI. The inflation probably eroded real land prices, in the short run, because the inflation over the period 1914-1920 was unexpected. Furthermore, house buyers probably had little understanding of the inflationary consequences of the sharply increasing money supply following the suspension of the Gold Standard at the outbreak of WWI. The increase in the dwelling *W-Y* ratio in the interwar period was mainly a result of the return of real house prices to their 1900 level, and the increase in the post-1982 period has been a result of increasing urbanization, reduced cost of capital, and easier access to credit, coupled with a relatively inelastic supply of urban land.

Gold and silver were important sources of wealth in the 500-year period from 1280 to the onset of the Napoleonic war in 1794 and the *W-Y* ratio fluctuated around 30% during this period (Figure 7).

Gold and silver gradually lose their economic status after 1794 as other financial instruments take over as means of goods exchange, saving and credit provision (the spike in 1938 was a result of an outflow of gold from the US in the fear of another devaluation of the USD by the Roosevelt administration, Irwin, 2012). The ultra-long-term declining trend in the gold and silver *W-Y* ratio has been dominated by declining real prices of gold and silver. Contrary to popular belief, the real CPI deflated gold prices have lost almost 4/5 of their value over the period 1210-2013.

The consumer durable *W-Y* ratio declines slowly but steadily over the period from 1225 to 1905, drops rapidly over the period 1905-1918 and stays low at around 6% until 1950 (Figure 9). It has since been increasing and has grown enormously since 1995, particularly driven by the demand for luxury cars, and it is currently at the 20% level, signifying that it is now a semi-significant storage of wealth.

Finally, Figure 10 shows that the share of non-reproducibles in total wealth was relatively constant in the range of 0.5-0.8 up to 1790 as farmland was the main source of wealth. Thereafter, the non-reproducibles share declines massively from 0.8 to its nadir of 0.2 in 1920 as the economy industrializes and fixed capital stock gradually takes over from agricultural land as the most important non-human factor of production. Since the beginning of the 1980s the ratio has more than doubled as the economy has entered a new phase of economic development in which the service sector has taken over from manufacturing as the prime mover of growth and, therefore, increased the pressure on urban land prices as professionals have been attracted by the inner city circles.

3.4 Comparison with the data of P&Z

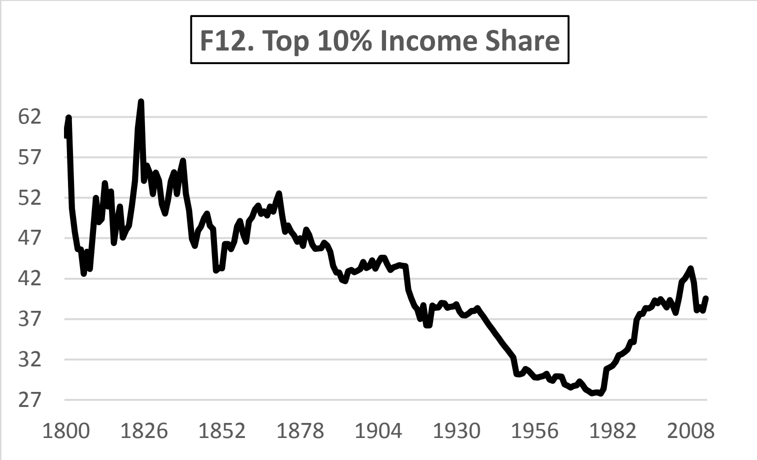
Figure 3 shows that the P&Z time profile of the *W-Y* ratio differs from mine, particularly the timing of the great decline in the *W-Y* ratio over the past two centuries. While the decline in the *W-Y* ratio started in around 1833 in my estimates, it started in 1913 in P&Z's data. My *W-Y* ratio decreases by approximately 220 percentage points over the period 1830-1902 and thereafter increases up to 1932. P&Z's *W-Y* ratio, in contrast, is relatively stable at around 700% up to 1913 and decreases no less than 300 percentage points over the period 1913-1920. There are two main reasons for this massive difference in the timing and the size of the great decline. The first reason pertains to the *W-Y* ratio for farmland and corporate fixed capital stock and the other pertains to the method used by P&Z to merge 'old' wealth estimates with newer national account estimates.

The *W-Y* ratio for farmland used here starts its decent around 1833, whereas the principal decline in P&Z's data is over the period 1913-1920. The data for the value of farmland used here have been cross checked with several data sources and historical narratives in the Data Appendix, and it is concluded that the data used in this paper is probably of better quality than that used by P&Z,

particularly because the data here are created from time-series data that have been carefully constructed using consistent sampling methods and actual market values. The data sources utilized by P&Z are mostly created independently over time from earnings records that are converted to wealth under some assumptions about 'years of purchase' and, as such, are only approximations that may be time-inconsistent.

