The paper studies how high household leverage and crises can arise as a result of changes in the income distribution. Empirically, the periods 1920-1929 and 1983-2008 both exhibited a large increase in the income share of high-income households, a large increase in debt leverage of the remainder, and an eventual financial and real crisis. The paper presents a theoretical model where higher leverage and crises arise endogenously in response to a growing income share of high-income households. The model matches the profiles of the income distribution, the debt-to-income ratio and crisis risk for the three decades prior to the Great Recession.

JEL: E21, E25, E44, G01, J31

Keywords: Income inequality; wealth inequality; debt leverage; financial crises; wealth in utility; global solution methods; endogenous default

I. Introduction

The United States experienced two major economic crises over the past century—the Great Depression starting in 1929 and the Great Recession starting in 2008. A striking and often overlooked similarity between these two crises is that both were preceded, over a period of decades, by a sharp increase in income inequality, and by a similarly sharp increase in debt-to-income ratios among lower- and middle-income households. When debt levels started to be perceived as unsustainable, they contributed to triggering exceptionally deep financial and real crises.
In this paper, we first document these facts, both for the period prior to the Great Depression and the period prior to the Great Recession, and we then present a dynamic stochastic general equilibrium model in which a crisis driven by greater income inequality arises endogenously. To our knowledge, our model is the first to provide an internally consistent mechanism linking the empirically observed rise in income inequality, the increase in debt-to-income ratios, and the risk of a financial crisis. In doing so it provides a useful framework for investigating the role of income inequality as an independent source of macroeconomic fluctuations.

The model is kept as simple as possible in order to allow for a clear understanding of the mechanisms at work. The crisis is the ultimate result, after a period of decades, of shocks to the income shares of two groups of households, top earners who represent the top 5% of the income distribution, and whose income share increases, and bottom earners who represent the bottom 95% of the income distribution. The key mechanism is that top earners, rather than using all of their increased income for higher consumption, use a large share of it to accumulate financial wealth that is backed by loans to bottom earners. They do so because, following Carroll (2000) and others, financial wealth enters their utility function directly. By accumulating financial wealth, top earners allow bottom earners to limit the drop in their consumption following their loss of income, but the resulting large and highly persistent increase of bottom earners’ debt-to-income ratio generates financial fragility that eventually makes a financial crisis much more likely. The crisis is the result of an endogenous and rational default decision on the part of bottom earners, who trade off the benefits of relief from their growing debt load against income and utility costs associated with default. Lenders fully expect this behavior and price loans accordingly. The crisis is characterized by large-scale household debt defaults and an abrupt output contraction, as in the recent U.S. financial crisis.

We show that, under a plausible baseline calibration of our model, a series of near-permanent negative shocks to the income share of bottom earners, of exactly the magnitude observed in the data, generates an increase in the debt-to-income ratio of bottom earners very close to the magnitude observed in U.S. data over the 1983-2008 period. Furthermore, this leads to a significant increase in crisis risk, of approximately the magnitude found in the recent empirical study of Schularick and Taylor (2012). A number of changes in the utility function parameters of top earners leads to faster growth of debt and crisis risk relative to the baseline, including a lower intertemporal elasticity of substitution in consumption, a lower curvature of the utility function with respect to wealth, and a greater weight on utility from wealth.

When a rational default occurs, it does of course provide relief to bottom earners. But because it

1In a more detailed model additional physical investment is a third option. See Kumhof and Ranciere (2010).
is accompanied by a collapse in real activity that hits bottom earners especially hard, and because of higher post-crisis interest rates on the remaining debt, the effect on their debt-to-income ratios is small, and debt quickly starts to increase again if income inequality remains unchanged.

When the shock to income inequality is, consistent with the time series properties of U.S. top 5% income shares, permanent or near-permanent, the preference for wealth of top earners is key to our results on debt growth. By contrast, when the shock to income inequality is modelled as transitory, the same model parameterized without preferences for wealth among top earners can generate an increase in debt and crisis risk purely based on a consumption smoothing motive.

The rest of the paper is organized as followed. Section II relates our paper to the literature. Section III discusses key stylized facts for the Great Depression and the Great Recession. Section IV presents the model. Section V shows model simulations that study the effects of increasing income inequality on debt levels and crises. Section VI concludes.

II. Relation to the Literature

The central argument of the paper links two strands of the literature that have largely been evolving separately, the literature on income and wealth distribution, and the literature on financial fragility and financial crises. In addition, our modeling approach takes elements from the literature on preferences for wealth, to explain the rise of household debt leverage when the increase in income inequality is permanent, and from the literature on rational default, to endogenize financial crises.

The literature on income and wealth distribution is mostly focused on accurately describing long run changes in the distribution of income and wealth (Piketty and Saez (2003), Piketty (2011)). One of its main findings is that the most significant change in the U.S. income distribution has been the evolution of top income shares. This feature is reflected in our model, which contains two groups representing the top layer and the remainder of the U.S. income distribution. Our paper focuses only on the macroeconomic implications of increased income inequality, rather than taking a stand on the fundamental factors shaping the change in the income distribution in the United States over the last thirty years.2

Turning to the literature on financial fragility and financial crises, popular explanations for the origins of the 2008 crisis focus on domestic and global asset market imbalances. For example, Keys et al. (2010) stress the adverse effects of increased securitization on systemic risk, Taylor (2009) claims that the interaction of unusually easy monetary policy with excessive financial liberalization caused the crisis, and Obstfeld and Rogoff (2009) claim that the interaction of these factors with

2A large literature focuses on these fundamental factors. For a partial review, see Kumhof and Ranciere (2010).
global current account imbalances helped to create a “toxic mix” that helped to set off a worldwide crisis. Typically these factors are found to have been important in the final years preceding the crisis, when debt-to-income ratios increased more steeply than before. But it can also be argued, as done in Rajan (2010), Reich (2010) and this paper, that much of this was simply a manifestation of an underlying and longer-term dynamics driven by income inequality.

Rajan and Reich suggest that sizeable increases in borrowing have been a way for the poor and the middle class to maintain or increase their level of consumption at times when their real earnings were stalling. But these authors do not make a formal case, in the form of a general equilibrium model, to support that argument. We think that this matters. The reason is that the debate about the driving forces behind the historical increase in U.S. household debt has been conducted among competing partial equilibrium views. On the one hand, Rajan (2010) emphasizes the role of (government supported) credit demand. His argument is that growing income inequality created political pressure, not to reverse that inequality, but instead to encourage borrowing to keep demand and job creation robust despite stagnating incomes. On the other hand Acemoglu (2011) claims that the main driving force was an increase in credit supply that was caused by financial deregulation. Our model can reconcile these views in a general equilibrium framework. Specifically, we find that the relative importance of credit demand and credit supply channels depends on the nature of preferences and of shock processes. When shocks to income inequality are transitory, and when top earners have no preferences for wealth, both credit demand and credit supply increase, due to a consumption smoothing motive. But when shocks to income inequality are permanent, as they have been in recent U.S. data, any debt increase must be exclusively due to higher credit supply generated by the preferences for wealth of top earners.

The stylized facts section of our paper documents a strong comovement between increases in income inequality and increases in household debt-to-GDP ratios in both the period prior to the Great Recession and the period prior to the Great Recession. In our model an increase in debt among bottom earners, which empirically has been the main driver of the overall increase in household debt in the period prior to the Great Recession, leads to an increase in crisis risk. This is consistent with the results of Schularick and Taylor (2012) who, using a sample of 14 developed countries over the period 1870 - 2008, find that increases in debt-to-GDP ratios have been a powerful predictor of financial crises. Furthermore, in our model the crisis itself is associated with a partial default

3Relatedly, Bertrand and Morse (2013) show empirically that U.S. income inequality is negatively correlated with the saving rate of the middle-class, and positively correlated with personal bankruptcy filings.
4See also Levitin and Wachter (2012).
5Note that we focus only on the two largest financial crises in post-WWI U.S. history, which were both preceded by historically unique, decades-long build-ups in household debt-to-income ratios. Bordo and Meisner (2012) look instead at a sample that includes many much smaller crisis episodes.
of bottom earners, accompanied by output costs of default. This mechanism is consistent with the results of Midrigan and Philippon (2011), who show that U.S. regions (states, counties) that had experienced the largest increases in household debt leverage suffered the largest output declines during the Great Recession.

