Steady state
Golden Rule: $f'(k) = n$

Dynastic guys $\rightarrow f'(k) = n + \theta$ with θ = discount rate.
$\rightarrow$ gov^t can push k accumul. ↑ with subsidy $\rightarrow$ how would it look?
Piketty '01?
Max $u(c_\infty, \ell_\infty)$ Steady state $f'(k) = n = r$
$\rightarrow$ accumulates if Steady $f'(k) = (1-\tau_k)\frac{f'(k)}{n} = \delta$ $\boxed{1-\tau_k = \frac{\delta}{n}}$
But can erase inequality entirely.

Chapter 1

Savings and Taxation

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1.1 INTRODUCTION

The way in which taxes affect savings behaviour has attracted much recent attention both theoretically and empirically. Developments in the theory of optimal taxation and econometric studies of the role of interest rates in the consumption function have stimulated thinking in this area, and both are highly relevant to issues currently at the centre of policy debate about tax reform. In 1978 the Meade Committee published a lengthy and detailed report analysing the failures of the British tax system and recommended that it be converted gradually into a personal-expenditure tax system (Meade Committee, 1978).

One of the most instructive features of the report was that it did not rely on the traditional arguments for an expenditure tax (that expenditure is a more just tax base than income and that an expenditure tax avoids the efficiency losses associated with the double taxation of savings implied by an income tax) but instead based its case on more practical grounds. The present tax system is neither an income tax nor an expenditure tax system, and its problems stem from this hybrid nature. The real case for an expenditure tax, so the argument runs, is that an expenditure tax would be much simpler to operate than a pure income tax and, indeed, that the latter would be almost infeasible. (For a detailed examination of these arguments, see Meade Committee, 1978, and Kay and King, 1978.) Similar propositions have been put forward in the United States by the report of the US Treasury (1977) and in Sweden by Lodin (1978).

Although the Meade Committee's report does not contain an evaluation of the efficiency arguments, it does not reject these as potentially insignificant; rather, the compelling arguments for a change lie elsewhere. Nevertheless, it is clearly important to assess the efficiency gains or losses that might be involved in any major reform. In this chapter we shall try to explore, in the context of a very simple model, the efficiency arguments for and against an expenditure tax and to assess some recent claims that the welfare gains from abolishing capital income taxes would be very large.

In section 1.2 we shall examine the optimal taxation of capital and labour incomes in a simple growth model and derive formulae for the optimal tax rates. These are used in section 1.3 to evaluate claims that abolishing capital income taxes would lead to large welfare gains. Inflation is introduced in section 1.4, and alternative approaches to modelling savings behaviour are discussed in section 1.5. Finally, we shall look briefly at some of the empirical evidence on the effects of taxes on savings.

Our analysis will be highly simplified. We shall ignore many of the issues stressed by the Meade Committee, such as the complex interaction between personal and corporate taxation, the sheer diversity of tax rates currently imposed on different forms of saving, and the portfolio aspects of personal saving. The relationship between expenditure on durables and saving and the effect of social security on consumption will also be left to one side, and we shall say little about the production side of the economy. (For surveys of the effects of taxes on investment, see Helliwell, 1976; King, 1977; and von Furstenberg and Malkiel, 1977.) Despite these omissions the model captures the essential features necessary to an evaluation of the efficiency arguments.

1.2 THE OPTIMAL TAXATION OF SAVINGS

The optimum taxation of savings, or equivalently, capital income, has long been the subject of debate by economists in terms of the choice of the tax base. Both a comprehensive income tax, in which capital income is taxed at the same rate as labour income, and an expenditure tax, in which capital income is not taxed at all, have fervent supporters and even more fervent opponents. With the development of the theory of optimal indirect taxation it is natural to apply the results of these theoretical investigations to the problems of intertemporal choice.

Ramsey's (1927) classic paper on optimal commodity taxes does discuss this question; he concluded that capital income should be taxed at lower rates than labour income but should not be exempt from tax altogether. But only very recently have economists tackled this question in more detail, and at this early stage it is perhaps not surprising that 'the subject remains clouded in confusion' (Feldstein, 1978a). Before examining a formal model we may relate the question to two traditional arguments often deployed in favour of an expenditure tax as opposed to an income tax.

A frequent argument in favour of an expenditure tax is that it is more 'just' to tax someone on the value of what he takes out of society, in terms of the goods and services that he consumes, than on the value of what he contributes to society, whether in the form of wage income in return for labour services or as capital income in return for the supply of capital services. It is common to cite the quotation of Hobbes (1651), popularised by Kaldor:

> What reason is there, that he which laboureth much and sparing the fruit of his labour, consumeth little, should be more charged, than he that liveth idly, getteth little, and spendeth all he gets; seeing the one hath no more protection from the commonwealth than the other?

Unfortunately, this quotation from Hobbes seems to have been misunderstood by many. First, there is simply no obvious reason for believing that a tax on actual consumption is more just than a tax on potential consumption. Is it just to treat a disabled beggar and a wealthy miser in the same way? Secondly, Hobbes's example is misleading. The reason for the unfairness is that, although one individual enjoys a good deal of leisure ('liveth idly') while his neighbour does not ('laboureth much'), this is not reflected in his tax payments. But this has nothing to do with the distinction between income and consumption. Under an expenditure tax the man who worked hard, saved and wanted to spend his money at some point in the future would face a heavy tax liability. Both an income tax and an expenditure tax fail to tax leisure and so provide a disincentive to work effort. Unless we believe that lump sum taxes are feasible, there is nothing that we can do about this. In assessing the economic effect of alternative systems of taxing capital income we must not overlook the efficiency loss involved in the distortion of the work-leisure choice.

Another traditional argument often used to advance the cause of an expenditure tax is that an income tax is inefficient because it gives rise to the 'double taxation of savings'. With an expenditure tax the rate of tax on consumption is (for a given rate schedule) the same regardless of the year in which the individual chooses to consume. An income tax, on the other hand, reduces the rate of return on personal savings below the market rate of interest. This produces an efficiency loss because, so that argument runs, the market rate of interest is equal to the rate of return on investment, and hence the income tax places a wedge between the intertemporal rates of substitution in consumption and transformation in production.

However, the market rate of interest may not be equal to the rate of return on investment, and one of the principal causes of a divergence here is the corporate tax system. The effects of the corporate tax system on the cost of capital are important to an evaluation of the efficiency loss resulting from an income tax, and it is possible to design a corporate tax system that largely offsets the effects of income tax. (For a detailed discussion of this, see King, 1977, ch. 8.) In this chapter, however, we shall assume that the pretax rate of interest equals the marginal product of capital.

Even if an income tax did produce substantial intertemporal inefficiency, standard 'second-best' arguments suggest that this would not be an overwhelming argument for an expenditure tax. The 'optimal' tax on income from savings must take into account both intertemporal inefficiency and also the static welfare loss arising from the distortion of the work–leisure choice by both income and expenditure taxes, which has been discussed above.

To explore this issue formally we shall begin by considering a simple two-period model similar to that analysed by Samuelson (1958) and Diamond (1965) and, in the context of taxes, by Atkinson and Sandmo (1979) and Auerbach (1978).[1] We shall make the following assumptions:

(1) Each individual lives for two periods. They work in period 1, supplying L units of labour (the length of the period is normalised to unity), and retire in period 2. They consume in both periods, and all individuals within a generation are identical (thus we ignore the problem of distribution within a generation). Preferences are described by the direct utility function

$$U(C_1, C_2, L) \tag{1}$$

where C_i is *per capita* consumption in the ith period of life.

(2) There are no bequests, and saving in period 1 is solely to provide consumption in period 2. (The taxation of bequests is discussed further by Atkinson in Chapter 2 below.)

(3) In any given period two overlapping generations are alive. There is a constant growth rate of population n, and so the younger generation is larger than the elder by a fraction $(1 + n)$. There is no technical progress, and there is steady state growth at the rate n.

(4) Output is given by the following constant-returns-to-scale production function, and for convenience we shall assume that capital does not depreciate; that is,

$$Y = LNf(k) \tag{2}$$

where k is capital per man-hour, N is the number of workers and Y is total output.

