

# Very Long-Run Discount Rates

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## Abstract

We provide the first direct estimates of how agents trade off immediate costs and uncertain future benefits that occur in the very long run, 100 or more years away. We find that very long-run discount rates are low, much lower than those routinely assumed by economic theory. We estimate these discount rates by exploiting a unique feature of residential housing markets in England, Wales and Singapore, where residential property ownership takes the form of either leaseholds or freeholds. Leaseholds are temporary, tradable ownership contracts with maturities between 50 and 999 years, while freeholds are perpetual ownership contracts. The difference between leasehold and freehold prices represents the present value of perpetual rental income starting at leasehold expiry. We estimate these discounts for varying leasehold maturities via hedonic regressions using proprietary datasets of the universe of transactions in each country. Agents discount very long-run cash flows at low rates, assigning high values to cash flows hundreds of years in the future. For example, 100-year leaseholds are valued up to 15% less than otherwise identical freeholds. This suggests that both long-term risk-free discount rates and long-term risk premia are low. Our results provide a new testing ground for asset-pricing theories, and have direct implications for climate-change policy and long-run fiscal policy.

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**Keywords:** Climate Change, Asset Pricing, Hyperbolic Discounting, Bubbles.

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Long-run discount rates play a central role in economics (Cochrane, 2011). For example, much of the debate around the optimal response to climate change centers on the trade-off between the immediate costs and the long-term benefits of policies that aim to reduce global warming: how much should be spent today to avoid environmental damage 100 or more years from now? Unfortunately, there is little direct empirical evidence on how households discount payments over such long horizons (Pindyck, 2013). We mostly observe prices of either claims to perpetual streams of payments, for example equity securities, or claims to payments that occur in the short to medium run, such as 10-year bonds.

We provide the first direct estimates of households' discount rates for payments very far in the future. We find very long-run discount rates to be low, much lower than what most economic models would predict. To estimate these long-run discount rates, we exploit a unique feature of residential housing markets in England, Wales and Singapore, where property ownership takes the form of either very long-term leaseholds or freeholds. Leaseholds are ownership contracts with maturity ranging between 50 and 999 years, while freeholds are perpetual ownership contracts. There are active secondary markets for both leaseholds and freeholds. The difference in price between leaseholds and freeholds on otherwise identical properties represents the present value of perpetual rental income starting at leasehold expiry.<sup>1</sup>

The main contribution of this paper is to estimate the term structure of the discounts between leaseholds of varying lease length (50 to 1000 years) and freeholds, and to draw implications for economic theory and policy. We find that agents discount very long-run cash flows at very low rates; for example, 100-year leaseholds are valued up to 15% less than otherwise identical freeholds. Discounts are even greater at shorter maturities. For example, they grow to 30% for leaseholds with 50 to 70 years remaining. The discounts are zero for leaseholds with maturities of more than 800 years. We show that, even under conservative assumptions on the long-term growth rate of rental income, these results are indicative of discount rates for cash flows that arise in the very long run that are substantially lower than those routinely assumed in economic theory. This is because standard exponential discounting assigns a low value to distant payoffs even at moderately low discount rates.

Our empirical analysis is based on proprietary datasets of residential property sales in England,

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<sup>1</sup>Consider a hypothetical property for which a buyer is offered either a 100-year lease or the freehold. In a frictionless market, the value of the freehold is equal to the present value of all future rental income, while the value of the leaseholds is the present value of the rents for the first 100 years only. Therefore, the difference in value between the two contracts is the present value of rents starting 100 years from now and extending to the infinite future.

Wales and Singapore. These datasets contain information on transaction prices, contract terms (including the lease length for leasehold properties), as well as property characteristics such as location and structural attributes. Our datasets cover the universe of private-market residential housing transactions in England and Wales for the 2009-2013 period and in Singapore for the 1995-2013 period. We estimate long-run discount rates by comparing the prices of leaseholds with different maturities to the prices of freeholds across otherwise identical properties. Since we only observe transaction prices for properties that are either freeholds or leaseholds, we employ hedonic regression techniques to control for possible heterogeneity between properties offered as leaseholds and properties offered as freeholds. Conditioning on observed property characteristics allows us to identify the discounts due to differences in lease length.

Our empirical results are consistent across both England-and-Wales and Singapore, two housing markets with otherwise very different institutional settings. We minimize the concern that our results could be driven by unobservable quality differences between freehold and leasehold properties or by institutional differences between the two types of contracts by showing that there is no price difference between leaseholds with more than 800 years remaining and freeholds that have similar observed structural characteristics. Similarly, our results are not driven by potential frictions that might be important for short-maturity leasehold properties (50-70 years), such as financing frictions, since discounts to freeholds remain substantial even for leaseholds with 150 or even 250 years of maturity.