Another recent literature has related increases in income inequality to increases in household debt (Krueger and Perri (2006), Iacoviello (2008)). In these authors' approach an increase in the variance of idiosyncratic income shocks across all households generates a higher demand for insurance through credit markets, thereby increasing household debt. This approach emphasizes an increase in income inequality experienced within household groups with similar characteristics, while our paper focuses on the rise in income inequality between two household groups. There is a lively academic debate concerning the relative roles of within- and between-group factors in shaping inequality. But our paper only focuses on changes in one specific type of between-group inequality that can be clearly documented in the data, namely inequality between high-income households and everyone else. Furthermore, the observed increases in household debt-to-income ratios have also been strongly heterogenous between these two income groups. This suggests that heterogeneity in incomes should be explored in models of household debt and financial fragility.

In the baseline version of our model, top earners exhibit preferences for wealth. Wealth in the utility function has been used by a number of authors including Carroll (2000), who refers to it as the “capitalist spirit” specification, Reiter (2004), and Piketty (2011). The reason for introducing this feature is that models with standard preferences have difficulties accounting for the saving behavior of the richest households. For instance, Carroll (2000) shows, using data from the U.S. Survey of Consumer Finance, that the life cycle/permanent income hypothesis model augmented with uncertainty proposed by Hubbard, Skinner and Zeldes (1994) can match the aggregate saving behavior only by over-predicting the saving behavior of median households and by underpredicting the saving behavior of the richest households. By contrast, models featuring wealth in the utility function can match both the aggregate data and the wealth accumulation patterns of the wealthiest households. Piketty (2011) shows similar results for France. Reiter (2004) confirms the finding of Carroll (2000), but also suggests a role for the idiosyncratic return risk that rich households face from closely held businesses. Francis (2009) shows that introducing a preference for wealth into an otherwise standard life cycle model generates the skewness of the wealth distribution observed in

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6Zou (1994,1995) are the first papers introducing direct preferences for wealth, in a two-period model, to explain the link between savings and growth. Bakshi and Chen (1996) introduce preferences for wealth into an infinite horizon model to study the implications for asset pricing.

7An alternative model of saving behavior is the dynastic model (Barro (1974)), where dynasties maximize the discounted sum of utilities of current and future generations. Carroll (2000) surveys evidence suggesting that this model also does not do well in explaining the saving decisions of the richest households.
the U.S. data. Kopczuk (2007) shows that terminally ill wealthy individuals actively care about the disposition of their estates, but that this preference is dominated by the desire to hold on to their wealth while alive. Finally, Dynan, Skinner and Zeldes (2004) find little empirical support for models in which heterogeneities in saving behavior reflect only differences in rates of time preference.

Wealth in the utility function can represent a number of different saving motives. One is as a reduced form representation of precautionary saving, based on the fact that wealth provides security in the presence of uninsurable lifetime shocks. It can also represent the desire to leave a bequest. Our preferred interpretation, following Franck (1985), Cole, Mailath and Postelweite (1992), Bakshi and Chen (1996), and Carroll (2000), is that agents derive direct utility from the social status and power conferred by wealth.

Endogenous, rational default of borrowing households is a key feature of our model, with higher leverage leading to a higher crisis probability. Our paper is therefore naturally related to models of consumer bankruptcy as developed in Athreya (2002), Chatterjee et al. (2007) and Livshits et al. (2007). However, these papers are based on economies with a continuum of heterogeneous agents, as in Aiyagari (1994) and Huggett (1993), while our model focuses on aggregate relationships between two groups of agents. It is therefore much closer to the sovereign default literature of Eaton and Gersovitz (1981), Bulow and Rogoff (1989), Aguiar and Gopinath (2006) and Arellano (2008). But our model also exhibits significant differences to that literature. First, lenders in our model are risk-averse, rather than risk neutral as the rest-of-the-world investors of the partial equilibrium sovereign debt literature. Risk-averse investors are also assumed in Borri and Verdelhan (2010), Lizarazo (2013) and Pouzo and Presno (2012). Second, since only a fraction of households will default in any real-world crisis event, we assume that default in our two-agent economy occurs only on a fixed fraction of outstanding debt. More elaborate default setups with partial recovery are studied in Benjamin and Wright (2009), Yue (2010) and Adam and Grill (2013). Third, in much of the sovereign debt literature the state space exhibits a sharp boundary between regions of certain non-default and certain default. But according to Schularick and Taylor (2012), the probability of a major crisis in the United States, while increasing in the level of debt, has nevertheless always remained well below 10% in any given year. Our model replicates this feature by adding a stochastic utility cost of default, similar to Pouzo and Presno (2012). Fourth, the assumptions of partial default and of low default probabilities imply that the level of debt that can be sustained in equilibrium is higher than what is commonly obtained in the sovereign debt literature, even though our output penalties of default are lower than the output losses observed during the largest crises.

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8See Francis (2009) for a discussion on the differences between models with a bequest motive and models with capitalist spirit preferences.
III. Stylized Facts

This section documents a number of key stylized facts, for the United States economy, that characterize the periods prior to the Great Recession and, where available, the Great Depression. These facts are relevant to the nexus between inequality, leverage and crises, and will inform the specification and calibration of our model. Five empirical regularities are particularly important. First, there was a large increase in income inequality prior to both crises. Data for the period prior to the Great Recession indicate that this was highly persistent, and not compensated by an increase in income mobility. Second, household debt-to-income ratios exhibited a large aggregate increase, which was to a large extent due to higher debt levels among the bottom 95% of households by income, both for mortgages and for unsecured debt. Third, this bottom income group experienced not only higher debt levels but also a decline in its share of overall wealth. Fourth, there was a large increase in the size of the financial sector. Fifth, higher debt levels were accompanied by a higher risk of financial crises. In addition, the crises themselves were characterized by very high default rates on both mortgages and consumer loans.

A. Income Inequality and Aggregate Household Debt

In the periods prior to both major crises, rapidly growing income inequality was accompanied by a sharp increase in aggregate household debt.

Pre-Great-Recession The left panel of Figure 1 plots the evolution of U.S. income inequality and household debt-to-GDP ratios between 1983 and 2008. Between 1983 and 2007 income inequality experienced a sharp increase, as the share of total income commanded by the top 5% of the income distribution increased from 21.8% in 1983 to 33.8% in 2007. During the same period the ratio of household debt to GDP doubled, from 49.1% to 98.0%.

Pre-Great-Depression The right panel of Figure 1 plots the evolution of U.S. income inequality and household debt-to-GDP ratios between 1920 and 1929. Between 1920 and 1928, the top 5% income share increased from 27.4% to 34.8%. During the same period, the ratio of household debt to GDP more than doubled, from 16.9% to 37.1%.

B. Debt by Income Group

The periods prior to both major crises were characterized by increasing heterogeneity in debt-to-income ratios between high-income households and all remaining households. For the period prior to the Great Depression data availability is very limited, but some evidence exists, and is consistent with the period prior to the Great Recession.
Pre-Great-Recession. The left panel of Figure 2 plots the evolution of debt-to-income ratios for the top 5% and bottom 95% of households, ranked by income, between 1983 and 2007. In 1983, the top income group was more indebted than the bottom income group, with a gap of around 20 percentage points. In 2007, the situation was dramatically reversed. The debt-to-income ratio of the bottom group, at 147.3% compared to an initial value of 62.3%, was now more than twice as high as that of the top group. Between 1983 and 2007, the debt-to-income ratio of the bottom group therefore more than doubled while the ratio of the top group remained fluctuating around 60%. As a consequence almost all of the increase in the aggregate debt-to-income ratio shown in Figure 1 is due to the bottom group of the income distribution. This provides strong motivation for introducing income inequality between top and bottom earners into a model of household indebtedness and financial fragility.

Pre-Great-Depression. The inter-war era was a period of rapid increases in consumer debt. According to Olney (1991, 1999), the ratio of non-mortgage consumer debt to income increased from 4.6% in 1919 to 9.3% in 1929. Around two-thirds of this was installment debt for the purchase of durable goods, especially cars. Between 1919 and 1929, the percentage of households buying new cars increased from 8.6% to 24.0%. According to Calder (1999), while the wealthiest households could buy cars on cash terms, their diffusion to the middle class, at a time of growing income inequality, was critically dependent on the increasing availability of installment credit.