As far as the tax system is concerned, we shall assume that lump sum

taxes are infeasible and that the government imposes *proportional* taxes on labour and capital incomes at the rates t_w and t_r respectively. Of course, some lump-sum taxation is feasible, but we shall suppose that there is a limit to the amount that can be raised by this means, and the government's revenue requirement may therefore be regarded in what follows as net of lump sum taxes. To simplify the analysis we shall also ignore non-linear tax schedules.

If we denote the pretax values of the wage rate by w and the interest rate by r, we may write the individual's budget constraint as

$$C_1 + hC_2 + w(1 - t_w)(1 - L) = w(1 - t_w) \tag{3}$$

where the price of second-period consumption is defined by

$$h = \frac{1}{1 + (1 - t_r)r} \tag{4}$$

The term on the right hand side of (3) is exogenous income, sometimes known as 'full' income.

The first-order conditions for an individual's maximising (1) subject to (3) are

$$\frac{U_2}{U_1} = h \tag{5}$$

$$\frac{U_L}{U_1} = -(1 - t_w)w \tag{6}$$

where U_i denotes the partial derivative of U with respect to argument i. The solution to equations (3), (5) and (6) describes individual behaviour as a function of factor prices and the tax rates imposed by government.

The steady state level of capital per head is given by the equilibrium condition that planned investment equals planned savings. Net investment equals the savings of the generation at work minus the dissavings of the retired (which equals the existing capital stock because the retired leave no bequests). Hence,

$$nkL = (1 - t_w)wL - C_1 - kL \tag{7}$$

Therefore, from (3) we get

$$(1 + n)kL = hC_2 \tag{8}$$

We shall assume that the government wishes to raise in each period a fixed amount of revenue g per *worker*. The government's budget constraint is given by

$$g = t_w wL + \frac{hrt_r C_2}{1+n} \tag{9}$$

The government chooses values of the two tax rates to maximise its objective function subject to the constraints implied by (3), (5), (6), (8) and (9). There are two aspects of this decision. The first is that relative tax burdens imposed on wage and interest income will reflect the need to balance the distortions in individual work–leisure and present–future consumption choices. Equations (5) and (6) show how the tax rates introduce a wedge between factor prices and rates of substitution. This is the standard efficiency argument underlying the theory of optimal taxation. But there is a second aspect to the government's decision problem, which arises from the dynamic nature of the model and has profound implications for the analysis of the welfare effects of capital income taxation. From equation (8) we can see that one of the effects of taxation is to influence the equilibrium level of capital per man-hour. This determines the path of consumption over time. Individuals in our model are concerned only with their own consumption and ignore future generations. In contrast, we shall *assume* that the government takes a view about intergenerational equity.

As yet we have not specified the government's objective function, and the precise form that we adopt will determine the optimal path of consumption over time. A natural specification in a steady state framework is that the government maximises the steady state level of utility. The level of capital intensity that maximises this objective function is given by the well-known 'golden rule' condition that the marginal product of capital equals the growth rate:

$$f'(k) = n \tag{10}$$

If the government has no policy instruments other than the two tax rates, then in general it will be unable to achieve the golden rule condition, and the optimal tax rates will reflect the trade-off not only between the conventional efficiency losses but also between these and the losses resulting from the failure to achieve the dynamic optimality condition (equation 10).

We shall see below that the optimal tax rate on capital income is very sensitive to this second factor. This is rather disturbing, because it alerts

us to the possibility that our conclusions are sensitive to the assumptions that we make about policy instruments excluded from the model. For example, if the government could use debt policy to determine the rate of interest in the economy, it could attain the golden rule, and the optimal tax rates would be independent of dynamic considerations. Clearly, the welfare implications of particular tax changes (such as the replacement of the income tax by a consumption tax) depend upon the constraints that are, or are not, assumed to restrict the use of other policy instruments. Although this is a standard 'second-best' argument, it is very important to note that one of the principal sources of disagreement over the potential welfare gains from tax reform is not differing assumptions about the behavioural responses to taxes but rather the assumptions (often implicit) made about other policy instruments. We shall see examples of this below.

It will obviously be interesting to solve the general problem in which changes in capital intensity are taken into account. But we shall begin by tackling the much simpler problem in which the level of capital per man-hour is fixed exogenously at $k = \bar{k}$. This determines the levels of the wage and interest rates, which in a perfectly competitive economy are, respectively

$$w = f(k) - kf'(k) \tag{11}$$

$$r = f'(k) \tag{12}$$

The government will choose the values of the two tax rates to maximise the steady state level of utility, subject to (1) individual optimising behaviour and (2) its own budget constraint, taking the values of w, r and k as parameters of the problem. One reason for exploring this problem first is that we may exploit the properties of its solution to solve the more complicated problem in a rather simple manner.

Partly for this purpose, and partly for reasons of elegance, we shall find it easier to work with the indirect utility function, which corresponds to (1) and which (normalising such that the price of first-period consumption is unity)[2] we may denote by

$$V[h, w(1-t_w), y] \tag{13}$$

where y is exogenous income.

A well-known property of indirect utility functions (see, for example, Diewert, 1974) is that the demand functions are given by

$$C_2 = -\frac{V_1}{V_3} \tag{14}$$

$$1 - L = -\frac{V_2}{V_3} \qquad \left(\Leftarrow L = \frac{V_2 + V_3}{V_3}\right) \tag{15}$$

where V_i is the partial derivative of V with respect to the ith argument. We may form the Lagrangean

$$l = V[h, w(1 - t_w), y] + \lambda\left(g - t_w wL - \frac{hrt_r C_2}{1 + n}\right) \tag{16}$$

The first-order conditions (with respect to t_w and t_r) are[3]

$$-w(V_2 + V_3) = \lambda\left[wL + wt_w \frac{\partial L}{\partial t_w} + \left(\frac{hrt_r}{1 + n}\right)\frac{\partial C_2}{\partial t_w}\right] \tag{17}$$

$$rh^2 V_1 = \lambda\left[wt_w \frac{\partial L}{\partial t_r} + \left(\frac{rht_r}{1 + n}\right)\frac{\partial C_2}{\partial t_r} + \frac{rhC_2}{1 + n}(1 + rht_r)\right] \tag{18}$$

The first of these equations allows for the fact that exogenous income depends upon t_w; therefore, from (14), (15), (17) and (18) we get

$$\frac{rh^2 C_2}{wL} = \frac{\left[wt_w \frac{\partial L}{\partial t_r} + \left(\frac{rht_r}{1 + n}\right)\frac{\partial C_2}{\partial t_r} + \frac{rhC_2}{1 + n}(1 + rht_r)\right]}{\left[wL + wt_w \frac{\partial L}{\partial t_w} + \left(\frac{rht_r}{1 + n}\right)\frac{\partial C_2}{\partial t_w}\right]} \tag{19}$$

The derivatives of C_2 and L with respect to the tax rates indicate the responses of savings and labour supply to taxation. In the appendix it is shown that, if we use the Slutsky equations and define the compensated elasticities as σ_{ij} (i.e. σ_{ij} is the percentage change in the demand for i for a 1 per cent compensated change in the price of j), the above expression reduces to the rather simple formula

$$\frac{rt_r}{1 + n}(-\sigma_{22} + \sigma_{L2}) = \frac{t_w}{1 - t_w}(\sigma_{LL} - \sigma_{2L}) + \frac{r - n}{1 + n} \tag{20}$$

This equation explains the *relative* tax burdens imposed on wage and interest income. The absolute values of the tax rates will depend also on the size of the government's revenue requirement.

In the special case when the government can use such policies as debt finance to attain the golden rule with $r = n$, the optimal 'second-best' tax rates are given by

$$\frac{rt_r}{1 + r}(\sigma_{L2} - \sigma_{22}) = \frac{t_w}{1 - t_w}(\sigma_{LL} - \sigma_{2L}) \tag{21}$$

In this case dynamic considerations disappear, and the optimal tax rates reflect only the compensated demand elasticities of leisure and future consumption.

If we restrict ourselves for the moment to an examination of (21), is there any presumption for an expenditure tax rather than an income tax on efficiency grounds? An expenditure tax (which corresponds to $t_r = 0$) will be the optimal tax system if, and only if,[4]

$$\sigma_{LL} = \sigma_{2L} \tag{22}$$

This result was first proved, in a different context, by Corlett and Hague (1953).