For corporate capital stock, the principal difference pertains to the size of the decline over the period 1913-1920. The *W-Y* ratio for corporate fixed capital stock at market values used here only declines by 16% over the period 1913-1920 and subsequently recovers. The corporate capital data are based on Feinstein's (1972, 1988) estimates, which are considered to be the most authoritative sources of British historical statistics on fixed capital formation, and are converted into market values using Tobin's *q*. The relative changes in the *W-Y* ratio for the corporate capital used here are consistent with the changes in stock market capitalization-GDP ratio computed by Madsen and Ang (2016, forthcoming), which can be used as a rough indicator of the *W-Y* ratio for fixed corporate capital stock. The market capitalization-GDP ratio declines 60% (36%) from its highest recorded value in 1910 to 1918 and increases 90% (80%) over the period 1918-1925, where the numbers in parentheses are computed from the *W-Y* data for corporate capital used here. The evolution of the market CAP is, to a large extent, consistent with the data used here.

The year at which P&Z transit from using 'old' wealth estimates to national account data is more critical for the timing of the jump in the *W-Y* ratio. The key is that the 'old' wealth estimates, which are chiefly based on profit flows and assumptions about 'years of purchase', are well in excess of wealth data based on the method used in this paper and the post-1920 data used by P&Z. This implies that the jump in the *W-Y* ratio depends almost entirely on the choice of the transition period from the 'old' to the 'new' method. P&Z, for example base their 1913 figures on Campion (1939), while they use mostly national account data from 1920, in which non-residential capital stock, for example, is based on Feinstein's (1972) estimates. P&Z would have found a relatively unaltered *W-Y* ratio over the period 1913-1920 had they instead relied on the estimates of Campion (1939) in 1928 and 1934 or Feinstein's (1972) wealth estimates in Table 46 up to 1920 (the same conclusion would apply if other wealth estimates were used, as discussed in the Data Appendix). The upshot of this is that the decline in the *W-Y* ratio occurs at the time at which one transits from 'old' to 'new' wealth estimates and that the sudden and marked jump in the *W-Y* ratio over the period 1913-1920 in P&Z's data is an artefact of the data construction.



Finally, the time-profile of the top 10% income share data of Atkinson and Leigh (2013), backdated to 1800 by income share data, shown in Figure 12 is consistent with the finding in this paper that the *W*-*Y* ratio started to decline in the 1830s (the 5% and 1% top income share data show almost identical time-profiles). Furthermore, the decline in the top 10% top income over the period 1913-1918 does not particularly stand out. The 6.6 percentage point decline over this period was probably larger than the decline in the *W*-*Y* ratio because of significantly negative returns on gold, silver, government bonds, and net foreign assets during the same period (see the Data Appendix, for documentation).

3.5 Saving

Saving is estimated as follows:

$$S_t^P = I_t^{Tot} + I_t^{Live} + CA_t + I_t^{G\&S}, \tag{9}$$

$$S_t^N = S_t^P + S_t^G, \tag{10}$$

where S^P is gross private saving; S^N is gross national saving; S^G is government saving; I^{Tot} is total nominal gross fixed capital formation at acquisition costs; I^{Live} is nominal investment in livestock at acquisition costs; *CA* is the current account surplus at current prices; and $I^{G\&S}$ is the nominal investment in monetary and non-monetary gold and silver. Government saving is the sum of its primary surplus minus interest on its debt and investment in fixed assets. The saving given by Eq. (9) extends conventional saving estimates by investment in livestock, gold and silver to make saving consistent with the wealth estimates. Furthermore, investment in livestock, gold and silver is part of saving because they are generated from income and are not yet consumed.

Total fixed capital formation is available from Feinstein (1988) after 1760. Before then, I^{Tot} is computed as:

$$I_t^{Tot} = I_t^{Man} + I_t^{Ships} + I_t^{Dwel} + I_t^{Agr} + I_t^{Land}, \quad (11)$$

where I^{Man} is manufacturing investment; I^{Ships} is investment in ships; I^{Dwel} is investment in residential buildings; I^{Agr} is investment in agricultural buildings (barns/stalls/byres); and I^{Land} is investment in land improvement, enclosure and fencing. Investment in sector X, I^X, where X = *Man*, *Ships*, *Dwel*, *Agr* and *Land*, is recovered from the equation, $I_t^X = K_t^X - (1-\delta^X)K_{t-1}^X$, where δ^X is the depreciation rate for sector X's fixed capital stock. Details of the data construction are relegated to the Data Appendix.

A 9-year centered moving average of private saving rates is displayed in Figure 11, where savings rates are estimated as private gross saving divided by NNI. The pre-1830 saving rates fluctuate around the 7.5% mark and, for the years surrounding 1688, are consistent with King's (1936) estimate of 4.1% in 1688, and the increasing saving during the 1700s is consistent with Deane's (1961) proposition that saving was relatively abundant during that period. Using, as a benchmark, Lewis' frequently cited phrase, that an economy that was "…previously saving and investing 4 or 5 percent of its national income or less, converts itself into an economy where voluntary savings is running at about 12 to 15 percent of national income…" (Lewis 1954, p. 155), Britain's saving rate was spectacular in the pre-industrial period and a culture of thriftiness and investment may have contributed to laying the foundations for nation building and the British Industrial Revolution.