To interpret the economic magnitude of the observed discounts, we first compare our results to the standard Gordon-Growth valuation model ([Gordon, 1982](#)). This deterministic model discounts rental income, which is assumed to grow at rate  $g$ , at a constant rate  $r$ . While this model abstracts in many dimensions, it provides a simple framework to interpret our results. We then strengthen the intuition by considering the impact of risk and frictions in more general models. To calibrate the return to housing ( $r$ ) and the growth rate of rents ( $g$ ) in the Gordon-Growth model, we estimate unconditional expected housing returns and rent growth in the U.S., the U.K. and Singapore. Real rates of rent growth are low, 0.2% a year, and very similar across countries. Expected real returns to housing are relatively high, between 6% and 9% a year, and primarily driven by high rental yields. These results are consistent with [Shiller \(2006\)](#), who argues that both real house prices and real rents have low growth over very extended periods of time. The Gordon-Growth model suggests that even

with a conservative rate of return of 5.5% and aggressive rent growth of 2% the discount in price of leaseholds of 100 or more years relative to freeholds should be essentially zero. This simple model highlights that the challenge for economic theory is to *jointly* rationalize a high expected return to housing (6 – 9%) with the low discount rates necessary to match the observed discounts for long-dated leaseholds relative to freeholds. We call this the “long-run valuation puzzle.”

We then consider whether the risk properties of housing can help explain the long-run valuation puzzle. The leading asset pricing models were not specifically designed to match the prices of very long-run cash flows, and imply an upward sloping term structure of discount rates for housing cash flows (given the observed expected returns on housing and rent growth rates). As noted in [Binsbergen, Brandt and Kojen \(2012\)](#) for the case of equity cash flows, the external habit formation model of [Campbell and Cochrane \(1999\)](#) and the long-run risk model of [Bansal and Yaron \(2004\)](#) produce an upward sloping term structure of discount rates, while the variable rare disaster model of [Barro \(2006\)](#), [Martin \(2013\)](#) and [Gabaix \(2012\)](#) generates a flat term structure of discount rates. These models thus produce a tension between rents that are sufficiently risky to generate a high average expected return to housing and the fact that, as rents become riskier, long-term cash flows are discounted at progressively higher rates thus generating smaller discounts for leaseholds with respect to freeholds. Extending the Gordon-Growth model by taking into account that rents are risky and housing commands a nontrivial risk premium therefore exacerbates the long-run valuation puzzle.<sup>2</sup>

We then consider the possible role played by financing frictions for leaseholds in explaining the long-run valuation puzzle. There are two opposing forces. On the one hand, shorter leases could be attractive to buyers that are liquidity constrained. Consider for example a young couple with high future income. Since the couple cannot borrow against their human capital, they might prefer to buy a shorter lease and extend as their human capital is monetized. This effect makes leaseholds more desirable compared to freeholds, decreases the discounts, and thus exacerbates the long-run valuation puzzle. On the other hand, mortgage lenders typically require 30 years of unexpired lease term to remain at the end of the mortgage, suggesting that leaseholds have to be financed with shorter maturity mortgages once the lease length falls below 60 years. While this effect certainly can contribute to lower valuations for short-term leases through the loss of collateral value, we show that it cannot quantitatively affect the discounts for longer-term leases. Intuitively, a lease that has 200 years left

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<sup>2</sup>See also [Lustig, Van Nieuwerburgh and Verdelhan \(2013\)](#).

today will only incur potential losses to its collateral value 140 years from now. Any losses that occur that far into the future have little impact on present values at conventional discount rates.

We analyze the features required of a model to rationalize the long-run valuation puzzle. We find our estimates to be consistent with a downward sloping term structure of discount rates. Discount rates have to be sufficiently high in the short to medium run to contribute to high average expected returns on housing, but also sufficiently low in the long run to match the observed discounts applied to long-run cash flows. Two existing classes of models can potentially generate this feature. Models of hyperbolic discounting, along the lines of [Laibson \(1997\)](#) and [Luttmer and Mariotti \(2003\)](#), generate a downward sloping term structure via a declining rate of time preference. The reduced form model of [Lettau and Wachter \(2007\)](#) generates a downward sloping term structure of discount rates for risky assets because claims to long-run cash flows have lower exposure to the unexpected innovation to rents (or dividends), which is the priced shock in the model.