For this period, only two BLS surveys are available to study differences in borrowing across income groups. They were conducted in 1917/1919 and 1935/1936, respectively. An obvious disadvantage of the 1935/1936 survey is that it was taken several years after the crisis of 1929. But the data are nevertheless informative, for two reasons. First, the top 5% income share in 1936 was still high (32.5%), implying that income inequality had only very partially been reversed since 1929. And second, by 1936 the number of new cars sold and the percentage of households buying cars on installment, after having collapsed between 1929 and 1934, had bounced back to reach very comparable levels to those of 1927 (Olney (1991)).

We use the income thresholds provided by Piketty and Saez (2003) to classify the respondents of both surveys into either the top 5% or the bottom 95% of the income distribution. To make the results of the two surveys comparable, we confine the analysis to installment credit. The surveys do not report stocks of debt but rather flows of new debt associated with installment purchases in the last twelve months. The right panel of Figure 2 presents the results. In the 1917/1918 survey, the new installment debt to income ratios are both low and similar for both groups, at 3.0% and 3.8% for the top and bottom income groups. In the 1935/1936 survey, the ratios are much higher.
and, more importantly, much more dissimilar across income groups, at 6.8% and 10.9% for the top and bottom income groups. In 1935/1936, the ratio of average incomes between borrowers in the top and bottom groups was 3.25, while the ratio of average amounts borrowed was only 1.6. To the extent that these data are representative of other years during this period, it indicates a significantly higher growth in debt-to-income ratios among the bottom income group.

Alternative sources for the period immediately prior to 1929 exist and are broadly consistent with the 1935/1936 survey, but they are much more limited in quality or scope. Plummer (1927) uses information from a trade journal study of 532 families and concludes that 25% of middle-class families, but only 5% of the well-to-do, bought goods on installment.° A 1928 BLS survey documents the cost of living of 506 families of federal employees in 5 cities. It shows that the ratio of interest payments on non-mortgage debt to income for federal employees was 1.5 times larger for the bottom income group that for the top income group.

C. Wealth by Income Group

In the periods prior to both major crises, the rise in income inequality was associated not only with divergent debt levels across income groups, but also with divergent shares of overall wealth.

Pre-Great-Recession The left panel of Figure 3 plots the share of wealth held by the top 5% and the bottom 95% of the income distribution between 1983 and 2007. Except for a brief period between 1989 and 1992, the wealth share of the top 5% income group increased continuously, from 42.6% in 1983 to 48.6% in 2007.

Pre-Great-Depression Data on the distribution of wealth along the income distribution do not exist for the period prior to the Great Depression. The closest available data are shares of wealth held by the top 1% of the wealth distribution. In the absence of wealth surveys, these shares are computed using estate tax returns. The right panel of Figure 3 reports the most recent estimates by Kopczuk and Saez (2004), along with previous estimates by Wolff (1995).° Wolff (1995) reports that the share of net worth of the top 1% increased from 36.7% in 1922 to 44.2% in 1929. The series of Kopczuk and Saez (2004) shows the share of the top 1% of the wealth distribution increasing from 35.2% in 1921 to 39.1% in 1927. It declined to 36.8% in 1929, but reached 40.3% in 1930.

°The classification between middle-class and well-to-do families is imprecise, as it is done according to the neighborhood of the family residence.

°°Results of that survey by income brackets are presented in Bureau of Labor Statistics (1929).

°°°In Kopczuk and Saez (2004) the top 1% refers to the wealth distribution among invididual adults, while in Wolff (1995) it refers to the wealth distribution among all households.
D. Size of the Financial Sector

In our model, the increase in income inequality generates an increasing need for borrowing and lending. The counterpart of this in the data is a larger size of the financial sector. One way to proxy this is through household debt-to-GDP ratios, as in Figures 1 and 2. An alternative measure, the value added share of the financial sector in GDP, is reported in Figure 4. This has the advantage of being consistently available for both periods of interest, due to the work of Philippon (2012). These data confirm that, in the periods prior to both major crises, the size of the financial sector increased significantly relative to GDP.

Pre-Great-Recession The left panel of Figure 4 shows that, between 1983 and 2007, the share of the financial sector in U.S. GDP increased from 5.5% to 7.9%. This indicates a roughly 50% increase in the size of the financial sector relative to GDP, considerably less than the approximate doubling of private debt levels in Figure 1.

Pre-Great-Depression The right panel of Figure 4 shows that, between 1920 and 1928, the share of the financial sector in GDP increased from 2.8% to 4.3%, again a roughly 50% increase.

E. Leverage and Crisis Probability

To quantify the link between household leverage and crisis probabilities, we use the dataset of Schularick and Taylor (2012), which contains information on aggregate credit and crises for 14 countries between 1870 and 2008. We follow these authors’ methodology by running, on the full dataset, a logit specification in which the binary variable is a crisis dummy, and the latent explanatory variables include five lags of the aggregate loans-to-GDP ratio. The estimated logit model is used to predict crisis probabilities over the full cross-section and time series sample. Figure 5 reports the estimated probabilities for the United States. We observe that, in the periods prior to both major crises, there was a sizeable increase in crisis probabilities, even though a crisis remained a low probability event.

Pre-Great-Recession The left panel of Figure 5 shows that the estimated crisis probability started at 2.0% in 1983 and increased to 5.2% by 2008. Between 2001 and 2008, the crisis probability increased by two percentage points.

Pre-Great-Depression The right panel of Figure 5 shows that the estimated crisis probability almost doubled between 1925 and 1928, from 1.6% to 3.0%, before reaching 3.9% in the year of the crisis.12

12The lack of credit data between 1913 and 1919 prevents estimation with five lags before 1925.
F. Household Defaults During Crises

Both major crises were characterized by high rates of default on household loans. Their magnitudes matter because the share of loans defaulted on in a crisis is a key parameter of our model.

**Great Recession** We compute delinquency rates as ratios of the balance of delinquent loans (defined as being past due by 90 days or more) to the total loan balance.\(^{13}\) Between 2006 and 2010 mortgage delinquency rates increased dramatically, from 0.9% to 8.9%, and so did unsecured consumer loans delinquency rates, from 8.8% to 13.7% for credit card loans, from 2.3% to 5.3% for auto loans, and from 6.4% to 9.1% for student loans (the delinquency rate for student loans reached 11.7% in 2013). For the period 2007-2010, mortgage foreclosure rates, at the Metropolitan Statistical Area level, display a strong positive correlation (0.54) with the median mortgage loan-to-income ratio prior to the crisis. This fact, combined with the evidence in Section III.B on higher leverage among bottom earners, suggests that default rates among bottom earners are higher than the above-mentioned figures, which apply to all households.\(^{14}\)

**Great Depression** The crisis of 1929 was followed by a wave of defaults on automobile installment debt contracts (Olney (1999)). The percentage of cars repossessed increased from 4.1% in 1928 to 10.4% in 1932. Furthermore, repossession rates were significantly higher for used cars than for new cars (13.2% versus 5.7% in 1932). Combined with the fact that wealthy households were much more likely to buy new cars than middle and lower class households (Calder, 1999), this suggests that default rates on installment debt were higher among the bottom income group.

G. Additional Stylized Facts for the Great Recession

This paper is motivated in part by the many similarities between the periods prior to the Great Depression and the Great Recession. But our model is calibrated in reference to the Great Recession. For this purpose we discuss a small number of additional stylized facts that are not available for the period prior to the Great Depression.

Income Mobility and Inequality Persistence

In theory, if increasing income inequality was accompanied by an increase in intra-generational income mobility, the dispersion in lifetime earnings might be much smaller than the dispersion in annual earnings, as agents move up and down the income ladder throughout their lives. However, a recent study by Kopczuk, Saez and Song (2010)\(^{15}\), using a novel dataset from the Social Security

\(^{13}\)Source: Federal Reserve Bank of New York, Household Debt and Credit Reports.

\(^{14}\)Source: Reality Trac for foreclosure rates, and Federal Financial Institutions Examination Council for loan-to-income ratios.