It is possible to show (after some manipulation of equations A1–A8 that, for (22) to hold, then more generally

$$\sigma_{1L} = \sigma_{2L} = \sigma_{LL} \tag{23}$$

This equation says that the necessary condition for an expenditure tax to be optimal is that a compensated change in the wage rate leads to equal proportionate changes in first-period consumption, in second-period consumption and in labour supply. This requires that the ratio of consumption in the two periods is independent of the wage rate.[5] Although this may appear a not implausible specification of preferences, there is no *a priori* reason for adopting it and little empirical evidence to enable us to test the hypothesis. Indeed, equation (22) should warn us that, if we are to use empirical evidence to assess the optimality of an expenditure tax, we must be very careful not to assume (implicitly) the truth of (22) but to allow the data to choose whether or not to reject the hypothesis that the utility function satisfies (22). Unfortunately, many common specifications of the individual utility function imply (22) and so cannot be used to provide independent evidence about the values of the relevant elasticities.

Consider, for example, the Cobb–Douglas indirect utility function

$$V = \frac{h^{a_1} x^{a_2}}{y} \qquad (a_1 + a_2 < 1) \tag{24}$$

where

$$x = w(1 - t_w)$$

The Cobb-Douglas form lies behind much empirical work on the life cycle hypothesis and the aggregate consumption function, because it implies that current consumption is a constant fraction of lifetime income.

From (3), (14) and (15) the demand functions are

$$C_1 = (1 - a_1 - a_2)y \tag{25}$$

$$C_2 = \frac{a_1 y}{h} \tag{26}$$

$$L = 1 - \frac{a_2 y}{x} \tag{27}$$

It is clear from these equations that the Cobb-Douglas specification implies a number of special properties. First, consumption in period 1 is a constant fraction of the value of the consumer's endowment. Secondly, savings are a constant fraction of the value of the consumer's endowment (from equation 26) and hence, in this model, are independent of the interest rate. Finally, the assumption that the endowment consists of potential labour supply in the first period means that actual labour supply will be constant and independent of factor prices.

Inverting the equations (A1)–(A4) and solving for the values of the compensated demand elasticities (and noting that in this model a change in x implies an equal change in y), we get

$$\sigma_{22} = a_1 - 1 \tag{28}$$

$$\sigma_{2L} = a_2 \tag{29}$$

$$\sigma_{L2} = \frac{-a_1 a_2}{1 - a_2} \tag{30}$$

$$\sigma_{LL} = a_2 \tag{31}$$

In this case it is clear that an expenditure tax is optimal.

It is equally clear that empirical investigations into the value of the elasticities based upon the assumption of a Cobb-Douglas utility function (or any other function satisfying equation 22) would provide no evidence on the question. We shall discuss the empirical evidence on savings and taxation below, but we may note that the values of the optimal tax rates

depend upon the cross-elasticities of saving with respect to the wage rate and of labour supply with respect to the interest rate. Typically, empirical studies ignore these elasticities. Suppose that the cross-elasticities are zero; then (we are still in the golden rule world)

$$t_r = -\left(\frac{1+n}{n}\right)\left(\frac{t_w}{1 - t_w}\right)\frac{\sigma_{LL}}{\sigma_{22}} \tag{32}$$

Typical values of the own price-elasticities are $\sigma_{LL} = 0{\cdot}4$ and $\sigma_{22} = -1{\cdot}2$, and we may take the growth rate over a generation to be unity. Hence,

$$t_r = \frac{2}{3}\left(\frac{t_w}{1 - t_w}\right) \tag{33}$$

If the tax rate on labour income is one-third (to meet the revenue requirements), the value of the tax on capital income is also one-third, and a comprehensive income tax is optimal. The choice between alternative tax bases is very sensitive to estimates of the relevant elasticities, and equally 'plausible' values for the parameters have very different implications for tax policy.

It is not difficult to imagine econometric studies producing point estimates which could be used to justify either an expenditure tax or a comprehensive income tax. A few more examples will illustrate the point. Let us retain the assumption that the cross-elasticities are zero and that the tax rate on labour income is one-third. If the value of σ_{22} is only $-0{\cdot}8$ instead of $-1{\cdot}2$, the optimal capital income-tax rate is 50 per cent; but if σ_{22} remains at $-1{\cdot}2$ and the estimate of σ_{LL} is revised downwards to $0{\cdot}2$, the capital income-tax rate is only one-sixth, exactly one-half of the tax rate on labour income. If we now relax the assumption that the cross-elasticities are zero and take $\sigma_{L2} = -0{\cdot}3$ and $\sigma_{2L} = 0{\cdot}25$, and hold σ_{LL} and σ_{22} at their original values, the optimal tax rate on capital income turns out to be 10 per cent. A value of σ_{2L} greater than σ_{LL} would imply that capital income should be subsidised. All of this goes to show that when basing policy recommendations on econometric estimates of the elasticities one must be confident that the standard errors of the estimates are small.

Equation (21) gives the optimal *relative* tax rates on capital and labour income. Their absolute values depend upon the revenue requirement. Suppose that the share of capital income in national income is c and that the government's revenue requirement as a fraction of the national income is $\bar{g}$; then

$$ct_r + (1 - c)t_w = \bar{g} \tag{34}$$

The value of c will in general depend upon the tax rates (except in the special case of a Cobb-Douglas production function), and (32) and (34) are the two implicit equations determining the values of the two tax rates.

In this section we have concentrated on the distortions caused by taxes to individual choices, but a very important effect of capital taxes is their impact on the aggregate level of savings. It is to this that we now turn.

1.3 TAXATION AND AGGREGATE SAVINGS

In the previous section we have examined the optimality of the tax system in terms of its distortion of individual choices between work and leisure and between present and future consumption. We have seen that in the light of our present knowledge about the values of the relevant elasticities this analysis provides little by way of clear-cut recommendations to policy-makers. But one of the arguments often employed by proponents of an expenditure tax is that such a tax would raise aggregate savings. To explore this issue we must analyse the optimal tax structure when the level of capital per head is allowed to vary in the model. In this section we shall assume that k is chosen by the government which does not necessarily have access to policy instruments (such as debt) that would enable it to attain the golden-rule growth path.

At first sight it seems that the case in which k is variable is considerably more complicated than the fixed k case of section 1.1, but this is not so. From (16) we see that k enters into the constrained maximisation problem as a parameter, and so by the envelope theorem the first-order condition for the fixed k case is also a first-order condition for the variable k case, provided that it is evaluated at the optimal value of k. Hence, the optimal tax rates in the general case are given by equation (20), where the value of r is the optimal value and would be expected to exceed n because of the failure of each generation to take into account the welfare of future generations. By appealing to the envelope theorem, we may solve the general problem by tackling first the more restricted case examined above.[6]

In the case when the government can achieve the golden rule path by the use of other policy instruments there is, we have argued, no presumption for either an income tax or an expenditure tax, since seemingly plausible estimates of the parameters could be used to defend either tax system. The question at issue here is whether allowing for the endogenous choice of capital intensity shifts the balance of the argument in favour of an expenditure tax. This is equivalent to asking if the optimal tax formula in the general case (equation 20) is more favourable to an expenditure tax than in the restricted golden-rule case (equation 21). Rearranging these formulae, and denoting the golden rule regime by subscript 1 and the variable k case by subscript 2, we get

$$t_{r1} = \mu\left(\frac{t_{w1}}{1 - t_{w1}}\right) \tag{35}$$

$$t_{r2} = \mu\left(\frac{t_{w2}}{1 - t_{w2}}\right) + \frac{r - n}{n\theta}(1 - \theta t_{r2}) \tag{36}$$

where

$$\mu = \left(\frac{1 + n}{n}\right)\left(\frac{\sigma_{LL} - \sigma_{2L}}{\sigma_{L2} - \sigma_{22}}\right)$$

$$\theta = \sigma_{L2} - \sigma_{22}$$

These equations do not determine the tax rates, because the absolute values of the rates will depend also upon the revenue requirement and the assumptions made about the production side of the economy.