Finally, our data and estimates are uniquely suited to directly test the classic theories of infinitely-lived rational bubbles of [Blanchard and Watson \(1982\)](#) and [Froot and Obstfeld \(1991\)](#). These theories study bubbles that in expectation grow faster than the discount rate and therefore imply a failure of the terminal condition that would normally impose the present value of a payment occurring infinitely far into the future to be zero. We can directly test this condition by verifying whether on average leaseholds of very long maturity, 800 or more years, are valued identically to freeholds on otherwise similar properties. We stress that our tests are model independent. Contrary to most of the empirical literature on bubbles, we do not need to assume a specific model of the "fundamental" value of the asset because *all* models, that assume the absence of infinitely-lived rational bubbles, imply the same value for a claim to a payment at infinite maturity, namely zero.

Focusing our analysis on real estate has several advantages. Real estate constitutes the most significant asset in most households' portfolios. Therefore, the term structure of discount rates applied to real estate cash flows contains important information about the time preferences of households over long horizons, as well as the risks perceived over different time horizons and the corresponding risk premia. The importance of real estate investment decisions for households also minimizes concerns about the reliability of our estimated discount rates compared to those that are more commonly derived in experimental or survey settings. Finally, real estate is the only major asset class for which we have liquid markets in which agents trade finite-horizon contracts spanning hundreds of years.

As such, it opens a new opportunities to study time and risk preferences over the very long run.

**Implications** Our paper contributes to three broad areas of economics and finance: asset pricing, environmental policy and real estate economics.

First, our results are informative for asset pricing theory. Our empirical evidence provides a new testing ground for the leading theoretical models of asset pricing as well as an input into the development of new theories. We view our paper as complementary to the recent and innovative contribution of [Binsbergen, Brandt and Kojen \(2012\)](#).<sup>3</sup> They study the term structure of equity discount rates by focusing on dividend strip securities and their prices. They show that the term structure of expected returns on equities is downward sloping. Our results are complementary in three dimensions. First, we focus on a different asset class: real estate instead of equity. Both are important components of households' portfolios. Second, our estimates are directly informative about (very) long-run discount rates, i.e. 80-250 years, while their estimates focus on (relatively) short-run discount rates (1-3 years in the original reference, and extended to 1-10 years in [Binsbergen et al. \(2013\)](#)). Finally, our methodology is different and provides independent evidence on downward-sloping term structures of discount rates in major asset classes.<sup>4</sup>

The literature on environmental policy has forcefully argued the importance of long-run discount rates in assessing the benefits of policies such as reducing carbon emissions ([Gollier and Weitzman, 2010; Pindyck, 2013; Barro, 2013](#)). For example, [Stern \(2007\)](#) called for immediate policy action to reduce future environmental damage based on the assumption of very low discount rates (essentially zero). The authors argue that while agents discount the future over their lifetimes, they have an ethical impetus to care about future generations (even very distant ones) leading to very low long-run discount rates. The Stern report has been criticized, amongst others by [Weitzman \(2007\)](#) and [Nordhaus \(2006\)](#), who argues that “the Review’s radical revision arises because of an extreme assumption about discounting [...] this magnifies enormously impacts in the distant future and rationalizes deep cuts in emissions, and indeed in all consumption, today.” A significant part of the critique hinged on the observation that asset markets reveal discount rates much higher than zero and often close to 6%, the private return to capital. However, such estimates of expected returns are based on claims to

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<sup>3</sup>A nascent literature motivated by this contribution includes [Belo, Collin-Dufresne and Goldstein \(2012\)](#) and [Boguth et al. \(2012\)](#).

<sup>4</sup>While each methodology is subject to potential criticism (see [Boguth et al. \(2012\)](#) comment on [Binsbergen, Brandt and Kojen \(2012\)](#)), we find it reassuring that different methodologies and data lead to consistent conclusions.

infinite streams of cash flows and, as such, are not directly informative of long-run discount rates.

We contribute to this literature by providing direct empirical evidence on long-run discount rates for an important asset, housing, that is arguably a good proxy for future environmental outcomes.<sup>5</sup> It is beyond the scope of this paper to develop a full quantitative integrated assessment model to formally analyze the impact of our results on optimal environmental policy. However, we note that our long-run discount rates are higher than those in the Stern report but substantially smaller than those suggested if one simply applied the unconditional return to the capital stock, the equity market, or even housing.