The trend savings rate increases from the onset of the First Industrial Revolution around 1760 up to 1913 and, thereafter, fluctuates around 20%. The increase may have been partly driven by the technological progress that, through Tobin's *q* mechanism, leads to capital accumulation. Institutional quality may also have played a role in the increasing saving rates, as highlighted by North and Weingast (1989), in that credible institutions with adequate contract enforcement increased investor confidence. Most importantly, as demonstrated by Laitner (2000), the growing share of reproducibles in total wealth during the industrialization period has increased the need for saving through deferred consumption. Households built up their asset positions through capital gains on farmland in the pre-industrialization period so there was less need for saving through deferred consumption. However, in the absence of capital gains on reproducibles, as shown in Section 2, households need to build up their asset positions almost entirely through deferred consumption in an economy in which reproducibles dominate wealth.

4. **Empirical estimates**

This section examines whether 1) there is a significant relationship between W-Y and $s/(g_t + \delta_t - \lambda_t\phi_t^{N-R})$; and 2) whether the growth in the W-Y ratio is governed by $s/(g_t + \delta_t - \lambda_t\phi_t^{N-R})$ and the deviation of W-Y from its steady state. OLS and 2SLS regressions are presented.

4.1 Level regressions

The following stochastic counterpart of the log-transformation of Eq. (3) ($W/Y = s/(g_t + \delta_t - \lambda_t\phi_t^{N-R})$) is estimated over the period 1221-2013:

$$ln\beta_t^X = \beta_0 + \beta_1 ln\beta_{t-1}^X + \beta_2 \ln(s_t^X + \kappa_1) + \beta_3 \ln \Psi_t + \varepsilon_{1,t}, \tag{12}$$

where

$$\Psi_t = g_t + \delta_t - \lambda_t\phi_t^{N-R} + \kappa_2.$$

Here, s^X is the national, (s^N), or the private, (s^P), gross saving rate; $\phi^{N\text{-}R}$ is the fraction of non-reproducible wealth (farmland, private and public urban land, gold and silver) in total wealth in market values; δ_t is the weighted average of depreciation rates of fixed capital stock excluding consumer durables; κ_1 and κ_1 are positive constants that are sufficiently large to ensure that the terms are positive at all times before they are logged; and ε is a stochastic error term.

The depreciation rate is allowed to change over time along with the changing composition of fixed capital, where the depreciation rate is, for example, set to 3% for building and construction stock and 17% for machinery and equipment stock (detailed depreciation rates are provided in the Data Appendix). Although gold and silver are in some sense reproducibles, they are included as a part of non-reproducibles because they are not reproducible at factor costs and, therefore, their price does not follow the general price level. The model is estimated using annual overlapping data in 10-year averages to smooth out random shocks and to allow for adjustment towards the steady state. Robust standard errors are used to correct for heteroscedasticity and the 9-year moving average created in the error terms. A 10-year lag of the dependent variable is included in the regression to capture the persistence in the W-Y ratio.

4.1.1 OLS regressions

The results of estimating Eq. (12) are presented in Table 1. The coefficients of the saving rates are positive and highly significant in the baseline regressions in columns (1) and (2), and the long-run elasticities are 0.53 for s^N and 0.40 for s^P, which are approximately half of their predicted values;

probably reflecting an errors-in variables bias. The coefficients of Ψ are also highly significant and have their expected negative signs. The long-run elasticity of Ψ is -1.27 in the β^N-regression in column (1), which is not far from its predicted value of one. For private saving, β^P, is -1.90 in the long run (column (2)); however it is much closer to its expected value in the IV regressions below.

Regressing instead the P&Z model in which the $\lambda_t \phi_t^{N-R}$term is omitted from Ψ, yields the regressions in columns (3) and (4) (the P&Z and Piketty models are, strictly speaking, formulated in net terms; here it is in gross terms). The coefficients of s^X remain significant in one case and the coefficients of $(g_t + \delta_t)$ are significant in both cases and have their expected negative signs; however, the statistical significance of the coefficients of $(g_t + \delta_t)$ is markedly lower than that of Ψ in the baseline regressions in columns (1) and (2). Furthermore, the long-run coefficients of $(g_t + \delta_t)$ are on average -0.17, which is well below the predictions of the P&Z model. Extending the P&Z model with the $\lambda_t \phi_t^{N-R}$ term yields highly positive coefficients of $\lambda_t \phi_t^{N-R}$, as predicted by the hypothesis of this paper (columns (5) and (6)), while the parameter estimates of $(g_t + \delta_t)$ and s_t remain unaffected by the inclusion of the interaction term.

Table 1. Parameter estimates of Eq. (12).