However, it can easily be seen that, if there is no presumption for low tax rates on capital income in the golden rule case, this is even more true of the general case. The following verbal argument illustrates this. Suppose that the compensated demand elasticities (and hence both μ and θ) are constants. In regime 2 (variable k) the output per man-hour is lower than in regime 1 (golden rule), because r exceeds n, and so to meet the fixed revenue requirement the average tax rate must be higher than in the golden rule case. If the second regime is to be characterised by a lower ratio of the tax rate on capital income to the tax rate on labour income, the absolute value of t_{w2} must exceed that of t_{w1}. (This is because the average value of t_{w2} and t_{r2} must be greater than the average value of t_{w1} and t_{r1}; and whatever are the appropriate weights, this must imply a higher absolute value for t_{w2}.) Hence,

$$t_{r2} = t_{r1} + \alpha + \frac{r - n}{n\theta}(1 - \theta t_{r2}) \tag{37}$$

where α is positive. This equation almost certainly implies that t_{r2} is greater that t_{r1}. Even with implausibly high estimates of the absolute value of σ_{22} the equation could not be satisfied by a value of t_{r2} lower than t_{r1}, except perhaps for very high values of the capital income-tax rate. This suggests that allowing for the endogeneity of aggregate savings does not in itself provide a strong argument for an expenditure tax. Of course, it must be stressed that none of these results support high tax rates on capital income. They merely emphasise that the structure of the 'optimal' tax system is very sensitive indeed to the precise values of the relevant elasticities.

It would, however, be wrong to conclude from this discussion that allowing for the response of aggregate savings is quantitatively unimportant. To demonstrate this, consider the case in which the individual utility function satisfies (22), in which case we know that along the golden rule path the optimal tax rate on capital income is zero. In the general variable-k case the optimal tax rate is

$$t_r = \frac{r - n}{r(\sigma_{L2} - \sigma_{22})} \tag{38}$$

A not implausible point estimate of the value of $(\sigma_{L2} - \sigma_{22})$ would be unity, in which case the optimal tax rate on capital income would be

$$t_r = 1 - \frac{n}{r} \tag{39}$$

Even though an expenditure tax would be optimal along the golden rule path, a substantial tax on capital income may well be optimal if the government has no independent policy instrument to influence the level of capital intensity in the economy. For example, a value of the rate of profit equal to twice the growth rate would imply an optimal tax rate of 50 per cent. Clearly, the relationship between the profit and growth rates at the optimum has a major bearing on the pattern of optimal tax rates. The value of r in equations (38) and (39) will be a function of t_r, and the form of the relationship will depend upon (1) the production function relating the marginal product of capital to the level of capital intensity, and (2) the function relating the aggregate supply of savings to the net-of-tax rate of return. The nature of the function relating r to t_r is likely to be complex, and it is difficult to make general statements about the value of t_r that will prove optimal, except to reiterate the point made above that in a wide range of cases it will probably exceed the value that would be optimal on the golden-rule growth path.

The main conclusions to emerge from this discussion are that the values of the optimal tax rates are very sensitive to the precise values of the various elasticities involved and that they depend on two different aspects of savings behaviour: the individual compensated demand elasticity of second-period (or 'retirement') consumption with respect to the interest rate, and the general equilibrium response of aggregate savings to changes in the net rate of return. The former effect concerns the efficiency loss of distorting individual choice between present and future consumption, and the latter concerns the ability of changes in capital income taxes to drive the economy nearer to the golden rule path of capital accumulation. These two aspects of savings behaviour are distinct, but both are important for the design of the tax system.

One of the virtues of a tax on capital income in the two-period model is that it helps to raise aggregate saving and hence pushes the economy nearer to the golden-rule growth path. At first sight this may seem paradoxical, but the explanation lies in the *timing* of tax payments. Ignore for the moment the substitution effects of taxes. Replacing a tax on labour income by a tax on capital income with the same present discounted value would have no effect on the individual's budget constraint, and this would not affect his chosen consumption levels. But the level of private saving would be affected, because the amount that would have been paid in labour income taxes in period 1 would be saved to meet the tax bill on capital income in period 2. In this way capital income taxes produce higher private saving. If the government could vary its own saving, by borrowing more, these changes in private saving would be of no consequence, and this is the case on the golden rule path. But where the government is unable to change its borrowing policy and is constrained to operate only on tax rates, any tax that raises revenue later in an individual's life rather than earlier will seem attractive. It is likely that capital income taxes would have a comparative advantage from this point of view, even if the model were extended to the more realistic case in which individuals lived for many periods, although the magnitude of the advantage would probably be less than in the rather extreme two-period model.

For illustration of this point, consider the example of Cobb-Douglas preferences. On the golden rule path we know that the optimal tax rate on capital income is zero; but if the tax system is to be used to encourage aggregate savings, this conclusion no longer holds. In the Cobb-Douglas case aggregate savings are (from equations 8 and 26)

$$S = \left(\frac{nN}{1 + n}\right) a_1 w(1 - t_w) \tag{40}$$

The level of savings is independent of the rate of interest and hence of the tax rate on capital income. A change in t_r can affect savings only via its effects on t_w, and the wage rate. An increase in t_r enables t_w to be reduced for a given revenue requirement, thus increasing savings directly. It can easily be checked that in the Cobb-Douglas case a reduction in t_w also results in a higher capital-labour ratio, implying a higher wage rate, which in turn increases savings. The direct and indirect effects reinforce each other.

Although this argument for taxes on capital income may seem somewhat artificial, in that we may normally assume that the government could offset any change in the timing of tax payments by changes in its own borrowing policy, it does have one important implication. This is that in the context of our model a labour income tax is not equivalent to an expenditure tax. The incentive effects of the two taxes are the same, but the timing of the tax payments to which they give rise is not. On the golden rule path this is irrelevant, but away from that path the expenditure tax has the advantage that tax payments occur later in life than under the labour income tax. This means that, if we impose a tax on consumption, the associated optimal tax rate on capital income will be less than if we imposed a tax on labour income. Consequently, the formulae derived above exaggerate the need for a capital income tax away from the golden rule, because the tax on labour income could have been imposed instead as a tax on consumption. Although we have argued above that the value of the optimal capital income-tax rate is very sensitive to the discrepancy between r and n, we can now see that it is sensitive also to the relative timing of tax payments under an expenditure tax and a capital income tax.

It is time, however, to question the assumption that government debt policy cannot be used to compensate for changes in the timing of tax payments. Ignoring debt policy can lead to absurd results. In the two-period model it would imply that the optimal tax was a lump sum tax collected on an individual's death bed, with his last action being to sign a cheque for the Inland Revenue for which he had spent most of his life saving up to pay. This unsavoury prospect can be disregarded once debt policy is brought into the picture, for the government may substitute public saving for private saving by running a surplus and investing the proceeds at the market interest rate. The government is now indifferent to the timing of tax payments and is concerned only to extract a given present discounted value of revenue from each generation. This means that the government's budget constraint becomes

$$g = t_w wL + \frac{hrt_r C_2}{1 + r} \qquad (9')$$

The difference between (9) and (9′) is that, whereas in the former cash receipts determine the constraint, in the latter the revenue requirement is perceived in present discounted value terms. The effect of this is that in all subsequent equations $(1 + n)$ is replaced by $(1 + r)$. It is a simple matter to check that with this change the optimal tax rates are given by equation (21), *even* when $r \neq n$ and we are not following the golden rule path. The reason is that the impact of taxes on aggregate saving can be offset by an appropriate debt policy, and so moving from one tax system to another does not move the economy near the golden rule path. Allowing for debt policy means therefore that the choice between income and expenditure taxes in terms of efficiency depends on the values of the elasticities that appear in (21) (see the earlier discussion of this).

Our conclusion that it is difficult to argue strongly for either an income or an expenditure tax on efficiency grounds seems to conflict with the views expressed recently by a number of economists about the large welfare gains that would result from the abolition of capital income taxes. For example, Boskin (1978) has claimed that capital income taxes in the United States impose an annual welfare loss amounting to an astounding \$60 billion (which is a present value close to \$1 trillion). Summers (1978) found that the present value of the welfare gains from replacing capital income taxes by an expenditure tax is at least the equivalent of five years' gross national product (GNP), which in the United States is about \$2·5 trillion. On a more modest scale Feldstein (1978a) obtained estimates of the annual gain from replacing capital income taxes by higher labour income taxes equal to about 1 per cent of GNP (\$50 billion in the United States). Whatever criterion one uses, it is clear that these are large figures.