\(^{15}\)See also Bradbury and Katz (2002).
Administration, shows that short-term and long-term income mobility in the United States has been either stable or slightly falling since the 1950s. These authors find that the surge in top earnings has not been accompanied by increased mobility between the top income group and other groups, as the probability of remaining in the top 1% group after 1, 3 or 5 years shows no overall trend since the top share started to be coded in Social Security Data (1978).

In addition, Kopczuk, Saez and Song (2010) directly measure the persistence of earnings inequality, defined as the variance of annual log earnings. They show that virtually all of the increase in that variance over recent decades has been due to an increase in the variance of permanent earnings (five-year log-earnings) rather than transitory earnings (five-year log earnings deviation). DeBacker et al. (2013) obtain similar results by using an alternative source, a 1-in-5000 random sample of the population of U.S. tax payers. They find that the increase in cross-sectional earnings inequality over 1987-2006 was permanent for male earnings and predominantly permanent for household income.

These studies, while not reflecting a complete consensus in the literature\(^ {16}\), suggest that the evolution of contemporaneous income inequality is close to the evolution of lifetime income inequality. This provides support for one of our simplifying modeling choices, the assumption of two income groups with fixed memberships. It also provides additional support for our calibration, whereby shocks to the income distribution are near-permanent.

**Alternative Debt Ratios**

It is sometimes argued that the more recent increases in household debt (2000-2007), which consisted to a large extent of mortgage loans, represented borrowing against houses whose fundamental value had risen, so that net debt increased by much less than gross debt, and debt-to-net-worth ratios would give a better indication of debt burdens than debt-to-income ratios. There are three interrelated responses to this argument. First, a similar pattern to the debt-to-income ratios in Figure 2 is also observed in debt-to-net-worth ratios, as we show in Figure 6. Second, a similar pattern is also observed for unsecured debt-to-income ratios, as we show in Figure 7. Third, the direction of causation between credit and house prices is of critical importance. Several recent empirical papers (Mian and Sufi (2009), Favara and Imbs (2010), Ng (2013), Adelino, Schoar, and Severino (2012)) argue that causation ran from credit to house prices, specifically that credit supply shocks caused

\(^{16}\)The findings of Kopczuk et al. (2010) and DeBacker et al. (2013) differ from previous results, based on PSID data, of Gottschalk and Moffitt (1994) and Blundell, Pistaferri and Preston (2008), who attribute a much larger role to increases in the variance of transitory earnings. However, they confirm the results of Primiceri and Van Rens (2009) who, utilizing repeated cross-sections of income from the Consumer Expenditure Survey, find that all of the increase in household income inequality in the 1980s and 1990s reflects an increase in the permanent variance.
house prices to increase above fundamental values.\footnote{Mian and Sufi (2009) also document that, in U.S. counties with a high share of subprime loans, income growth and credit growth were negatively correlated.} In light of these facts our theoretical model abstracts from collateralized borrowing and focuses on overall debt-to-income ratios.

IV. The Model

The model economy consists of two groups of infinitely-lived households, referred to respectively as top earners, with population share $\chi$, and bottom earners, with population share $1 - \chi$. Total output $y_t$ is given by an autoregressive stochastic process

\begin{equation}
\label{eq:y}
y_t = (1 - \rho_y) \bar{y} + \rho_y y_{t-1} + \epsilon_{y,t},
\end{equation}

where a bar above a variable denotes its steady state value. The share of output received by top earners $z_t$ is also an autoregressive stochastic process, and is given by

\begin{equation}
\label{eq:z}
z_t = (1 - \rho_z) \bar{z} + \rho_z z_{t-1} + \epsilon_{z,t}.
\end{equation}

The standard deviations of $\epsilon_{y,t}$ and $\epsilon_{z,t}$ are denoted by $\sigma_y$ and $\sigma_z$.

A. Top Earners

Top earners maximize the intertemporal utility function

\begin{equation}
\label{eq:U_t}
U_t = E_t \sum_{k \geq 0} \beta_T^k \left\{ \frac{(c_{t+k}^T)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \varphi \frac{(1 + b_{t+k} \frac{1-\chi}{\chi})^{1-\frac{1}{\eta}}}{1 - \frac{1}{\eta}} \right\},
\end{equation}

where $c_{t}^T$ is top earners’ consumption, $b_t \frac{1-\chi}{\chi}$ is top earners’ per capita tradable financial wealth, which takes the form of loans to bottom earners, $\beta_T$ is the discount factor, $\sigma$ is the intertemporal elasticity of substitution in consumption, $\eta$ parameterizes the curvature of the utility function with respect to wealth, and $\varphi$ is the weight of wealth in utility. These preferences nest the standard case of CRRA consumption preferences for $\varphi = 0$. One implication of preferences for wealth is that a unique, stable steady state for financial wealth $b_t$ exists.

When top earners lend to bottom earners, they offer $p_t$ units of consumption today in exchange for 1 unit of consumption tomorrow in case bottom earners do not default. In case bottom earners do default, top earners receive $(1 - h)$ units of consumption tomorrow, where $h \in [0, 1]$ is the haircut.
parameter, the proportion of loans defaulted on in a crisis. Bottom earners default only rarely, because doing so entails large income and utility losses, as explained in Section IV.C. Consumption of each top earner is given by

\[ c^\tau_t = y_t z_t \frac{1}{\chi} + (l_t - b_t p_t) \frac{1 - \chi}{\chi}, \]

where \( b_t \) is the amount of debt per bottom earner issued in period \( t \) at price \( p_t \), to be repaid in period \( t + 1 \), while \( l_t \) is the amount of debt per bottom earner repaid in period \( t \). The decision to default is given by \( \delta_t \in \{0, 1\} \), where \( \delta_t = 0 \) corresponds to no default and \( \delta_t = 1 \) corresponds to default. Then we have

\[ l_t = b_{t-1} (1 - h\delta_t). \]

Top earners maximize (3) subject to (4) and (5). Their optimality condition is given by

\[ p_t = \beta \tau E_t \left[ \left( \frac{c^\tau_{t+1}}{c^\tau_t} \right)^{-\frac{1}{\sigma}} (1 - h \delta_{t+1}) \right] + \varphi \left( \frac{1 + b_t \frac{1-\chi}{\chi}}{c^\tau_t} \right)^{-\frac{1}{\sigma}}. \]

This condition equates the costs and benefits of acquiring an additional unit of financial wealth. The cost equals the current utility loss from foregone consumption. The benefit equals not only next period’s utility gain from additional consumption, whose magnitude depends in part on the parameter \( \beta \tau \), but also the current utility gain from holding an additional unit of financial wealth, whose magnitude depends in part on the parameter \( \varphi \).

**B. Bottom Earners**

Bottom earners’ utility from consumption \( c^b_t \) has the same functional form as that of top earners, and their intertemporal elasticity of substitution takes the same value \( \sigma \). They do not derive utility from wealth.\(^{18}\) Their lifetime utility is given by

\[ V_t = E_t \sum_{k \geq 0} \beta^b \left\{ \left( \frac{c^b_{t+k}}{c^b_t} \right)^{1-\frac{1}{\sigma}} \right\}. \]

\(^{18}\)Heterogeneity in preferences between lenders and borrowers is a common assumption in the literature, but has so far mostly taken the form of assuming different rates of time preference combined with borrowing constraints (e.g. Iacoviello (2005)). In Bakshi and Chen (1996), the preference for wealth is specific to a social-wealth index, which captures the social group of reference, and is assumed to be increasing with the income of the group.
Bottom earners’ budget constraint is

\[ c^b_t = y_t (1 - z_t) (1 - u_t) \frac{1}{1 - \chi} + (b_t p_t - l_t) \],

where \( u_t \) is the fraction of bottom earners’ endowment that is absorbed by a penalty for current or past defaults. The output penalty \( y_t (1 - z_t) u_t \) represents an output loss to the economy. The fraction \( u_t \) is given by

\[ u_t = \rho_u u_{t-1} + \gamma_u \delta_t, \]

where the impact effect of a default is given by \( \gamma_u \), while the decay rate, in the absence of further defaults, is \( \rho_u \).