Finally, our results are of direct relevance for real estate economics and the ongoing effort to understand house prices. We add to the recent research effort to understand the return properties of real estate ([Flavin and Yamashita, 2002](#); [Lustig and Van Nieuwerburgh, 2005](#); [Piazzesi, Schneider and Tuzel, 2007](#); [Favilukis, Ludvigson and Van Nieuwerburgh, 2010](#)) by focusing on a previously unexplored aspect of real estate: the term structure of house prices.

## 1 Housing Markets in Singapore and the United Kingdom

In this section we provide the basic institutional details about the housing markets of the UK and Singapore (especially in relation to the properties of freeholds and leaseholds). Appendix [A.1](#) provides additional information, including details on the property taxation regimes.

### 1.1 The UK Housing Market

Around 1.43 million properties are owned as leaseholds in England and Wales ([The Independent, 2013](#)). Owning a leasehold interest provides the right to live in the property or to rent it for a period of time up to the term of the lease. The initial term of a leasehold interest is typically 99, 125, 150, 250 or 999 years. Leasehold properties are often transacted in the private secondary market, in which case the buyer purchases the remaining term of the lease. Once the leasehold expires, the ownership reverts back to the freeholder, a process called “reversion”. However, it is common for leaseholders to purchase extensions of the outstanding terms ahead of maturity. Over time, a number of laws

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<sup>5</sup>Houses, or land, cannot by definition be relocated and are therefore naturally exposed to environmental changes in climate or sea and river levels. For example, the housing stock in Singapore is naturally exposed to the potential threat of rising sea levels.

described in Appendix A.1.1 have regulated the rights of leaseholders to extend their lease terms. For our sample periods, leaseholds had the *right* to lease extensions at market prices. If leasehold and freeholder cannot agree on the market price, a government-run leasehold valuation tribunal would determine the price.

## 1.2 The Singaporean Housing Market

Residential properties in Singapore are either sold as freeholds or leaseholds, where the latter have initial terms of 99 years or 999 years.<sup>6</sup> There is a vibrant private secondary market for leaseholds, where buyers purchase the remaining terms of the original leases.

At the expiry of the lease, the ownership interest reverts to the land owner (i.e. reversionary right). After this, the owner can either maintain her ownership interest, sell the freehold outright, or sell a new leasehold term to a lessee (either the same or a new one). Lessees may apply for a renewal of the lease with the SLA (Singapore Land Authority) before the expiry of the lease. The granting of an extension is decided on a case-by-case basis; considerations include whether the development is in line with Government's planning intentions, is supported by the relevant agencies, and results in land use intensification, the mitigation of property decay and the preservation of community. If the extension is approved, the Chief Valuer determines the land premium that will be charged. The new lease will not exceed the original, and it will be the shorter of the original or the lease in line with the Urban Redevelopment Authority (URA) planning intention.

## 2 Empirical Analysis

The estimation of the relative prices of leaseholds and freeholds is challenging because the underlying properties are heterogeneous assets. Since leasehold and freehold properties could, in principle, differ on important dimensions such as property size and location, comparing prices across properties requires us to control for these differences. To address this challenge, we use hedonic regression techniques, which allow us to consider the variation in price over time and across lease terms for different properties while controlling for key characteristics of each property such as size, location

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<sup>6</sup>There are also other types of less common lease structures. The first are private development 103-year leaseholds sold on freehold land. In addition, in November 2012 a plot of land at Jalan Jurong Kechil was the first to be sold for residential development under an initial 60-year lease agreement; though houses built there do not yet appear in our data.

and property age.

## 2.1 Analysis - United Kingdom

### 2.1.1 U.K. Residential Housing Data

We begin by analyzing data from the United Kingdom. We have obtained administrative transaction-level data on all residential housing transactions in England and Wales for the period from the 1st of January 2009 to the 31st of March 2013. This initial dataset provides us with a total of 2.2 million housing transactions. The data include a leasehold indicator (whether or not the property is a leasehold or freehold), the price paid as well as some characteristics of the house: house type (detached, flat, semi-detached or terraced), full address with postcode, and a new construction indicator. In addition to this, we have obtained a separate, proprietary dataset on details of each lease from the U.K. Land Registry - this provides information on the lease start date as well as the overall lease length. Figure 1 shows the distribution of remaining lease lengths for properties at their point of transacting. We can see that there are many transactions with remaining lease lengths between 100 and 250 years. This is interesting, since it allows us to trace out the term structure across different horizons. Finally, for a subset of the homes, we have been able to obtain information from “for sale” listings from a large U.K. property listings website. In particular, this provides us with information on the number of bedrooms, bathrooms and the number of total rooms. Overall we can match approximately 760,000 transactions to listings, allowing us to obtain hedonic characteristics. Table 1 provides key summary statistics on our U.K. transaction sample.