Why is it, therefore, that these studies appear to conflict with the ambiguous nature of our theoretical results? Interestingly enough, it appears to be the assumptions, explicit or implicit, that these studies make about labour supply that provide the key to the differences. Assumptions about savings behaviour do not seem to be responsible for the magnitude of the estimated welfare gains. For example, to calculate the welfare gains from abolishing capital income taxes Boskin implicitly used an estimate of $\sigma_{22} = -1{\cdot}533$ (see secton 1.5 below), but the main reason for the size of the gain is that he assumed that the taxes on capital income are replaced by lump sum taxes and not, as appears more plausible, by higher taxes on labour income. No weight is given to the efficiency loss associated with alternative sources of revenue.

The differential welfare gain from substituting taxes on labour income or consumption for capital taxes has been considered by both Feldstein and Summers. In Summers's case labour supply is assumed to be fixed, and so again there is no efficiency loss associated with labour income or expenditure taxation. He also placed great emphasis on the timing of tax payments in a complete life-cycle model, which, as we have seen above, is an issue that depends critically on the role afforded to government debt policy.

Feldstein has presented a very careful analysis of the net welfare gain from replacing capital income taxes by a higher rate of tax on labour income in a two-period model very similar to the one discussed above. The size of the welfare gain depends on the values of the compensated demand elasticities shown in equation (21). Feldstein assumed values of zero for the *uncompensated* elasticities of savings with respect to the interest rate and of labour supply with respect to both the wage rate and the interest rate. These are not taken as 'estimates of the true values but as simply illustrative values that are likely to understate the welfare cost of capital income taxation'. Unfortunately, the particular parameter estimates chosen by Feldstein happen to imply that an expenditure tax is in fact optimal, as we shall now show. The assumption that the uncompensated elasticity of labour supply with respect to the wage rate is zero implies (from equations A1 and A5) that

$$\sigma_{LL} = -w(1-t_w)\frac{\partial L}{\partial y} \tag{41}$$

If the uncompensated elasticity of labour supply with respect to the interest rate is zero, then (from equation A3 and the symmetry condition on substitution effects)

$$S_{L2} = -S_{2L} = C_2\frac{\partial L}{\partial y} \tag{42}$$

$$\therefore \quad \sigma_{2L} = -w(1-t_w)\frac{\partial L}{\partial y} \tag{43}$$

Hence, $\sigma_{LL} = \sigma_{2L}$, and we know from our earlier analysis that this is the condition for an expenditure tax to be optimal.

The assumption of zero uncompensated elasticities is more powerful than their description of 'illustrative values' may imply. The welfare gain computed by Feldstein is the gain that would result from adopting an expenditure tax *if* that tax were optimal. Again it is instructive to note that the assumptions that are responsible for this result concern not the response of savings but labour supply behaviour. One may be tempted to

think that, because the optimal tax system is unlikely to be exactly equal to an expenditure tax, Feldstein's estimates provide an upper bound on the welfare gains of moving from the present system to an expenditure tax. Even this, however, is untrue. It may well be that the optimal tax system requires a subsidy to capital income, and in such a case adopting an expenditure tax might bring large welfare gains even though it would move us only part of the way towards the optimum. Nevertheless, it is not clear that Feldstein's estimates 'are likely to understate the welfare cost of capital income taxation'. The correct conclusion appears to be that the size of the efficiency losses associated with the present tax system is very uncertain, but the fact that, as Feldstein has demonstrated, they may be substantial alerts us to the need for a much greater understanding of the response of savings and labour supply to taxation.

1.4 INFLATION AND CAPITAL INCOME TAXES

In our theoretical model we have assumed that capital income taxes are imposed only on real interest income; in essence we have assumed either that there is no inflation or that the tax system is fully indexed. But in practice capital income taxes are levied on nominal capital income, and one of the most forceful arguments for an expenditure tax is the practical problems that would be entailed by complete indexation of the tax system. In this section, therefore, we shall examine the optimal tax structure, given the constraint that taxes on capital income can be charged only on receipts of nominal interest income. Since tax authorities have been reluctant to get embroiled with indexation, the choice between expenditure taxes and unindexed capital income taxes may represent the realistic alternatives.

The introduction of inflation implies two changes in the basic model. First, the price of second-period consumption becomes

$$h' = \frac{1+\pi}{1+(1-t_r)(r+\pi+\pi r)} \tag{44}$$

where π is the rate of inflation and r is again the real rate of interest before tax. Secondly, the government's budget constraint changes; and if we take the discounted value form of the constraint (equation 9′), it is

$$g = t_w wL + \frac{h'C_2 t_r(r+\pi+\pi r)}{(1+\pi)(1+r)} \tag{45}$$

It can easily be verified that with these changes the optimal tax rates are given by a modified version of (21), in which the real interest rate is replaced by the nominal interest rate; that is,

$$r' = r + \pi + \pi r \tag{46}$$

It is intuitively clear that this is the case because the base of the capital income tax is the nominal interest rate and not the real rate of interest. If the government uses debt policy to determine the real rate of interest, and hence the optimal level of capital accumulation, the real rate of interest can be regarded as exogenous.[7]

With this assumption it is easy to show that the optimal tax rate on nominal (unindexed) capital income is such as to have no effect on real behaviour nor on the optimal tax rate on labour income. For this to be so it is necessary that

$$\frac{rt_r}{1+r} = \frac{r't_r'}{1+r'} \tag{47}$$

From (46) this gives

$$t_r' = t_r\left(\frac{r(1+\pi)}{r+\pi+\pi r}\right) \tag{48}$$

This value of t_r' implies that individual behaviour is unaltered because $h = h'$, and it also satisfies the government's budget constraint.

The introduction of inflation into the model means that the optimal tax rate on capital income is multiplied by the factor shown in (48). It is clear that the tax rate should be lower in the presence of inflation. For example, if as above we take the real rate of interest over a generation to be unity, then with a rate of inflation equal to the rate of interest the optimal tax rate is reduced to two-thirds of its previous value. The failure to index the taxation of capital income makes a significant difference to the relative claims of an income tax and an expenditure tax. This brings us back to one of the arguments deployed by the Meade Committee (1978) in favour of an expenditure tax, namely, that the practical difficulties of indexing capital income taxation are as large, if not larger, as the transitional problems involved in shifting to an expenditure tax. The theoretical arguments of this section certainly support the contention that the optimal tax rate on capital income is very sensitive to the feasibility of indexation.

1.5 VARIATIONS ON THE BASIC MODEL

In this section we shall discuss the effects of relaxing some of the crucial assumptions of the simple two-period model analysed above. The most important of these is that labour is supplied only in the first period, which implies that an individual's initial wealth is independent of interest rates. In practice, his endowment consists of a future stream of potential receipts, and the present value of this stream will depend upon the interest rate. For many people the most significant component of the stream is future labour income. Once we allow the value of wealth, especially future earnings, to depend on the interest rate, changes in interest rates have two effects on savings: the direct effect analysed above, and an indirect effect via changes in wealth. An increase in the interest rate will reduce the present value of wealth, thus lowering present consumption and stimulating savings. The interest elasticity of savings with respect to the interest rate may be much higher than is suggested by the two-period model. This point was first made by Hall (1968) and later stressed by Summers (1978), who, on the basis of numerical simulations, concluded that the size of the interest elasticity of savings is dominated by the wealth effect and that the elasticity of substitution between present and future consumption is much less important.

It is possible to illustrate this phenomenon by extending our model to include an endowment in period 2. To simplify matters we shall assume that each individual receives in period 2 a fixed endowment of value e. (This is sufficient to make the point and avoids the complications of modelling variable labour supply in period 2.) The individual budget constraint is now given by

$$C_1 + hC_2 = w(1 - t_w)L + he \tag{49}$$

We may also write the budget constraint as

$$C_1 + S = w(1 - t_w)L \tag{50}$$

where S is the value of personal savings per worker. This form of the constraint applies to both the original and amended forms of the model. The government budget constraint (in present discounted value terms) may also be written as

$$g = t_w wL + \left(\frac{rt_r}{1+r}\right)S \tag{51}$$

If we now maximise steady state utility *per capita* subject to this budget constraint, expressed in terms of savings rather than second-period consumption, we obtain suitably modified versions of the first-order conditions (17) and (18). Together with the appropriate Slutsky equations, these conditions enable us to derive the optimal tax rates in terms of compensated *savings* elasticities. It is straightforward to show that the relationship between the optimal tax rates is given by

$$\frac{t_w}{1-t_w}(\sigma_{LL}-\sigma_{SL})=\frac{rt_r}{1+r}(\sigma_{LS}-\sigma_{SS}+1) \tag{52}$$

where σ_{SL} = compensated elasticity of savings with respect to the wage rate,
σ_{LS} = compensated elasticity of labour supply with respect to the price of second-period consumption h, and
σ_{SS} = compensated elasticity of savings with respect to the price h.