Bottom earners maximize (7) subject to (8) and (9). Their optimality condition for consumption is given by

\[ p_t = \beta_b E_t \left[ \left( \frac{c^b_{t+1}}{c^b_t} \right)^{-1} \left(1 - h \delta_{t+1}\right) \right]. \]

**C. Endogenous Default**

At the beginning of period \( t \) bottom earners choose whether to default on their past debt \( b_{t-1} \). This, together with the haircut parameter \( h \), defines the amount \( l_t \) that bottom earners repay during period \( t \), according to equation (5). Their lifetime consumption utility \( V_t \) is a function of the state of the economy \( s_t = (l_t, y_t, z_t, u_t) \), and is recursively defined by

\[ V_t(s_t) = \frac{(c^b_t)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + E_t [V(s_{t+1})]. \]

The decision to default \( \delta_t \) is a rational choice made at the beginning of the period, given a predefault state \( \hat{s}_t = (b_{t-1}, y_t, z_t, u_{t-1}) \), by comparing the lifetime consumption utility values of defaulting \( V_t^D = V(\hat{s}_t, \delta_t = 1) \) and not defaulting \( V_t^N = V(\hat{s}_t, \delta_t = 0) \). Bottom earners default when \( V_t^D - V_t^N \) is higher than an i.i.d. additive utility cost of default \( \xi_t \), as in Pouzo and Presno (2012). We can
therefore write the decision to default as:

\[
\delta_t = \arg\max_{\delta_t \in \{0, 1\}} \{ V_t^D - \xi_t, V_t^N \},
\]

where \(V_t^D = V(b_{t-1} (1-h), y_t, z_t, \rho_u u_{t-1} + \gamma_u)\) and \(V_t^N = V(b_{t-1}, y_t, z_t, \rho_u u_{t-1})\). The distribution of \(\delta_t\) depends upon the distribution of \(\xi_t\). We have the simple formula

\[
\text{prob}(\delta_t = 1 | \hat{s}_t) = \Xi (V_t^D - V_t^N),
\]

where \(\Xi\) is the cumulative distribution function of \(\xi_t\). We assume that \(\Xi\) takes the modified logistic form

\[
\Xi(x) = \begin{cases} 
\frac{\psi}{1 + e(-\theta x)} & \text{if } x < \tilde{x}, \\
1 & \text{if } x \geq \tilde{x}.
\end{cases}
\]

We assume that \(\tilde{x}\) is an arbitrarily large positive number, and that \(\psi < 1\). Together this implies that, over the economically relevant range, default occurs with positive probability but never with certainty. The parameters \(\psi\), \(\theta\), \(\gamma_u\) and \(\rho_u\) are calibrated to match the empirical evidence for the probability of crises, but with \(\gamma_u\) and \(\rho_u\) in addition constrained by the need to at least approximately match the evidence on the depth and duration of such crises. The parameter \(\psi\) helps to determine the mean level of crisis probability over the sample, while \(\theta\) determines the curvature of crisis probability with respect to the difference \(V_t^D - V_t^N\). The latter in turn can be shown to depend on debt levels. It can then be shown that for larger \(\theta\) the crisis probability is more convex in debt levels in the neighborhood of the original, low-debt steady state.

\[\textbf{D. Equilibrium}\]

In equilibrium top earners and bottom earners maximize their respective lifetime utilities, the market for borrowing and lending clears, and the market clearing condition for goods holds:

\[
y_t (1 - (1 - z_t) u_t) = \chi c_t^T + (1 - \chi) c_t^b.
\]

\[\text{Note that this differs from the probability of default that prevailed at the time the debt maturing at time } t \text{ was negotiated, which was conditional on } s_{t-1}, \text{ and which determined the interest rate risk premium.}\]
E. Analytical Results

A small number of key parameters of our model affects the speed at which bottom earners’ debt accumulates following a drop in their income share. Before explaining the calibration, it is therefore useful to analytically derive some relationships that clarify the role of these parameters.

Debt Supply and Debt Demand

In the deterministic steady-state, top earners’ and bottom earners’ Euler equations (6) and (10) can be interpreted as the hypothetical prices at which top earners and bottom earners would be willing to buy and sell debt while keeping their consumption constant. These equations therefore represent steady-state supply and demand functions for debt. Bottom earners’ demand price as a function of debt, \( p(b) \), is flat at

\[
(15) \quad p(b) = \beta_b,
\]

while top earners’ supply price as a function of debt is implicitly given by

\[
(16) \quad p(b) = \beta_T + \frac{\varphi \left( \frac{1}{\chi} \bar{z} + b(1-p(b)) \right)^{\frac{1}{\sigma}}}{\left(1 + \frac{1}{\chi} b \right)^{\frac{1}{\sigma}}}.
\]

By combining (15) and (16), one obtains the steady state relationship

\[
(17) \quad \frac{\beta_b - \beta_T}{\varphi} = \frac{\left( \frac{1}{\chi} \bar{z} + \bar{b}(1-\beta_b) \frac{(1-\chi)}{\chi} \right)^{\frac{1}{\sigma}}}{\left(1 + \frac{1}{\chi} \bar{b}(1-\chi) \right)^{\frac{1}{\sigma}}}.
\]

The numerator on the right-hand side, which equals \((\bar{c}^{\bar{c}^\bar{c}})^{1/\sigma}\), is always positive because consumption utility satisfies the Inada conditions. Equation (17) therefore shows that for any capitalist spirit model \((\varphi > 0)\), a steady state with positive debt of bottom earners \((\bar{b} > 0)\) requires the condition \(\beta_b > \beta_T\), which is therefore satisfied in our calibrations. However, this does not mean that top earners are more impatient than bottom earners. The reason is that the effective impatience of top earners is given by \(p(b)\) rather than simply by \(\beta_T\). Effective impatience is therefore endogenous to the level of debt, and the effective steady state impatience of top earners is equal to the impatience of bottom earners.

\[20\] The simplification of abstracting from default, for the purpose of this exercise, is justified by the fact that default has a negligible effect on the Euler equations in the neighborhood of the original steady state.
The left subplot of Figure 8 shows debt demand (15) and debt supply (16) for the baseline calibration, and then varies the weight of wealth in top earners’ utility function $\varphi$ while adjusting the parameter $\beta_\tau$ to remain consistent with an unchanging level of steady state debt. In other words the point at which the debt demand and debt supply schedules intersect, as well as steady-state consumption, remain unchanged. We observe that a higher $\varphi$ increases the slope of the credit supply schedule. If debt starts out below the equilibrium, a higher $\varphi$ therefore implies that top earners are willing to more aggressively lower interest rates on debt (raise the price of debt) to move towards an unchanged equilibrium.

**Response of Steady State Debt to Steady State Income**

Differentiating (17), we see the effect of an increase in top earners’ output share $\bar{z}$ on the steady-state debt level $\bar{b}$,

$$\frac{\partial \log(\bar{b})}{\partial \log(\bar{z})} = \frac{\frac{1}{\sigma} \left(\bar{y} \bar{z} \frac{1}{\chi}\right)}{\frac{1}{\eta} \frac{\bar{b} (1-\chi)}{\chi} \bar{c}^\tau - \frac{1}{\sigma} \frac{\bar{b} (1-\beta_b) (1-\chi)}{\chi}},$$

which is positive\(^{22}\) for any plausible calibration, implying that an increase in income inequality raises the steady state equilibrium level of debt. The right subplot of Figure 8 illustrates this point for an increase in top earners’ income share of 5 percentage points.

Equation (18) shows that, starting from an initial steady state $\bar{b}$, the extent of long-run financial wealth accumulation by top earners in response to a permanently higher income share depends on the key parameters $\sigma$ and $\eta$, but not on $\varphi$. A lower intertemporal elasticity of substitution $\sigma$ leads to more financial wealth accumulation. The reason is that a lower $\sigma$ increases the rate at which marginal utility falls in response to increases in consumption, while not affecting the response of marginal utility to increases in wealth. Following an increase in their income, top earners therefore limit the increase in their consumption more strongly, in order to accumulate more wealth. The accumulation of wealth also generates additional interest income, which amplifies the increase in wealth. This effect can be seen in the second term in the denominator of (18). A higher $\eta$, meaning a lower curvature of the utility function with respect to wealth, has the same effect as a lower $\sigma$. In other words, what matters is the relative size of the two.