### 2.1.2 Price Variation by Lease Length Remaining

In this section we estimate the relative prices paid for leaseholds of varying remaining duration and freeholds for properties in England and Wales. Given the support of the “remaining lease length” distribution (see Figure 1) we construct a number of *MaturityGroup* buckets for different remaining lease lengths. In particular, our buckets are 70-84 years, 85-99 years, 100-124 years, 125-149 years, 150-300 years and 300+ years. We then run regression (1). The unit of observation is a transaction  $i$  of a property of type  $g$  (e.g. detached, semi, terraced, flat/maisonette) in postal district  $h$  (of which we have 1,165 unique ones in the data) at time  $t$ . We assign each leasehold to one of the *MaturityGroup*

buckets depending on the number of years remaining on the lease at the point of sale. The excluded category are freeholds, so that the  $\beta_i$  coefficients capture the log-discount of leaseholds with that maturity relative to otherwise similar freeholds. Since we do not observe the size of the individual properties, our primary specification uses  $\log(Price)$  as our dependent variable. In a second set of results, we include  $\log(Price/Room)$  as the dependent variable.<sup>7</sup>

$$\log(Price_{i,h,t,g}) = \alpha + \sum_{i \in MaturityGroup} \beta_i \mathbf{1}_{\{Maturity \in i\}} + \gamma Controls_i + \xi_h \times \psi_t \times \phi_g + \epsilon_{i,h,t,g} \quad (1)$$

We control for average prices in a property's geography by including postal district by time of sale by property type fixed effects. This means that we are identifying leasehold discounts by comparing leaseholds to freeholds for the same type of property that was sold in the same area and at the same time. We also include control variables for whether the property is a new construction, as well as dummy variables for the number of bedrooms, bathrooms, and the number of total rooms. Standard errors are clustered at the level of the fixed effects.<sup>8</sup>

Table 2 shows the results from regression (1). In column (1) we control for the time of sale in the interacted fixed effects by including the quarter of sale. in column (2) by including the month of sale. In column (3), our preferred specification, we also interact our fixed effect with the number of bedrooms of the properties. This increases the number of fixed effects to 253,000. Here the identification of the  $\beta_i$  leasehold discount coefficients comes from comparing two properties of the same type with the same number of bedrooms sold in the same district and at the same time. The results show that freeholds and leases with maturities of more than 300 years (most of which have a maturity of more than 800 years) trade at approximately the same price: the coefficient on  $\beta_{300+Years}$  is small and statistically indistinguishable from zero. However, leaseholds with shorter maturities trade at significant discounts to otherwise identical freeholds: leaseholds with 100 to 125 years remaining trade at a 15% discount to freeholds. Leaseholds between 150 and 300 remaining trade at a 7% discount.<sup>9</sup>

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<sup>7</sup>We are in the process of obtaining additional hedonic property characteristics such as property size and age from a number of different sources for the next draft of this paper. As such, the current results should be considered as preliminary in that respect.

<sup>8</sup>Clustering standard errors addresses possible concerns about the correlation of regression residuals across different transactions within the unit of clustering. If this correlation was driven by unobserved characteristics or events that affected all properties within the level of fixed effects the same way, the fixed effects would already pick this up and robust OLS standard errors would be consistent. Therefore, given the large number of fixed effects, this is a very conservative strategy to estimate standard errors. See Petersen (2009) for details.

<sup>9</sup>The percentage discount is calculated as  $1 - e^{\beta}$ .

In columns (4) - (6) we include  $\log(Price/Room)$  as the dependent variable. The estimated log-discount of leasehold properties remains the same: while leases with 300+ years maturity remaining trade at the same price as freeholds, for shorter maturity leases there is a significant discount to the prices of freeholds. Figure 2 plots the coefficients  $\beta_i$  from regression (1). The top panel includes  $\log(Price)$  as the dependent variable, the bottom panel includes  $\log(Price/Room)$ .<sup>10</sup>