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Dep Var	$\mathbf{ln}\boldsymbol{\beta}_t^N$	$\mathbf{ln}\boldsymbol{\beta}_t^P$	$\mathbf{ln}\boldsymbol{\beta}_t^N$	$\mathbf{ln}\boldsymbol{\beta}_t^P$	$\mathbf{ln}\boldsymbol{\beta}_t^N$	$\mathbf{ln}\boldsymbol{\beta}_t^P$	$\mathbf{ln}\boldsymbol{\beta}_t^R$	$\mathbf{ln}\boldsymbol{\beta}_t^{N-R}$
Ln β_{t-1}^N	0.85*** (44.6)	0.80*** (34.7)	0.87*** (39.1)	0.79*** (29.6)	0.95*** (41.1)	0.87*** (32.7)	0.89 (66.0)	0.89 (39.0)
Ln s_t^X	0.08*** (7.72)	0.08*** (4.40)	0.05*** (4.78)	0.01 (0.63)	0.05*** (5.68)	0.01 (0.99)	0.11*** (11.0)	0.03* (1.74)
Ln Ψ_t	-0.19*** (11.5)	-0.38*** (12.0)					-0.10*** (5.77)	-0.26*** (8.88)
$\text{Ln}(g+\delta)_t$			-0.02*** (2.31)	-0.04*** (3.51)	-0.02** (1.96)	-0.04*** (3.09)		
$\lambda_t \phi_t^{N-R}$					0.39*** (9.14)	0.34*** (8.32)		
R^2	0.84	0.76	0.80	0.70	0.84	0.74	0.93	0.89
# Obs	783	783	783	783	783	783	783	783
Est. Period	1231-2013	1231-2013	1231-2013	1231-2013	1231-2013	1231-2013	1231-2013	1231-2013

Notes. The numbers in parentheses are absolute *t*-statistics based on standard errors that are robust to serial correlation and heteroscedasticity. The saving rate, s^X, is measured as private saving, s^P, in the regressions where $ln\beta_t^P$is the dependent variable, and as national saving, s^N, in the regressions where $ln\beta_t^N$is the dependent variable. β_t^R = reproducible wealth-income ratio; β_t^{N-R} = non-reproducible wealth-income ratio. The data are measured in 10-year moving averages.

These results have two implications. First, the P&Z model extended with the $\lambda_t \phi_t^{N-R}$-term is empirically a vast improvement over the simple model without the $\lambda_t \phi_t^{N-R}$ term and, therefore, that real capital gains on non-reproducibles are influential for the evolution of the *W*-*Y* ratio. Second, the result that the parameter estimates of s^X are independent of the inclusion of the $\lambda_t \phi_t^{N-R}$ term implies that the effects of saving and capital gains on the *W*-*Y* ratio can be analyzed in isolation.

The dependent variable is decomposed into reproducibles, β^R, and non-reproducibles, $\beta^{N\text{-}R}$, in the regressions in columns (7) and (8). The coefficient of saving is highly significantly positive and its long-run elasticity is exactly one in the β^R regression; however, it is barely significant in the $\beta^{N\text{-}R}$ regression, which is consistent with the model because changes in $W^{N\text{-}R}$ is almost entirely driven by real capital gains. Although significantly negative in both regressions, the coefficient of Ψ is substantially more negative in the $\beta^{N\text{-}R}$ regression than the β^R regression as we would expect, since the $\lambda_t\phi_t^{N-R}$ term can only affect non-reproducibles. The $\beta^{N\text{-}R}$ and β^R regression results have the important implications that 1) the P&Z model needs to be extended with the $\lambda_t\phi_t^{N-R}$ term; and 2) causality predominantly goes from savings to β^N and not the other way around; otherwise the coefficients of saving would have been the same in both regressions.

4.1.2 IV regressions

Thus far the explanatory variables have been assumed to be exogenous and, therefore, the causality runs from saving and Ψ to the *W-Y* ratio. It is, however, possible that the coefficients of saving and Ψ are biased because of endogeneity and measurement errors. There is particularly likely to be negative feed-back effects from the *W-Y* ratio to saving triggered by capital gains on stocks and land that simultaneously increase the *W-Y* ratio and reduce saving rates. The saving rate and Ψ are instrumented to deal with these issues. Although the instruments introduced below are not ideal, they will nevertheless give some more insight into the direction of causality and alleviate potential errors-in-variables biases.

The fraction of non-reproducibles in total wealth, ϕ^{N-R} and the depreciation–income ratio, *Dep*/*Y* are used as instruments for the saving rate, where *Dep* is depreciation of fixed capital. The depreciation-income ratio, *Dep*/*Y*, which is computed as the gross saving rate minus the net saving rate, is quite different from the depreciation rate used in the Ψ term because it uses income instead of wealth in the denominator. It is exogenous from the perspective that it is determined by past investment decisions and, particularly, by the composition of fixed capital stock. The depreciation-income ratio was at a relatively low level before the 1800s when the total fixed capital stock with low depreciation rates was composed almost entirely of buildings. The share of machinery and equipment stock in total fixed capital stock has since been increasing and is now almost a half of total fixed capital stock.

The share of non-reproducibles in total wealth is used as an instrument for saving following Laitner's (2000) aforementioned analysis in which he shows that the national accounting saving rate is inversely related with real capital gains on non-reproducibles. The non-reproducible wealth share is mainly determined by the production structure and the shortage of urban and agricultural land; factors that are quite independent of the *W-Y* ratio. Both instruments for saving are likely to satisfy the exclusion restrictions as there is no theory linking depreciation rates or the share of non-durables in total wealth directly to the *W-Y* ratio.