\(^{21}\)Recall that $\pi^\tau = \bar{y} \bar{z} \frac{1}{\chi} + \bar{b} (1-\beta_b) \frac{(1-\chi)}{\chi}$.

\(^{22}\)It can be shown that the denominator is positive if and only if the level of debt is below some (typically very large) upper bound. In our baseline calibration, the upper bound is equal to 14111 while steady-state debt is equal to 0.49.
F. Calibration

The model is calibrated to match key features of the U.S. economy over the period 1983-2008. Because our study concerns longer-run phenomena, we calibrate the model at the annual frequency. Table 1 summarizes the calibration.

Steady state output is normalized to one, \( \bar{y} = 1 \). Top earners and bottom earners correspond to the top 5% and the bottom 95% of the income distribution, respectively, \( \chi = 0.05 \). The steady-state net real interest rate is fixed at 4% per annum, similar to values typically used in the real business cycle literature, by fixing bottom earners’ time discount factor at \( \beta_b = 1.04^{-1} \). The intertemporal elasticity of substitution, again following many papers in the business cycle literature, is fixed at \( \sigma = 0.5 \) for both top and bottom earners.

As explained in the previous subsection, the relative magnitudes of \( \eta \) and \( \sigma \) affect the long-run magnitude of top earners’ financial wealth accumulation in response to a permanent increase in their income share, while the magnitude of the parameter \( \varphi \) affects its speed. We set \( \eta = 1/0.7 \) and \( \varphi = 0.0135 \) to match the post-1983 profile of financial wealth accumulation, which equals bottom earners’ debt accumulation, and note that the relationship \( \eta > \sigma \) is consistent with the capitalist spirit specification of Carroll (2000). This leaves \( \beta \) to be determined so as to match the initial steady state debt-to-income ratio. Sensitivity analysis will be performed for all of these parameters.

For the calibration of the remainder of the model’s initial steady state, we note that one of our main objectives is to construct a scenario that, for the period from 1983 to 2008, exactly matches the evolution of the top 5% income share, and that approximately matches the debt-to-income ratio of bottom earners and the associated probability of a major crisis. We therefore calibrate the initial steady state top 5% income share and debt-to-income ratio to be equal to their 1983 counterparts.

For the 1983 value of the top 5% income share we use the data computed by Piketty and Saez (2003, updated). These data are based on annual gross incomes derived from individual tax returns, where gross income includes interest payments received by households on their fixed income assets, but excludes interest payments made on their debt liabilities. The counterpart of this measure in the model includes, in the numerator, an exogenous component, the output share of top earners, and an endogenous component, the interest payments received by top earners on non-defaulted financial assets. The denominator is the sum of total output net of default penalties and the interest payments received by top earners. The formula is given by

\[
\tau_t = \frac{y_t z_t + (1 - \chi) l_t (1/p_{t-1} - 1)}{y_t (1 - u_t (1 - z_t)) + (1 - \chi) l_t (1/p_{t-1} - 1)}. \tag{19}
\]
We choose \( z \) to replicate \( \bar{\tau} \) in 1983, which equals 21.8%.

For the 1983 value of bottom earners’ debt-to-income ratio, we note that the model counterpart of that ratio is given by

\[
\ell_t = \frac{(1 - \chi) b_t}{y_t (1 - z_t) (1 - u_t)}.
\]

We choose \( \beta_\tau \) to replicate \( \bar{\ell} \) in 1983, which equals 62.3%.

We now turn to the calibration of the characteristics of default events. We begin with the haircut, or percentage of loans defaulted upon during the crisis, which we set to \( h = 0.1 \), consistent with the empirical evidence discussed in Section III.F. The probability of a crisis at any point in the state space depends on the output and utility costs of default, which are discussed in the following two paragraphs.

Output costs alone would sharply divide the state space into regions of certain non-default and certain default, separated by a default frontier where \( V_t^D = V_t^N \), while random utility costs give rise to a continuum of gradually increasing default probabilities, with the default frontier being replaced by the center of the default region, where bottom earners default for \( V_t^D = V_t^N \) with a probability that equals one half of the maximum probability (see below). As in the sovereign default literature, the size of output costs is calibrated so as to move the center of the default region towards plausible values. We calibrate the impact output cost at 4% of the income of bottom earners, \( \gamma_u = 0.04 \), and the decay rate at \( \rho_u = 0.65 \). Given that bottom earners’ income share equals less than 70% at the time of the crisis, this corresponds to a slightly less than 3% loss in aggregate output on impact, and a cumulative output loss of around 8% of annual output. We note that these output losses are lower than what was observed during the Great Recession. However, few observers had even anticipated the Great Recession, and once it had started most observers initially underestimated its full severity (Dominguez and Shapiro (2013)). With these output costs the center of the default region is located at a loan-to-income ratio of approximately 200%.

The random utility costs are calibrated to match the order of magnitude of the Schularick and Taylor (2012) probabilities of a major crisis over our sample period. To reproduce the fact that default probabilities generally remain below 10% according to the computations of Schularick and Taylor (2012), we set \( \psi = 0.2 \). This implies that the maximum theoretically possible default probability, at extremely high debt levels, is 20%,\(^{23}\) with actual default probabilities generally well below that. By setting \( \theta = 25 \) we ensure that the relationship between debt and default probability

\(^{23}\)In other words, in 80% of all periods the random utility cost for defaulting is prohibitively high.
is convex over the empirically relevant range, meaning that the default probability increases at an increasing rate as debt increases.

We estimate the exogenous stochastic process for output $y_t$, equation (1), using the detrended series of U.S. real GDP from 1983 to 2008 obtained from the BEA. Our estimates are $\rho_y = 0.669$ and $\sigma_y = 0.012$. For the years 1983-2008 of the baseline calibration we calibrate the shocks $\epsilon_{y,t}$ to exactly match the detrended data, while all subsequent shocks are set to zero for the simulations.

Next we specify the parameters $\rho_z$ and $\sigma_z$ of the exogenous stochastic process for the top 5% output share $z_t$ (equation (2)) such that the behavior of the top income share in equation (19) matches that of the updated Piketty and Saez (2003) data series from 1983 to 2008. Using standard tests, the hypothesis that this data series has a unit root cannot be rejected. In the baseline scenario, we therefore calibrate $\rho_z = 0.999$ and then estimate $\sigma_z = 0.008$ from the data. With $\rho_z = 0.999$ increases in income inequality are near-permanent, with a half-life for $z_t$ of around 700 periods. This makes the effects of an increase in income inequality over our 50-year simulation periods quantitatively indistinguishable from a fully permanent shock. Finally, as for output shocks, for

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**Table 1—Calibration of the Baseline Model**

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<td>Steady-State Top 5% Income Share $\bar{\tau}$</td>
<td>Data: 21.8% in 1983</td>
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<td>Steady-State Debt-to-Income Ratio $\bar{\ell}$</td>
<td>Data: 62.3% in 1983</td>
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<td>Haircut (% of Loans Defaulted)</td>
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<td>and Depth of Crisis</td>
<td>and Depth of Crisis</td>
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24With the preference for wealth specification, choosing $\rho_z < 1$ ensures that the model is locally stable, which greatly simplifies the analysis and computations.
the years 1983-2008 of the baseline calibration we calibrate the shocks $\epsilon_{z,t}$ such that $\tau_t$ exactly matches the data, with all subsequent shocks set to zero.

**G. Solution Method**

The above model has two features that advise against the application of conventional perturbation methods. The first is the presence of default, which implies large discrete jumps in state variables. The second is the fact that the stochastic process for income shares $z_t$ is extremely persistent, which in our simulations implies that bottom earners’ debt-to-income ratio drifts far away from its original steady state for a prolonged period.

We therefore obtain a global nonlinear solution using a variant of a time-iterative policy function algorithm, as described by Coleman (1991). This solution technique discretizes the state space and iteratively solves for updated policy functions that satisfy equilibrium conditions until a specified tolerance criterion is reached.