## 2.2 Data - Singapore

We next analyze data from Singapore. We have obtained transaction-level price data for all private residential transactions from the Urban Redevelopment Authority. We do not observe transaction prices for property sales by the HDB, which usually happen at below-market value (see section 1.2). In total we observe about 380,000 armslength transactions between 1995 and September 2013. For each transaction there is information on the transaction price and date, the lease terms, property characteristics such as size and age, as well as the precise location of the property. Table 3 provides an overview of the transaction sample used in the regressions. There are between 10,000 and 40,000 transactions per year. Many of transactions are for newly built apartments, with the average transacted home being less than 5 years of age. Between 30% and 60% of all private transactions each year are recorded for freehold properties. There is a fair amount of variance of the lease length remaining for transacting leaseholds. Figure 4 shows the frequency of remaining lease lengths for transacting properties. In the top panel we show the remaining lease length of leases initially written for 99 years. In the bottom panel we show the equivalent for leases of initially 999 years. There are essentially no transactions of leasehold properties with between 100 and 800 years remaining on the lease.

## 2.3 Analysis - Singapore

To analyze the relative price paid for leaseholds and freeholds we run regression (2). The unit of observation is a property  $i$  of type  $h$  (e.g., apartment, condominium, detached house, executive condominium, semi-detached house and terrace house), of title type  $s$  (either “strata” or “land”, see appendix A.1.2), in geography  $g$ , sold at time  $t$ . The key dependent variable is the price per square

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<sup>10</sup>Figure 3 plots the discounts for a robustness check in which we use the entire data sample, but replace the number of bedrooms, bathrooms and total rooms with a unique indicator for all properties that we cannot match to “for sale” listings. The results are very similar in this sample.

foot paid in the transaction.<sup>11</sup> As before, we split the 99-year leases into buckets with different groups of lease length remaining.<sup>12</sup> We form buckets of leases (*MaturityGroup*) with 50-69 years, 70-84 years, 85-89 years, 90-94 years and 95-99 years remaining. We also include a dummy variable for all 999-year leases, all of which have at least 800 years remaining when we observe the transaction. The excluded category are the freeholds.

$$\ln \left( \frac{\text{Price}}{\text{Sqft}} \right)_{i,h,s,g,t} = \alpha + \sum_{i \in \text{MaturityGroup}} \beta_i \mathbf{1}_{\{\text{Maturity} \in i\}} + \gamma \text{Controls}_{i,t} + \\ + \xi_h \times \rho_s \times \phi_g \times \psi_t + \epsilon_{i,h,s,g,t} \quad (2)$$

The results from this regression are shown in Table 4. In column (1) we control for 5-digit postcode by property type by title type by transaction quarter fixed effects. Beyond these 94,700 fixed effects, our other control variables include property age, size and type, as well as the total number of units in a development. As before, standard errors are clustered at the level of the fixed effect. The results are very consistent with our findings for the U.K. The price per square foot paid for freeholds and otherwise similar 999-year leases is economically and statistically identical. On the other hand, leases with durations of 100 years or less sell at a significant discount to otherwise identical freeholds. For example, a lease with 95-99 years remaining maturity trades at a 12.7% discount, a lease with 70-84 years remaining maturity trades at a 23% discount. The regression has an extremely high adjusted  $R^2$  of above 95%. This suggests that there remains no significant variation in prices that is not yet explained by our control variables, and that our discounts are thus unlikely to be driven by unobserved heterogeneity between freehold and leasehold properties.<sup>13</sup>

In column (2) we also interact the fixed effects with property type to further ensure that our results are not driven by observed differences between leasehold and freehold properties. In column (3) we further control for the transaction month rather than the transaction quarter. This is to address possible concerns that leaseholds and freeholds might transact at different times in the year, which, combined with aggregate market price movements over time could potentially explain our findings. While these additions increase the total number of fixed effects to approximately 98,000 and 140,000

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<sup>11</sup>In order to avoid our results being primarily driven by extreme outliers such as luxury condominiums, we winsorize the price per square foot at the 1% level. This adjustment has little effect on the estimated coefficients.

<sup>12</sup>See Figure 4 for a distribution of the lease length remaining at the time of sale in our dataset.

<sup>13</sup>The adjusted  $R^2$  remains at 95.3% if we exclude those instances where we only observe one transaction for a particular fixed effect, in which the fixed effects perfectly explains the transaction price.

respectively, the estimated discounts across all maturities remain the same in both specifications. Figure 5 plots the coefficients  $\beta_i$  from regression (2), as in column (3) of Table 4.<sup>14</sup> This provides a graphical display of the term structure of leasehold discounts.