Real food price inflation, π^F, and epidemics, *Epi*, are used as instruments for Ψ because they simultaneously impact on all the variables contained in Ψ and are, at least at cyclical frequencies, determined exogenously. Real food price inflation is driven by technological progress and population pressure at a secular level and fluctuates around this level as a result of crop failures, livestock diseases, and other adverse supply shocks. Positive π^F is likely to be associated with positive capital gains on farmland and low population growth through the Malthusian mechanism; thus rendering the relationship between Ψ and π^F negative. Epidemics reduce population growth while their effect on per capita income growth and capital gains on non-reproducibles is ambiguous. An epidemic will increase farmland labor productivity due to diminishing returns to labor; however, per capita income will be adversely affected by ill health through lower work effort and less investment in human capital. Finally, epidemics will probably reduce the returns to land because a reduced workforce lowers the marginal productivity of farmland. Epidemics, *Epi*, is based on the perpetual inventory method with a 10% depreciation rate applied to a dummy variable taking the value of one in years of epidemics and zero otherwise. Thus, *Epi* is a stock variable that increases in periods of epidemics, remains positive and gradually declines after an epidemic to account for its persistent effects on Ψ.

Table 2. 2SLS regressions.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
	First-Stage Regress.			**Structural Regressions**					
Dep Var	$\ln \Psi_t$	lns_t^P	lns_t^N	$ln\beta_t^N$	$ln\beta_t^P$	$ln\beta_t^N$	$ln\beta_t^P$	$ln\beta_t^N$	$ln\beta_t^P$
π_t^F	-0.02*** (7.92)								
$ln\ Epi_t$	-1.61*** (4.09)								
ϕ_t^{N-R}-		-1.26*** (8.89)	-0.74*** (3.38)						
$(Dep/Y)_t$		10.7*** (14.7)	10.0*** (14.2)						
Ln β_{t-1}^N				0.70*** (21.3)	0.80*** (34.7)	0.79*** (7.33)	0.79*** (7.33)	0.81*** (23.0)	0.79*** (7.33)
Ln s_t^X				0.20*** (8.00)	0.08*** (4.40)	0.41*** (3.86)	0.41*** (3.86)	0.08*** (4.45)	0.01 (0.49)
Ln Ψ_t				-0.76*** (9.05)	-0.38*** (12.0)	-1.36*** (5.49)	-1.37*** (5.50)	-0.27*** (4.50)	-0.18*** (2.80)
p-value	0.00	0.00	0.00						
R^2	0.20	0.64	0.56						
# Obs	793	793	793	793	793	380	783	414	414
Est. Per.	1221-2013	1221-2013	1221-2013	1221-2013	1221-2013	1221-1600	1221-1600	1600-2013	1600-2013

Notes. See notes to Table 1. The coefficient of *Epi* is multiplied by 1000. The *p*-value is the *p*-value for the joint test of excluded restrictions.

The first stage regressions are displayed in the first three columns in Table 2. The coefficients of the instruments are highly significant in the Ψ-regression in column (1). The coefficient of π^F is negative as it leads to positive capital gains on farmland and reduced population growth. The coefficient of *Epi* is negative, indicating that the negative population effects outweigh potential positive per capita growth effects. Gross savings rates are positively related to depreciation rates, as savers try to maintain the real value of fixed capital stock, and negatively related to the share of non-reproducible wealth in total wealth, consistent with Laitner's (2000) hypothesis. Common for all first-stage regressions is that the instruments are sufficiently correlated with Ψ and π^F to act as potentially good instruments.

The structural regression results are reported in columns 4-9 in Table 2 for the periods 1221-2013, 1221-1600 and 1600-2013, where 1600 marks the transition from a regime with minuscule growth rates to a regime of persistent growth in per capita income (Broadberry *et al*., 2015). The coefficients of savings are significant at the 1% level except in one case, and their long-run magnitudes are significantly closer to their theoretical values of one than their OLS-counterparts. The coefficients of Ψ are all significantly negative at the 1% level. Their long-run values are closer to the theory-predictions in the post-1600 estimates than in the pre-1600 regressions, probably reflecting distortions/omitted confounding variables from the outbreak of the Plague in 1348 that lasted more than two centuries.

4.2 Growth regressions

Thus far the empirical analysis has centered on the steady state relationship between β and s_t/Ψ_t and no attention has been paid to the movements of the *W-Y* ratio around its steady state and the extent to which the *W-Y* ratio converges to its steady state. A good *W-Y* model should be able to explain the steady-state equilibrium as well as convergence towards the steady state. Eq. (3) predicts that the *W-Y* ratio converges towards s_t/Ψ_t in steady state and, therefore, that shocks to the economy have only temporary effects on the growth in the *W-Y* ratio. To examine the short-run behavior of the *W-Y* ratio consider the following simple steps following the logic of the growth-model of Mankiw *et al*. (1992).