This formula holds irrespective of the size of any second-period endowment.

In the case analysed in section 1.2, in which there was no second-period endowment, savings were a constant multiple of second-period consumption, which implies that

$$\begin{aligned}\sigma_{SL}&=\sigma_{2L}\\ \sigma_{LS}&=\sigma_{L2}\end{aligned} \tag{53}$$

The own price elasticities are related, as

$$S=hC_2$$

$$\therefore\quad \frac{\partial S}{\partial h}=C_2+h\frac{\partial C_2}{\partial h} \tag{54}$$

Multiplying both sides by h/S, we get

$$\sigma_{SS}=1+\sigma_{22} \tag{55}$$

The elasticity of savings with respect to the price h is equal to the elasticity of second-period consumption plus unity. This is because savings represent current *expenditure* on future consumption (see Feldstein, 1978b).

It is easy to see that substituting (53) and (55) into (52) yields equation (21), which is the condition derived above for the case of a zero second-period endowment. But we may now extend the model to allow for a positive endowment in period 2 equal to e. In this case we have, as before, $\sigma_{LS}=\sigma_{L2}$, but the savings elasticities are now changed. From (49) and (50) we get

$$S=h(C_2-e) \tag{56}$$

$$\therefore\quad \frac{\partial S}{\partial w}=h\frac{\partial C_2}{\partial w}$$

$$\sigma_{SL}=\frac{w}{S}\cdot\frac{\partial S}{\partial w}=\left(\frac{C_2}{C_2-e}\right)\frac{w}{C_2}\cdot\frac{\partial C_2}{\partial w}$$

$$\therefore\quad \sigma_{SL}=\alpha\sigma_{2L} \tag{57}$$

where

$$\alpha=\frac{C_2}{C_2-e}>1$$

Similarly, it can be shown that

$$\sigma_{SS}=1+\alpha\sigma_{22} \tag{58}$$

In other words, allowing for a second-period endowment (a very simple way of introducing more realistic life-cycle considerations into the model) means that the elasticities of second-period consumption that appear in the formula for the optimal tax rates are multiplied by the factor α. The general formula is

$$\frac{t_w}{1-t_w}(\sigma_{LL}-\alpha\sigma_{2L})=\frac{rt_r}{1+r}(\sigma_{L2}-\alpha\sigma_{22}) \tag{59}$$

This reduces to (21) when α equals unity, but in general α will exceed unity and can be large if, as one might expect in a complete life-cycle model, most future consumption is financed from future earnings.

The significance of this extension may be illustrated by looking again at two of the examples examined in section 1.1. Consider first the case where utility functions are Cobb-Douglas. In the basic model the assumption of Cobb-Douglas utility functions implies that the optimal tax is an expenditure tax with $t_r=0$ (see above). Substituting from (28)-(31) for the values of the compensated elasticities in the Cobb-Douglas case, we see that in the extended model the optimal tax rates are given by

$$t_r = -\left(\frac{1+r}{r}\right)\left(\frac{t_w}{1-t_w}\right)\left(\frac{(\alpha-1)\,a_2(1-a_2)}{[a_1a_2(\alpha-1)+\alpha(1-a_1-a_2)]}\right) \tag{60}$$

It is clear from this expression that the optimal tax on capital income is in fact a *subsidy.*[8] As $\alpha \to 1$, the optimal tax system tends to the expenditure tax, but in general higher values of α will imply a capital subsidy, the size of which will depend on parameter values. Suppose that we take the case where $\alpha = 3$ (i.e. the second-period endowment is equal to two-thirds of desired second-period consumption), $r = 1$, $t_w = \frac{1}{3}$, $a_1 = \frac{1}{3}$ and $a_2 = \frac{1}{4}$; then $t_r = -0{\cdot}265$. That is, the optimal tax system consists of a 26·5 per cent subsidy to capital income. With $t_w = \frac{1}{2}$ the optimal subsidy to capital income is as high as 53 per cent.

The second example to consider is that in which a comprehensive income tax is optimal in the basic model. In this example the cross-elasticities are assumed to be zero, and the remaining parameter values are $\sigma_{LL} = 0{\cdot}4$, $\sigma_{22} = -1{\cdot}2$, $r = 1$ and $t_w = \frac{1}{3}$. When $\alpha = 1$, $t_r = \frac{1}{3}$, and the optimal tax system is a comprehensive income tax. Again, higher values of α lead to lower tax rates on capital income; for example, when $\alpha = 3$, the optimal tax rate on capital income is only 11 per cent.

As a final illustration of the need to consider an extended life-cycle model, consider the case when savings are very small. As savings tend to zero in this model, the value of α tends to infinity. In the limit the optimal tax rates are given by

$$\frac{t_w}{1-t_w} = \left(\frac{rt_r}{1+r}\right)\frac{\sigma_{22}}{\sigma_{2L}} \tag{61}$$

Provided that leisure in the working stage of the life cycle is a substitute rather than a complement for consumption in retirement (a not unreasonable assumption), the optimal tax on capital income is always a subsidy, regardless of the particular form of individual utility functions. In the Cobb–Douglas case considered above the optimal rate of subsidy would be 37·5 per cent.

The main lesson to be learned from the extended life-cycle model is that the results are sensitive to the characterisation of the individual's life cycle. It can be seriously misleading to model the life cycle as consisting of two quite-distinct stages, namely, 'work' and 'retirement', both of fixed length. A model of the optimal allocation of time between different activities would allow decisions about retirement or time spent in education to be endogenous. The retirement decision has been the subject of some

recent research (see, for example, Feldstein, 1974; Diamond and Mirrlees, 1978; Sheshinski, 1978; and Boskin and Hurd, 1978). Time spent in training and investment in human capital has been investigated by Heckman (1976).

Another reservation about our results is that we have ignored uncertainty. Uncertainty about future labour income, the rate of return on savings, the date of retirement (influenced possibly by ill health or the state of the labour market) and the date of death will all affect the optimal consumption and savings plan. The implications of this for the tax system are unclear, and we shall simply make two points. First, to analyse savings decisions under uncertainty in a tractable manner one may be tempted to make simplifying assumptions about individual utility functions, such as separability between consumption and leisure or between consumption levels over time. Unfortunately, as we have seen, assumptions such as these have very strong implications for the design of the tax system. Secondly, the presence of uncertainty has been used by some as an argument for the provision of social security (i.e. state pensions). For example, Diamond (1977), in an interesting (and non-technical) discussion of the rationale for social security, has pointed to the absence of securities offering riskless real rates of return and to the difficulty of insuring in the market against the effects of unemployment or ill health on retirement. These are offered as possible justifications for a compulsory social-security scheme. Leaving to one side the merits of the argument, we shall simply draw attention to the implications of the existence of social security for the tax system. In the model explored above we may interpret the second-period endowment e as an untaxed pension with the payments financed out of general tax revenues. As shown above, the existence of the pension scheme increases the individual elasticity of savings with respect to the interest rate and lowers the optimal tax rate on capital income. Unfunded pension schemes reinforce any case for an expenditure tax and against an income tax.

This leads us into the remaining set of questions, which are those concerning the government. It is clear that one of the roles of a state pension scheme is redistribution both within and between generations. We have examined the intergenerational distribution when considering the impact of taxation on the aggregate level of savings, but the assumption of identical individuals ruled out any role for intragenerational distribution. The source of differences in endowments in our model is twofold. First, potential wages may vary among individuals because of differing 'abilities'; in a model without savings this formulation is the optimal income-tax problem examined by Mirrlees (1971) and Wesson (1972). Secondly, the value of future labour earnings may vary because of differences in the opportunity cost of capital facing individuals. Imperfections in the capital market, such as constraints on borrowing or lending, will affect the value of an individual's endowment. In the absence of an explicit model it is difficult to assess the extent to which these considerations will influence the choice of the tax *base* as opposed to the optimal rate structure. Ordover and Phelps (1979) have shown that allowing for differences in abilities or wage rates does not alter the condition for optimality of the expenditure tax, which remains that each individual's utility function must satisfy 22. Imperfections in the capital market are a more difficult matter and require further research.