In column (4), rather than controlling for the age of the property directly, we only focus on the sale of newly-built properties. The estimates for 95-99 year leases are unaffected. For leases with shorter maturities the estimates move around. However, since most leases get topped up to 99-years when the property gets rebuilt, there are essentially no transactions to estimate the discount of new properties with 80 years lease length remaining. In column (5) we restrict the transactions to those where the buyer is not the HDB. The results are very similar to those in columns (1) - (3), suggesting that sales to the HDB generally happen at market value.

### 2.3.1 Time Series of Discounts

We also investigate the returns of different series of leasehold and freehold properties.<sup>15</sup> We do this analysis for Singapore only, since our time series extends back to 1995 (as opposed to 2009 for the UK). Analyzing time series movements of house prices is challenging, because the characteristics of houses sold may vary over time. This means that comparing average transaction prices across different time periods is inadequate. Many time series of house prices such as the Case-Shiller indices for the U.S. are thus constructed using a repeat-sales methodology. Assuming that the characteristics of individual houses do not change over time, this approach elicits market prices movements by analyzing the appreciation of individual properties. However, when analyzing the time series movements of leaseholds, a repeat sales approach is inadequate. This is because in between two sales of the same leaseholds the lease length has declined, so that the change in the transaction price would underestimate market-wide increases of prices holding all else fixed.

In order to analyze the time series properties of the returns series we therefore need to keep the lease length of the properties fixed over time. To do this we estimate regression (3). We include 4-digit postcode by property type by title type fixed effects. As before, we also control for the age of the property (by including a dummy variable for every possible age in years), the size of the property (by including a dummy for each of 40 equally sized groups capturing property size) and the total

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<sup>14</sup>Figure 5 shows the same evidence but with robust unclustered standard errors.

<sup>15</sup>Analogously, [Binsbergen, Brandt and Kojen \(2012\)](#) analyze time series returns for US equity dividend strip securities of various maturity.

number of units in the property. We therefore run the following regression separately for houses of given remaining lease maturity:

$$\log(P_{Sqft,i,h,s,g,t}) = \alpha + \sum_{t=1996}^{2013} \beta_t I_{(Year=t)} + \gamma ControlVars_{i,t} + \phi_g \times \xi_h \times \chi_s + \epsilon_{i,h,s,g,t} \quad (3)$$

The time series of  $e^{\beta_t}$  is the price index for that lease type. Figure 6 shows these price indices for the same buckets as in Figure 5.<sup>16</sup> While definitely correlated, the time series of the price series are different across lease length. In particular, the short-end of the maturity structure (50-70 years) seems to appreciate faster than leases of higher maturity.

To get a clearer picture of the average returns across maturities, Figure 7 plots average yearly returns by maturity bin with standard errors. While these graphs are obtained using only the capital gains series, rents conditional on observable characteristics are likely to be the same across maturities. This suggests that the pattern for capital gains will follow the pattern for average returns. The figure suggests a pattern of decreasing discount rates by maturity (with the exception of the very-long term leaseholds and freeholds). We interpret these results as suggestive that expected returns are decreasing across maturities, consistent with the results in [Binsbergen, Brandt and Koijen \(2012\)](#) who look at short-end dividend strips of maturity of up to 10 years. Due to the short time series for returns the standard errors around the estimates are very high (the picture shows 1 standard error bounds around the estimate). Because of this, in section 3 we use several models to investigate the term structure of discount rates rather than to rely on the return estimates directly.

### 3 Discussion and Interpretation

The previous section presented some novel facts about the pricing of freeholds and leaseholds of different maturities. Leaseholds with over 800 years of maturity remaining trade at the same price as freeholds for otherwise identical properties. Discounts on leaseholds with maturities of 70-250 years range from 25% for maturities of around 70 years to 12 – 15% at around 100 years to 6 – 8% at around 200 years. These facts were broadly consistent across housing markets in England and Wales and in Singapore. In this section we analyze what these discounts can tell us about preferences and discounts over long-run horizons.

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<sup>16</sup>Average lease length remaining within each bin remains approximately even over time.

### 3.1 Expected Returns, Leasehold Discounts, and Discount Rates

We begin by interpreting the results in light of the classic Gordon-Growth valuation model ([Gordon, 1982](#)). While this model is certainly unrealistic in many dimensions, it provides a simple framework to obtain an immediate intuition about our results. We will then extend the setup to incorporate both risk and frictions in Sections [3.3](#) and [3.2](#), respectively.