Approximating around the steady state the speed of adjustment is given by:

$$\frac{\partial ln\beta_t^X}{\partial t} = \gamma[ln\beta^* - ln\beta_t], \tag{13}$$

where $\gamma = (g + \delta - \lambda\phi^{N-R})$ is the convergence rate. Eq. (13) implies:

$$ln\beta_t - ln\beta_0 = (1 - e^{-\gamma t})[ln\beta^* - ln\beta_0].$$

Substituting for β^* yields:

$$\Delta ln\beta_t = (1 - e^{-\gamma t}) \ln s_t - (1 - e^{-\gamma t}) \ln(g_t + \delta_t - \lambda_t \phi_t^{N-R}) - (1 - e^{-\gamma t}) ln\beta_{t-1}, \qquad (14)$$

which is stochastically specified as:

$$\Delta ln\beta_t^X = \alpha_0 + \alpha_1 \ln(s_t^X + \kappa_1) + \alpha_2 \ln\ \Psi_t + \alpha_3 ln\beta_{t-1}^X + \varepsilon_{2,t}, \qquad (15)$$

where Δ is an 10-year difference operator and β_{t-1}^X is the initial *W*-*Y* ratio (the first year in the 10-year estimation interval). Here, β_{t-1}^X is measured in the first year over which $\Delta ln\beta_t^X$ spans, and $(s_t^X + \kappa_1)$ and Ψ_t are measured in 10-year averages. The advantage of Eq. (15) over the specifications used thus far is that it accounts for out-of-steady-state dynamics. Furthermore, it deals with feedback growth effects of the *W*-*Y* ratio when the economy is outside its steady state.

The theory predictions of the coefficients in Eq. (15) are as follows. The average value of $(g + \delta - \lambda\phi^{N-R})$ over the period 1211-2013 is 0.037, which implies that $-(1 - e^{-\gamma t}) = -(1 - e^{-0.37}) =$ - 0.31 and, therefore, that the economy is halfway towards its steady state after approximately 19 years.

Table 3. Explaining growth in the *W*-*Y* ratio (Eq. (1)).

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Dep Var	$\Delta ln\ \beta_t^N$	$\Delta ln\ \beta_t^P$	$\Delta ln\ \beta_t^N$	$\Delta ln\ \beta_t^P$	$\Delta ln\ \beta_t^N$	$\Delta ln\ \beta_t^N$	$\Delta ln\ \beta_t^N$	$\Delta ln\ \beta_t^N$	$\Delta ln\ \beta_t^N$	$\Delta ln\ \beta_t^P$
Ln s_t^X	0.15*** (14.6)	0.12*** (14.0)	0.17*** (15.1)	0.12*** (12.9)	-0.15*** (11.1)	0.10*** (5.88)	0.09*** (5.52)	0.12*** (5.41)	0.18*** (4.97)	0.14*** (4.06)
Ln Ψ_t	-0.54*** (32.8)	-0.51*** (34.9)	-0.57*** (18.7)	-0.51*** (20.0)	-0.52*** (18.1)	-0.41*** (7.42)	-0.74*** (28.5)	-0.80*** (35.2)	-0.62*** (6.96)	-0.54*** (7.00)
Ln β_{t-10}^X	-0.27*** (16.6)	-0.29*** (17.1)	-0.27*** (20.8)	-0.29*** (19.6)	-0.22*** (6.24)	-0.28*** (7.77)	-0.06*** (2.98)	-0.04* (1.86)	-0.28*** (6.55)	-0.29*** (5.67)
R^2	0.72	0.72			0.63		0.91			
# Obs	793	793	793	793	414	414	380	380	79	79
Frequency	Annual	Annual	Annual	Annual	Annual	Annual	Annual	Annual	10-year	10-year
Estimator	OLS	OLS	2SLS	2SLS	OLS	2SLS	OLS	2SLS	2SLS	2SLS
Est. Period	1221-2013	1221-2013	1221-2013	1221-2013	1600-2013	1600-2013	1221-1600	1221-1600	1221-2013	1221-2013

Note. See notes to Table 1.

The results of regressing Eq. (15) are presented in Table 3. IV and OLS regressions in addition to regressions in 10-year non-overlapping intervals are presented. The coefficients of saving are all statistically highly significant and positive regardless of whether the OLS or IV estimator is used or whether the regressions are in 10-year overlapping or non-overlapping intervals. The coefficients of saving are approximately half of the theory predictions of 0.31. The coefficients of Ψ are also all highly significant and have their expected negative signs. Except for the pre-1600 regressions, the coefficients of the initial *W*-*Y* ratio, β_{t-1}^X, are all highly significant and the sizes indicate a relatively speedy adjustment towards steady state. Except for the pre-1600 regressions, the magnitudes of the coefficients of Ψ and β_{t-1}^{X} are close to the predictions of the model; thus giving further credibility to the model.