The final point is that in practice governments are only too aware of the fact that tax reform must start from the system that we have today. The transition from today's initial position to the desired long-run state is important from both a theoretical and a practical point of view. An overnight switch from an income tax to an expenditure tax would result in a windfall loss for the generation just retired. Not only is this undesirable in principle, but the incentives to conceal wealth held on the changeover date make such a transition infeasible. A gradual transition designed so as to avoid significant windfall gains and losses is necessary on practical grounds (see the lengthy discussion on transitional arrangements in the Meade Report). To achieve this it would be necessary to keep constant the present value of the tax liabilities of those living at the start of the transition. This would require adjustments in government borrowing to compensate for changes in the timing of tax payments. It would be a mistake not to use debt policy as an instrument to aid a smooth transition; and if debt policy is an important part of the transition to a new state, it seems misplaced to rule it out in the long run also. This provides support for the view that the appropriate formulation of the government's budget constraint is in terms of the present discounted value of revenue received from each generation.

1.6 EMPIRICAL EVIDENCE ON SAVINGS AND RATES OF RETURN

Since a consumption function is an essential ingredient of all large econometric models, we might expect to find numerous empirical studies of the relationship between savings and interest rates. In fact there are remarkably few econometric investigations of the effects of relative prices on the intertemporal allocation of consumption, and even fewer that have used the appropriate explanatory variables.

The empirical study that is closest to the theoretical model explored above is that of Boskin and Lau (1978). They estimated a two-period life-cycle model in which individuals are able to choose their consumption levels in both the working and retirement stages of their lives and their labour supply in the working stage; retirement occurs at a fixed date. Individuals have identical preferences represented by an indirect utility function (equivalent to the function V used above), which is assumed to be of the homogeneous translog form. This gives the following demand function for the three goods: first-period consumption, second-period consumption, and leisure:

$$p_i x_i = \alpha_i + \sum_{j=1}^{3} \beta_{ij} \log p_j + z_i \qquad (i = 1, 2, 3) \tag{62}$$

where x_i = demand for good i,
p_i = normalised price of good i, (i.e. the price divided by exogenous income), and
z_i = vector of characteristics such as age, sex and life expectancy of consumer.

A good explanation of the derivation of this demand system (and other functional forms for the indirect utility function) is contained in Berndt, Darrough and Diewert (1977). They proved that the demand system in (62) is linear in the parameters only because of the assumption that preferences are homothetic. This is a strong assumption, because it implies that all full wealth elasticities are unity. It also imposes the following restrictions on the parameters:

$$\sum_{i=1}^{3} \alpha_i = 1 \qquad \sum_{i=1}^{3} z_i = 0 \qquad \sum_{i=1}^{3} \beta_{ij} = 0 \qquad \beta_{ij} = \beta_{ji} \qquad \text{for each } i \text{ and } j \tag{63}$$

Although the specification of (62) is derived from a model of individual household behaviour, Boskin and Lau have estimated the model on aggregate time-series data for the United States over the period 1929–69 (for a discussion of aggregation in models such as this, see Berndt, Darrough and Diewert, 1977).

Boskin and Lau reported estimates of the *uncompensated* elasticities, whereas our formulae for optimal tax rates are derived in terms of *compensated* elasticities. But in the special case of homothetic preferences (in consumption and leisure) it can be shown from equation (19) and the

Slutsky equations (A1)–(A4) that the following formula holds if we replace the compensated elasticities σ by the corresponding uncompensated elasticities η and denote the average propensity to save by s:

$$\frac{t_w}{1-t_w}(\eta_{LL}-\eta_{2L}+1)=\frac{rt_r}{1+r}(\eta_{L2}-\eta_{22}-s) \tag{64}$$

Taking prices and wealth as exogenously determined, Boskin and Lau obtained the following estimates for the elasticities (asymptotic t-statistics in parentheses):

$$\begin{array}{ll} \eta_{LL}=-0{\cdot}08 & \eta_{L2}=-0{\cdot}08 \\ \quad(-15{\cdot}64) & \quad(-3{\cdot}93) \\ \eta_{2L}=1{\cdot}11 & \eta_{22}=-1{\cdot}49 \\ \quad(39{\cdot}46) & \quad(-15{\cdot}71) \end{array}$$

These estimates have the striking implication that the optimal tax system comprises a substantial subsidy to capital income.

Taking s in the two-period case as 0·4, we get

$$t_r=-0{\cdot}19\left(\frac{t_w}{1-t_w}\right)\left(\frac{1+r}{r}\right) \tag{65}$$

The length of the period assumed by Boskin and Lau is twenty years. If we assume a real rate of return of 3 per cent per annum, a revenue requirement of 25 per cent of national income, and a share of capital income of 25 per cent, the optimal tax rates are

$$t_w=0{\cdot}405 \qquad t_r=-0{\cdot}216$$

The optimal tax system is a substantial subsidy to capital income. Moreover, the estimates (together with the assumptions about the rate of return and share of capital income) imply that the *maximum* revenue that the optimal tax system could generate would be only 34 per cent of national income. This would occur with a tax rate on labour income of 57·5 per cent.

Substituting (65) into (34) produces a non-linear relationship between the revenue raised as a proportion of national income and the optimal tax rate on earned income. This could be interpreted as an optimal tax version of the so called 'Laffer' curve relating tax revenue to the rate of tax. We have explicitly allowed for the optimal tax structure in order to find the reduced form solution for the relationship between revenue and tax rates.

The virtue of the Boskin and Lau study is that it shows clearly how crucial are the estimates of the cross-elasticities, which are usually ignored in studies that focus on the dependence of labour supply on the wage rate and of savings on the interest rate. An earlier study of the life cycle model by Diewert (1974) employs an indirect utility function defined over the prices of consumption, leisure and money for each of thirty periods. The model is estimated on aggregate annual US data over the period 1946–65. In order to limit the number of parameters to be estimated, Diewert imposed some *a priori* restrictions, including the assumption that the elasticity of future consumption with respect to the wage rate (σ_{2L}) is zero. Despite this restriction, his estimates also imply that a subsidy to capital income would be optimal.

A major difficulty with these models is the measurement of the price of future consumption, and this has recently emerged as a key issue in the estimation of conventional consumption functions incorporating interest rates There are two related issues here. The first is that, although it is clear that the relevant interest rate is the real net-of-tax rate of return, most of the few studies that include interest rates use nominal interest rates and ignore tax rates. This procedure leads to downward bias in the estimated interest elasticity of savings (Feldstein, 1970). A careful treatment of tax rates by Wright (1969) led to estimates of the interest elasticity of savings in the range 0·18 to 0·27. For empirical work it is necessary to decide on which of a whole array of interest rates to use and to estimate the average marginal tax rates on income from capital. We shall simply mention these questions and pass on.

The second issue that arises in measuring the rate of return is that it should be net of the expected rate of inflation. If the time horizon is of the order of twenty years (between the first and second periods of the stylised life cycle), we need to specify a model for the formation of expectations over a long period. The appropriate way to do this lies at the heart of the debate over the size of the interest elasticity of savings between Boskin (1978) and Howrey and Hymans (1978).

Boskin's aim was to estimate the interest elasticity of savings using a properly specified measure of the rate of return, net of both taxes and expected inflation. He used aggregate annual US data for the period 1929–69 and experimented with a variety of specifications and definitions of the variables. The preferred equation was estimated using instrumental variables to allow for simultaneous equations bias, and the sample period for this equation was 1929–66, excluding the war years 1941–6. This yielded the following estimated equation (standard errors in parentheses):

$$\log C_t = -5{\cdot}83 + 0{\cdot}55 \log Y_t + 0{\cdot}32 \log Y_{t-1}$$
$$\quad (1{\cdot}55) \quad (0{\cdot}13) \qquad (0{\cdot}23)$$
$$+ 0{\cdot}72 \log W_{t-1} - 0{\cdot}031 \log U_t - 2{\cdot}28\, R_t - 0{\cdot}36\pi_t \qquad (66)$$
$$(0{\cdot}03) \qquad (0{\cdot}014) \qquad (0{\cdot}62) \qquad (0{\cdot}21)$$
$$R^2 = 0{\cdot}99$$

where C_t = real *per capita* consumption,
Y_t = real *per capita* private disposable income,
W_t = real *per capita* wealth,
U_t = unemployment rate,
R_t = real post-tax interest rate, and
π_t = expected rate of inflation.