All variables are solved on a regularly spaced grid and interpolated using natural cubic splines, whose second-order derivatives are set to zero on the boundary of the grid. The state-space is approximated by a $50 \times 10 \times 10 \times 5$ rectangular grid over $(l_t, y_t, z_t, u_t)$, with boundaries $0 \leq l_t \leq 4$, $0.9 \leq y_t \leq 1.1$, $0.1 \leq z_t \leq 0.4$, and $0 \leq u_t \leq 1$.

At each iteration we solve for the optimal policies and the value function of bottom earners. We do so simultaneously on the entire grid, which produces large vectorization gains. The default decision is implicitly characterized by the value function of bottom earners. Fully documented MATLAB solution routines are available from the authors’ website.\textsuperscript{25}

**V. Results**

Figures 9–15 present simulation results that first explore the properties of the model, and then its ability to match the behavior of key historic time series that pertain to the inequality-leverage-crises nexus. Figure 9 displays regions of the state space with specified default probabilities. Figures 10–12 show impulse responses for output, income share and crisis shocks. Figure 13 presents our main scenario for the period 1983–2030, for the baseline model specification with preferences for wealth and the baseline model calibration. Figure 14 presents sensitivity analysis that varies aspects of that calibration. Figure 15 presents the same simulation for an alternative model specification without preferences for wealth, with debt accumulation solely due to consumption smoothing.

\textsuperscript{25}http://www.mosphere.fr/files/krw2013/doc/
A. Default Regions

In our model, due to random utility costs, the state space is divided into regions with a continuum of probabilities of default. Figure 9 contains a visual representation that divides the state space into regions whose boundaries represent default probabilities that increase in equal increments of 2 percentage points. Each subplot shows the debt-to-income ratio on the horizontal axis and output on the vertical axis. The effect of variations in the third state variable, the top income share, is illustrated by showing two separate subplots, corresponding to the 1983 top income share of 21.8% and the 2008 top income share of 33.8%.

We observe that higher debt levels imply a higher crisis probability, by increasing the benefits of defaulting without affecting the costs. Over the historically observed range of debt levels, the implied crisis probabilities range from around 1% to around 5%. As is standard in this class of models, default is more likely to occur when output is low, because at such times the insurance benefits of default are high while the output costs of default are low. Over the observed range of income and debt levels, these effects are not as large as the effects of higher debt. However, small drops in income start to have a significantly larger effect on default probabilities as we move from regions of low debt to regions of very high debt. In other words, with high debt levels the economy becomes more fragile and vulnerable to small output shocks that would not have mattered very much at lower debt levels. For the same reasons as for lower output, higher top income shares also lead to higher crisis probabilities. But their direct effect, beyond the effect that operates through higher debt accumulation, is modest.

Finally, the width of the default probability regions in Figure 9 shows the effects of the logistic distribution of the random utility cost. First, over the historically observed range of debt levels, the default probability is convex in the debt level, where the steepness of the rate of increase depends on the parameter $\theta$. Second, for extremely high debt levels the default probability becomes concave in the debt level, with a maximum at 20% that corresponds to the parameter $\psi$.

B. Impulse Responses

Figure 10 shows a one standard deviation positive shock to aggregate output $y_t$. This shock allows both top earners and bottom earners to increase their consumption, so that the equilibrium loan interest rate drops by around 70 basis points on impact, with a subsequent increase back to its long-run value that mirrors the gradual decrease in output. The drop in the interest rate represents an additional income gain for bottom earners relative to top earners, so that the top 5% income share falls by around 0.35 percentage points on impact and then gradually returns to its long-run
value. Bottom earners smooth their income gain over time by decreasing their debt-to-income ratio by just under one percentage point on impact, while still increasing their consumption by almost twice as much as top earners. The latter is due to their gains from the lower real interest rate.

Figure 11 shows a one standard deviation near-permanent shock to the output share \( z_t \). The top 5% income share immediately increases by around 1 percentage point, accompanied by a downward jump of around 0.5% in bottom earners’ consumption and an upward jump of around 3% in top earners’ consumption. The long-run increase in top earners’ consumption is even larger, because they initially limit their consumption in order to accumulate more financial wealth. The process of wealth and thus debt accumulation takes several decades, with bottom earners’ debt-to-income ratio increasing by well over 10 percentage points in the long run, accompanied by an increase in crisis probability of just under 0.2 percentage points. The real interest rate falls on impact by 5 basis points, due to the increase in credit supply from top earners that initially limits the drop in consumption of bottom earners. The top 5% income share \( \tau_t \), because it includes not only the output share \( z_t \) but also the interest earnings on increasing financial wealth, increases in the long run by approximately another 0.5 percentage points, and top earners’ long-run increase in consumption is correspondingly larger. Similarly, the long-run decline in bottom earners’ consumption is larger than in the short run. We note that the one standard deviation income distribution shock in Figure 11 is small compared to what occurred in the three decades since the early 1980s.

Figure 12 shows the impulse response for a crisis shock. Bottom earners default on 10% of their loans, but they also experience a 4% loss in income due to the output costs of default, which are suffered exclusively by this group of agents. As a result their debt-to-income ratio only drops by around 1.5 percentage points. The impact effect on the real economy is a 3% loss in GDP, followed by a V-shaped recovery. The real interest rate mirrors developments in output, with an initial increase of almost 2.5 percentage points followed by a return to the original interest rate level after about a decade. Consumption of top earners and bottom earners follows an almost identical profile after the crisis. Top earners experience no change in their endowment income. They suffer a loss on their financial wealth, but in terms of income this is more than compensated by the temporary increase in the real interest rate, so that the top 5% income share increases by around 1.5 percentage points on impact. Top earners lend after the crisis, in order to return their financial wealth to the desired level. The counterpart of this is reborrowing by bottom earners.26 As a result

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26 A model with borrowing constraints would limit reborrowing. However, data from the crisis period show that, while the crisis stopped mortgage debt from increasing further, unsecured debt kept increasing. In 2009, the Federal Reserve Board re-surveyed the same households that were surveyed in the 2007 Survey of Consumer Finance. The resulting panel data show that, between 2007 and 2009, the ratio of mortgage debt to income of the bottom 95% remained virtually unchanged, while the ratio of unsecured debt to income increased from 25.2% to 28.1%.
of reborrowing, bottom earners’ debt-to-income ratio is almost back to its original level after about one decade.

C. Baseline Scenario

Figure 13 shows the central simulation of the paper. The variables shown are the same, and are shown in the same units, as in the impulse responses in Figures 10-12. The horizontal axis represents time, with the simulation starting in 1983 and ending in 2030. The red lines with circular markers represent U.S. data, while the black lines represent model simulations. The data for GDP and for the top 5% income share are used as forcing processes that pin down the realizations of the shocks $\epsilon_{y,t}$ and $\epsilon_{z,t}$ between 1983 and 2008. Additional data for GDP and the top 5% income share are shown for 2009 and 2010, but are not used as forcing processes. We assume that a crisis shock hits in 2009. Thereafter, and starting in 2009, the model is simulated assuming a random sequence of utility cost shocks, but no further nonzero realizations of output or output share shocks. Because the preceding shocks imply further increases in debt after 2009, this means that future endogenous crises remain a possibility, and in fact become increasingly likely. The subplots also report 1983-2010 data for bottom earners’ debt-to-income ratio, and a time series for crisis probability computed using the methodology of Schularick and Taylor (2012). These are not used as forcing processes, but rather are used to illustrate the ability of the model to approximately replicate the behavior of these variables.

The key forcing variable is the increase in the top 5% income share from 21.8% in 1983 to 33.8% in 2008. One effect is an increasing wedge between top earners’ and bottom earners’ consumption, with the former increasing their consumption by more than a cumulative 56% until just prior to the Great Recession, and the latter reducing their consumption by a cumulative 8%. In addition, top earners, in order to increase their wealth in line with their near-permanent increase in income, initially limit the increase in their consumption in order to acquire additional financial wealth, in other words to lend to bottom earners. Bottom earners’ debt-to-income ratio therefore increases from 62.3% in 1983 to 143.2% in 2008, accompanied by an increase in crisis probability from initially around 1.5% in any given year, to 4.9% in 2008. The simulations of both the debt-to-income ratio and the crisis probability match the behavior of the data very closely.