From standard statistical criteria, the pre-1600 regressions are fine; however, as mentioned above, the magnitudes of the coefficients of Ψ and β_{t-1}^{X} fall a bit outside the theory predictions. The absolute values of the coefficients of β_{t-1}^{X} are on average 0.06, suggesting a very slow adjustment towards equilibrium and, like the estimates in Table 2, are probably heavily influenced by the extended disequilibrium induced by the Plague. The *W*-*Y* ratio is well above its steady-state value during the approximate period 1350-1500 based on computational values from Eq. (3), and it is likely that confounding forces, unaccounted for in the model regressions, have led to biased coefficients of β_{t-1}^{X}. Conversely, the coefficients of Ψ are significantly more negative than the model predictions of approximately -0.3 in the pre-1600 estimates. The coefficient estimate of Ψ is probably driven down by the extraordinary decrease in the $(g + \delta - \lambda\phi^{N-R})$ term and a simultaneous, but marked, upward shift in the *W*-*Y* ratio during the Plague. The marked increase in the *W*-*Y* ratio in the period 1350-1500 may have been exaggerated by measurement errors.

5. Decomposing wealth accumulation to saving and capital gains effects

This section decomposes the growth in private wealth into saving and capital gain/loss effects, where the saving effect is wealth accumulation derived from of *net* saving, while the capital gains effect is capital-gain-induced wealth accumulation. This decomposition is not only important because it gives insight into the sources of wealth accumulation through British history; it also gives insight into the importance of the $\lambda\phi^{N-R}$ term in the *W*-*Y* model.

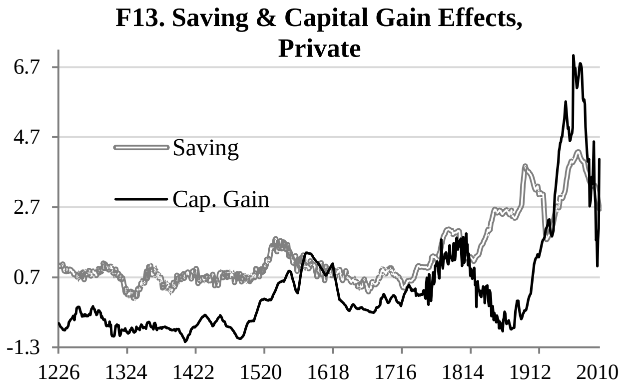
The following identity is used to decompose private wealth accumulation into net saving and capital gains effects:

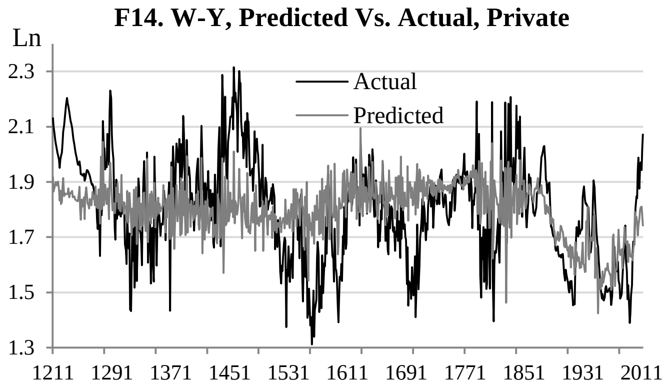
$$\Delta W_t^P = S_t^P - \delta_t \ W_{t-1}^P + r_t^{Cg} W_{t-1}^P, \tag{16}$$

where r^{Cg} is real capital gains on private wealth. The capital gains are found residually from Eq. (16) as:

$$r_t^{Cg} = \frac{\Delta W_t^P}{W_{t-1}^P} - \frac{S_t^P - \delta_t \ W_{t-1}^P}{W_{t-1}^P}, \tag{17}$$

where the second right-hand-term is the ratio of net saving and wealth.





Notes. Figure 13 decomposes the growth rate in nominal private wealth into capital gains/losses and net saving/dissaving. The data are smoothed by 31year (saving) and 51-year (capital gains) centered moving average, where the length of the years of the moving average is stepwise reduced at the endpoints. The predictions in Figure 14 are based on regressing Eq. (12) without the lagged dependent variable.

The growth rate in private wealth is decomposed into capital gains and net saving effects in Figure 13. The figures are smoothed out by long centered moving averages. The capital gains and saving-induced wealth accumulation rate is increasing over time mainly to compensate for the increasing NNI growth rate. Inspection of the graphs gives the impression that capital gains and savings have been approximately equally important drivers of wealth over the past eight centuries. The standard deviation for capital gains exceeds that of net saving, suggesting that capital gains have been the most important driver of wealth at high frequencies, noting that the standard deviation of the data depends on the time-span over which the first difference spans since net saving and capital gains do not follow random walks. Three epochs can be distinguished: 1210-1500, 1500-1833 and 1833-2013.

The pre-industrial era, 1210-1500, is characterized by low saving and real capital losses on land, where the capital losses have been heavily influenced by the Plague that reduced the marginal productivity of land through reduced labor inputs. The increasing population results in significantly positive capital gains on wealth throughout the period 1500-1833. However, the heydays of landowners come to a halt around 1833, and the wealth accumulation is subsequently driven by saving up to WWI as industrialization gains momentum and, consequently, leads to the rapidly increasing share of reproducibles in total wealth. Capital gains take over as the chief driver of wealth accumulation after WWII caused by the massive increase in urban population that pushed urban land values up. Remarkably, the strongest capital gains occur in the approximate period 1955-1992 and not in the post-1992 period during which the *W-Y* ratio escalated substantially.