This equation implies a significant interest elasticity of savings of about 0·4 evaluated at the mean values of the variables. In fact the total elasticity of savings is higher than this, because the value of 0·4 is computed ignoring any effect of interest rate changes on wealth.

We shall not discuss the rationale of the specification of a consumption function of the type shown in (66) but concentrate on the estimate of the interest rate coefficient. The interest rate that Boskin used was a post-tax nominal rate minus the expected inflation rate. This latter variable was 'estimated from an adaptive expectations model of price expectations, truncated after 8 years, with varying speeds of adjustment' (p. S11). It is clear that this is not the only possible specification, and Howrey and Hymans (1978) have challenged Boskin's results by claiming that the significance of the interest rate coefficient is very sensitive to the particular measure of the expected rate of inflation and to the sample period. For example, by omitting a single year (i.e. 1934) they found that, using Boskin's original data, the interest rate coefficient becomes insignificant.

This is an important point, and it suggests that there may be insufficient information contained in annual observations to enable us to identify at all precisely the size of the savings elasticity. Unfortunately, Howrey and Hymans's own estimates are flawed by their use of single-equation estimation methods and a very narrow definition of savings. They preferred a concept of 'loanable-funds' saving to the more conventional national-income accounting definition, thus excluding saving in pension funds, owner-occupied housing and other durables. The resulting definition of saving covers only 14 per cent of private saving, amounting in 1975 to $36·0 billion out of a total figure for gross private saving of $259·4 billion.

On this basis Howrey and Hymans found an insignificant interest elasticity of savings.

Leaving aside the econometric difficulties of estimating the savings elasticity, we can see that Boskin is right to emphasise that even small values for the interest elasticity of savings imply large values for the elasticity of retirement consumption with respect to its own price. To relate estimates obtained from annual data to the two-period theoretical model we shall redefine the price of second-period consumption to allow for continuous compounding; thus,

$$h = e^{-rT} \qquad (67)$$

where r is the net-of-tax real interest rate per annum and T is the length of time between the periods, and

$$\frac{\partial h}{\partial r} = -hT \qquad (68)$$

Since $S = hC_2$, the compensated derivatives are related by

$$\frac{\partial S}{\partial r} = -hTC_2 + h\frac{\partial C_2}{\partial r} \qquad (69)$$

$$\therefore \quad \sigma_{sr} = -rT + \sigma_{2r} \qquad (70)$$

But

$$\sigma_{2r} = -rT\sigma_{22} \qquad (71)$$

$$\therefore \quad \sigma_{sr} = -rT(1 + \sigma_{22}) \qquad (72)$$

Using Boskin's average estimate of 0·4 for σ_{sr}, and values of 3 per cent for the interest rate and 25 years for the period, we obtain a point estimate of −1·533 for σ_{22}. Even if $\sigma_{sr} = 0$, then $\sigma_{22} = -1$, and it is evident that the efficiency arguments for or against an expenditure tax are only marginally affected by whether the interest elasticity of savings is zero or 0·4.

1.7 CONCLUSIONS

The main contentions of the Meade Report (Meade Committee, 1978) are (1) that there is probably little to choose between an expenditure tax and a comprehensive income tax on efficiency grounds, and (2) that the telling argument for an expenditure tax is that it represents the only practicable alternative to the present mess, which has arisen, at least partly, from an unworkable distinction between capital and income. To bolster this argument the Meade Committee has drawn attention to the enormous range of effective tax rates on capital income according to the type of asset and financial medium through which savings are channelled. This emphasis on the 'portfolio' nature of saving and its tax treatment contrasts with the simple model of life cycle saving examined above. In one sense our analysis confirms the Meade Report's judgement that on conventional efficiency grounds it is difficult to argue strongly for an expenditure tax. Yet, the analysis shows also that the efficiency losses of the present system may be large. Perhaps the most striking result is that the 'optimal' tax system is very sensitive to the characterisation of the life cycle in our model and much less so to the precise value of the interest elasticity of savings. The strongest arguments for an expenditure tax, within the framework of our model, are the failure to index the income tax base for inflation and the existence of unfunded pension schemes. Academic debate over the size of a single elasticity (i.e. the interest elasticity of savings) is only a part, and a small part at that, of the set of considerations that are relevant to the very important practical problem of tax reform.

APPENDIX

The expression for the optimal relative tax rates on wages and interest income is derived by substituting the Slutsky equations into equation (20). Using y for income and S_{ij} for the substitution terms, we get the Slutsky equations

$$\frac{\partial L}{\partial t_w} = -w\left(L\frac{\partial L}{\partial y} + S_{LL}\right) \tag{A1}$$

$$\frac{\partial C_2}{\partial t_w} = -w\left(L\frac{\partial C_2}{\partial y} + S_{2L}\right) \tag{A2}$$

$$\frac{\partial L}{\partial t_r} = rh^2\left(-C_2\frac{\partial L}{\partial y} + S_{L2}\right) \tag{A3}$$

$$\frac{\partial C_2}{\partial t_r} = rh^2\left(-C_2\frac{\partial C_2}{\partial y} + S_{22}\right) \tag{A4}$$

The compensated *elasticities* σ_{ij} are defined by

$$\sigma_{LL} = \frac{(1-t_w)wS_{LL}}{L} \tag{A5}$$

$$\sigma_{2L} = \frac{(1-t_w)wS_{2L}}{C_2} \tag{A6}$$

$$\sigma_{L2} = \frac{hS_{L2}}{L} \tag{A7}$$

$$\sigma_{22} = \frac{hS_{22}}{C_2} \tag{A8}$$

We now substitute (A1)-(A4) for the partial derivatives in (19), to get

$$\begin{aligned} rh^2C_2\left[wL - w^2t_w\left(L\frac{\partial L}{\partial y} + S_{LL}\right) - \frac{wrht_r}{1+n}\left(L\frac{\partial C_2}{\partial y} + S_{2L}\right)\right] \\ = wL\left[wt_wrh^2\left(-C_2\frac{\partial L}{\partial y} + S_{L2}\right) + \frac{r^2h^3t_r}{1+n}\left(-C_2\frac{\partial C_2}{\partial y} + S_{22}\right)\right. \\ \left. + \frac{rhC_2}{1+n}(1+rht_r)\right] \end{aligned} \tag{A9}$$

The income effects cancel out, and we may replace the substitution terms by the elasticities (A5)-(A8), using the symmetry condition that

$$S_{2L} = -S_{L2} \tag{A10}$$

This gives

$$1 - \left(\frac{t_w}{1-t_w}\right)\sigma_{LL} + \left(\frac{rt_r}{1+n}\right)\sigma_{L2} = \frac{1+r}{1+n} + \left(\frac{rt_r}{1+n}\right)\sigma_{22} - \left(\frac{t_w}{1-t_w}\right)\sigma_{2L} \tag{A11}$$

After rearrangement this becomes equation (20) of the text.

NOTES: CHAPTER 1

1 The model is based on Atkinson and Sandmo (1979), but the results and their derivation are somewhat different.
2 Those who prefer to work with the direct utility function will find that the mathematics is only slightly more cumbersome.
3 It can be shown that the second-order conditions for an optimum are satisfied if the revenue from each tax is an increasing function of the tax rate.
4 The 'only if' follows provided that all elasticities are finite.
5 It should be noted that (23) is not equivalent to the condition that the utility function be weakly separable into labour supply and some function of the consumption levels in the two periods except when the function of consumption levels is homothetic (see Atkinson and Stiglitz, 1976; and Auerbach, 1979).
6 This enables us to obtain a result and proof both of which appear to be much simpler than those in the literature. The envelope theorem is discussed in, for example, Dixit (1976).
7 We are assuming here that the incentive to invest depends only on the real rate of interest, which will not be true if the corporate tax system is unindexed.
8 To be strictly accurate, equation (60) shows that t_w and t_r have opposite signs. But from (34) it can be shown that, when the tax rates have opposite signs, it is the tax on labour income that is positive, provided that the share of capital income is less than $b/(1 + b)$, where b is the ratio of the absolute magnitudes of t_w to t_r. This condition is satisfied in all our examples and in all plausible cases.

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