The simulated top 5% consumption share, which is shown in the same subplot as the top 5% income share, also increases between 1983 and 2008, but by less than the income share. This is a necessary consequence of the fact that bottom earners are borrowing, and are therefore maintaining higher consumption levels than what their income alone would permit. There is an ongoing
debate in the empirical literature about the relative evolution of consumption inequality and income inequality.\textsuperscript{27} The results of this literature are however not directly comparable to our results, because it has so far not produced an empirical estimate of the top 5\% consumption share that would correspond to our model simulations.

The crisis event in 2009 has only a modest effect on bottom earners’ debt-to-income ratio in the model, which drops by around 4 percentage points but then immediately resumes its upward trajectory. The reason is that the income share of bottom earners deteriorates, both because of the output costs of the crisis and because of higher real interest rates. For 2009 the model in fact simulates a worse income share for bottom earners than what we see in the data. One reason is that our model omits monetary policy, which has clearly played a major role in preventing the major increase in real interest rates following the crisis that is predicted by the model.

For the future, the model predicts a further increase of the income share of top earners, not because of further increases in their output share z_t but rather because of further increases in debt and associated interest charges. Bottom earners’ simulated debt-to-income ratio increases from around 150\% to over 200\% over the post-crisis decade, accompanied by an increase in crisis probability from around 5\% to well over 10\%. Under the random sequence of utility cost shocks used in our simulation, the model generates two subsequent crises, in 2023 and 2028.\textsuperscript{28}

\section{D. Sensitivity Analysis}

Figure 14 illustrates how a key quantitative result of our baseline scenario, the growth of bottom earners’ debt-to-income ratio during the pre-crisis period 1983-2008, depends on the structural parameters of the model. In each case we vary one parameter of interest, the intertemporal elasticity of substitution in consumption $\sigma$, the curvature of top earners’ utility function with respect to wealth $\eta$, and the weight on wealth in top earners’ utility function $\phi$, while adjusting $\beta_\tau$ in order to keep the initial steady state debt-to-income ratio of bottom earners unchanged.

The results in Figure 14 are consistent with the discussion in Section IV.E. Top earners’ response to near-permanently higher income is to allocate the additional income to either higher consumption or to additional wealth accumulation, in proportions that ensure that the marginal benefits of the two are equalized. Lower $\sigma$ or higher $\eta$ imply that the curvature of the utility function with respect to consumption becomes stronger relative to the curvature with respect to wealth. This implies

\textsuperscript{27} Krueger and Perri (2006) argue that consumption inequality increased by much less than income inequality between 1983 and 2003. These results have recently been challenged by Aguiar and Bils (2012), who estimate that the increases in consumption and income inequality mirror each other much more closely. Attanasio et al. (2012) confirm the results of Aguiar and Bils (2011). By contrast, Meyer and Sullivan (2013) find that the rise in income inequality has been more pronounced than the rise in consumption inequality.

\textsuperscript{28} The model simulations for the very long run see the economy returning to its initial steady state (recall that $\rho_z = 0.999$).
that top earners will allocate a smaller share of their additional income to consumption, in order to accumulate a permanently higher level of financial wealth. The interest rate temporarily drops to ensure that this higher credit supply is taken up by bottom earners, who end up with a higher debt-to-income ratio. These effects are clearly visible in Figure 14. A higher $\varphi$ on the other hand implies that top earners have a stronger desire to eliminate differences between their current and long-run levels of financial wealth, while the desired long-run level of financial wealth does not change relative to the baseline scenario. This is also clearly visible in Figure 14, which shows that the transition is faster for higher $\varphi$.

E. Pure Consumption Smoothing and Shock Persistence

In our baseline scenario the increase in bottom earners’ debt is due to increased credit supply from top earners. The reason is that shocks to the income distribution are near-permanent ($\rho_z = 0.999$), so that bottom earners have a quantitatively negligible incentive to smooth consumption, while top earners have a strong incentive to accumulate wealth.

While the near-permanence of income distribution shocks is consistent with the evolution of income inequality between 1983 and 2008, and with the evidence discussed in Section III.G., it is nevertheless interesting to ask what quantitative role pure consumption smoothing, in the complete absence of a wealth accumulation motive, could play if shocks to the income distribution were perceived to be more temporary. In that case top earners have no motive to accumulate wealth for its own sake, while both bottom and top earners have a stronger incentive to borrow and lend to smooth consumption. In other words, in such a world there is an increased role for credit demand relative to credit supply.

For this exercise we perform another variation of our baseline simulation in which wealth does not enter the utility function of top earners, and both income groups have the same discount factor ($\varphi = 0, \beta_b = \beta_T = 1.04^{-1}$). The initial values of all endogenous variables are identical to the baseline case. Under this calibration top earners no longer wish to gradually accumulate more wealth due to a preference for wealth. However, as long as shocks to the income distribution are temporary, this is now replaced by a consumption smoothing motive, as bottom (top) earners borrow (lend) in anticipation of future higher (lower) incomes. We set the persistence of the inequality process to $\rho_z = 0.98$, because under this calibration, as shown in Figure 15, the effects of consumption smoothing are quantitatively similar to those of preferences for wealth in the baseline. The simulation in Figure 15 would however change dramatically for less persistent shocks to

\[^{29}\]This implies a half-life of income inequality shocks of 35 years, rather than 700 years as in the baseline.
In that case the consumption smoothing motive would become very much stronger, because it would imply that bottom earners continually expect their income to revert to a much higher level over a fairly short period. The cumulative effect of this perception, which of course would represent large and one-sided forecast errors over the 1983-2008 period, would be a much larger build-up of debt.

VI. Conclusions

This paper has presented stylized facts and a theoretical framework that explore the nexus between increases in the income share of high-income households, higher debt leverage among poor and middle-income households, and vulnerability to financial crises. We provide evidence which suggests that this nexus was prominent prior to both the Great Depression and the Great Recession. In our baseline theoretical model higher debt leverage arises as a result of near-permanent positive shocks to the income share of high-income households who, due to preferences for wealth, recycle part of their additional income back to the remaining households by way of loans. This increase in credit supply allows poor and middle-income households to sustain higher consumption levels. But the result is that loans keep growing, and therefore so does the probability of a crisis that, when it happens, is accompanied by a contraction in the real economy. This contraction in turn implies that the effect of a crisis on debt leverage and therefore on the probability of further crises is quite limited.

It is possible to use our framework to simulate alternative scenarios for the future of the U.S. economy. One alternative, studied in Kumhof, Rancière and Winant (2013), looks at the consequences of a reversal of the post-1983 increase in income inequality over a period of 10 years. We find that this would lead to a sustained reduction in leverage that would significantly reduce the probability of further crises.
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Figure 1. Income Inequality and Household Leverage


Figure 2. Debt-to-Income Ratios by Income Group

Wealth is measured by net worth, which equals the difference between the total value of household assets and the stock of all outstanding household debt liabilities.

**Figure 3. Wealth Inequality**

**Figure 4. Size of the Financial Sector (Value Added/GDP)**
Source: Authors’ calculation based on Schularick and Taylor (2012).

**Figure 5. Leverage and Crisis Probability**

Source: Survey of Consumer Finance (triennial), 1983-2007. Debt corresponds to the stock of all outstanding household debt liabilities. Net worth corresponds to the difference between the total value of household assets and the stock of all outstanding household debt liabilities.

**Figure 6. Debt-to-Net-Worth Ratios by Income Group**

Source: Survey of Consumer Finance (triennial), 1983-2007. Unsecured debt corresponds to the difference between the stock of all outstanding household debt liabilities and the amount of outstanding household debt liabilities secured by residential properties. Income corresponds to annual income before taxes, in the year preceding the survey.

**Figure 7. Unsecured Debt-to-Income Ratios by Income Group**

Source: Survey of Consumer Finance (triennial), 1983-2007. Unsecured debt corresponds to the difference between the stock of all outstanding household debt liabilities and the amount of outstanding household debt liabilities secured by residential properties. Income corresponds to annual income before taxes, in the year preceding the survey.
Figure 8. Steady-State Debt

Figure 9. Default Regions
Figure 10. Impulse Responses - Output Shock

Figure 11. Impulse Responses - Income Distribution Shock
Figure 12. Impulse Responses - Crisis

Figure 13. Baseline Scenario
Figure 14. Sensitivity Analysis

Figure 15. Consumption Smoothing Scenario