Two implications can be drawn from Figure 13. First, on average, capital gains effects have been 0.40% annually, while the saving effect has been 1.34%, suggesting that saving has been the chief driver of wealth accumulation in the long run; however, since saving is much more stable than capital gains at medium term frequencies, capital gains can result in larger shifts in the *W-Y* ratio than saving and, therefore, income distribution. The significance of capital gains/losses for the evolution of the *W*-*Y* ratio reinforces the importance of including the $\lambda\phi^{N-R}$ term in the *W*-*Y* model. This result stands in contrast to the model and the findings of P&Z that the *W* accumulation is driven predominantly by net saving in steady state and, therefore, that the gross-version of the P&Z model, $\beta^S = s/(g+\delta)$ dominates Eq. (3), $\beta = s/(g+\delta-\lambda\phi^{N-R})$. Note, however, that the results here cannot be compared with P&Z's decomposition of wealth accumulation because they use *gross* as opposed to *net* saving in their decomposition.

A second implication of Figure 13 is that saving was the primary source of wealth accumulation during the period 1800-1915, when the British industrialization gained momentum, while capital gains and saving have been almost equally important drivers of wealth since then. This raises the question whether we are now back to the pre-industrial era, 1500-1800, during which capital gains were essential parts of wealth accumulation. The answer to this question is affirmative. Capital gains are likely to be a vital part of wealth accumulation in the future because the increasing demand for food at the world level coupled with stalling land productivity in the advanced countries is likely to result in increasing prices of farmland. Furthermore, the increasing income and increasing share of professional jobs is likely to put continuous pressure on the housing market.

6. Can the model explain the *W*-*Y* path?

The logarithm of the actual and the predicted *W*-*Y* ratio is displayed in Figure 14. The predictions are based on the regression in column 9 of Table 1 in which the lagged dependent variable is excluded from the regression to prevent an almost perfect visual fit, driven predominantly by the lagged dependent variable. The model tracks the actual *W*-*Y* ratio well after 1700 and reasonably well before 1700. The model mostly fails to explain the ascending *W*-*Y* ratio following the Plague and the subsequent decline, which is likely to, in part, have been driven by the reduction in the returns to wealth from 5.3% to 2.6% over the period 1332-1472 and the increase up to 7.0% in 1610 (the returns are computed as 18-years centered moving average).

7. Conclusion

In their influential work Piketty (2014) and Piketty and Zucman (2014) document a U-shaped *W*-*Y* path in Britain, Germany, the US and France over the past century and propose that the saving-growth ratio can be used as a framework to explain the past and project the future.

This paper has extended their work in three dimensions. First, annual data on the *W*-*Y* ratio, growth and saving are constructed for the Britain over the past eight centuries. These data not only give insight into the evolution of the W-Y ratio in the ultra-long run, they are also a major improvement over the data collected P&Z. Second, P&Z's $s/(g+\delta)$ model is extended to allow for capital gains on non-reproducibles. The extension distinguishes between reproducibles, in which capital gains are non-existent, and non-reproducibles, in which capital gains and losses prevail in steady state because their supply is inelastic. Third, the annual data are used to test the capacity of the models of P&Z and the extended model to explain the low frequency movements in the W-Y ratio and the dynamic adjustment of the W-Y ratio towards the steady state.

The construction of the W-Y ratio gave two key insights. First, that the W-Y ratio fluctuates substantially at medium-term frequencies of one and two centuries' duration and tends towards a constant of 600% in the ultra-long-run. Second, the largest fraction of wealth accumulation in Britain over the past eight centuries has been real capital gains on non-reproducibles as opposed to savings. Since values per unit of capital cannot be driven down to acquisition costs on non-reproducibles, it follows that r and W-Y will be independent for non-reproducibles and, therefore, that the elasticity of substitution between capital and labor or land and labor need not be higher than one for capital's income share to be increasing in the W-Y ratio. This finding is important because the elasticity of substitution needs to exceed one before the models of Karababounis and Nieman (2014) and Piketty and Zucman (2014) can explain the recent increase in capital's share.

The long data was used to test whether the evolution of the W-Y ratio over the past eight centuries could be explained by the extended P&Z model. Time-varying depreciation rates, the share of non-reproducibles in total wealth, real food prices and epidemics were used as instruments for saving and growth to cater for endogeneity and errors-in-variables. The regression results showed a highly significant relationship between W-Y ratio and its determinants in the extended model and the results were robust to model specification, estimation period and identification. The simple $s/(g+\delta)$ model, by contrast, did not adequately explain the movements in the W-Y ratio. Furthermore, it was shown that the extended model was able to explain the movements in the W-Y ratio around its steady state well. Thus, The $s/(g+\delta)$ model needs to be extended to allow for capital gains on non-reproducibles before it can be used to predict the inequality path over the rest of this century.

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