The Gordon-Growth model assumes that rents are deterministic, markets are frictionless, and agents discount the future using a discount rate (required rate of return)  $r$ . Rents ( $D$ ) grow at rate  $g$  and evolve according to:  $D_{t+s} = D_t e^{gs}$ . Assume  $r - g > 0$ . A claim to the rents for  $T$  periods, the T-term leasehold, is valued at:

$$P_t^T = \int_t^{t+T} e^{-r(s-t)} D_t e^{gs} ds = \frac{D_t}{r-g} (1 - e^{-(r-g)T}). \quad (4)$$

It follows that the freehold is priced at:

$$P_t = \lim_{T \rightarrow \infty} P_t^T = \frac{D_t}{r-g}.$$

The discount for a T-term leasehold with respect to the freehold ( $Disc_t^T$ ) is:

$$Disc_t^T \equiv \frac{P_t^T}{P_t} - 1 = -e^{-(r-g)T}.$$

Note that in this model  $r$  corresponds to the expected (average) return on the housing claim, and  $g$  corresponds to the expected growth rate of rents. It follows that estimating the real expected return on housing ( $r$ ) and the real growth rate of rents ( $g$ ) is important to interpreting the magnitude of discounts implied by the Gordon-Growth model. We document below the estimates for real housing returns and rental growth rates for England-and-Wales and Singapore. At the estimated benchmark value of  $r - g = 6\%$ , the discount at 100 years would be  $Disc_t^T = -e^{-0.06*100} = -0.25\%$ . In other words, the 100-year leasehold would be valued only 0.25% less than the freehold. The discount we find in the data is 12%. After discussing our estimates of  $r$  and  $g$  in the subsections below, Section [3.1.4](#) shows that the simple numerical example provided above is robust to a number of alternative calibrations.

### 3.1.1 Real Housing Returns

In this section we estimate  $r$  and  $g$  for the US, the UK and Singapore. We describe briefly our methodology here, and provide the details of the data and estimation procedure in Appendix A.2. We employ three complementary approaches to estimating average returns to housing. The first approach, which we call the balance-sheet approach, is based on the total value of the residential housing stock and the total value of housing services consumed (the dividend from that stock). We obtain this information from countries' national accounts.<sup>17</sup> We control for the growth of the housing stock over time in order to back out the return series for a representative house. The second approach, which we label the price-rent approach, starts from the price-rent ratio estimated in a baseline year and constructs a time series of returns by combining a house price index and a rental income index. This approach focuses on a representative portfolio of houses and, therefore, does not need to correct for changes in the housing stock. For Singapore, we also have access to a time series of median house prices and median rents by region; these are actual values and not indexes. In this case, therefore, we use a direct approach and estimate the return by summing the capital gain and the dividend yield period by period. Therefore, for Singapore only, we replace the balance-sheet methodology with this direct approach. After adjusting for inflation, all three methods provide estimates of the gross real returns to housing ( $E[R^G]$ ). To compute net returns, we subtract maintenance costs and depreciation ( $\delta$ ) and any tax-related decreases in return ( $\tau$ ). We estimate net returns as  $r = E[R] = E[R^G] - \delta - \tau$ .

The top panel of Table 5 presents the estimated average housing returns for the US, England-and-Wales, and Singapore. Our estimates for housing returns in the US follow Favilukis, Ludvigson and Van Nieuwerburgh (2010).<sup>18</sup> While U.S. housing returns are not the focus of this paper, they provide a useful benchmark because they have been the subject of an extensive literature (Gyourko and Keim, 1992; Flavin and Yamashita, 2002; Lustig and Van Nieuwerburgh, 2005; Piazzesi, Schneider and Tuzel, 2007). The balance-sheet and the price-rent approaches provide similar estimates for the average annual real gross return ( $E[R^G]$ ): 10.9% and 8.2% respectively.<sup>19</sup> We calibrate the impact

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<sup>17</sup>To determine the total consumption of housing services, these measures impute the value of the owner-occupied equivalent rents, the housing services consumed by individuals from living in their own house. See Mayerhauser and Reinsdorf (2006) and McCarthy and Peach (2010) for a description of the construction of these measures.

<sup>18</sup>We thank Stijn van Nieuwerburgh for sharing the data and for insightful discussions on how to estimate housing returns.

<sup>19</sup>The returns actually refer to two overlapping but different time periods, since in each case we used the longest possible sample. When we apply the balance-sheet approach to the same sample used for the price-rent approach, we recover a 9.19% estimate.

of maintenance and depreciation ( $\delta$ ) at 1.5% and the property tax impact  $\tau$  at 0.67%.<sup>20</sup> We conclude that average real net returns in the U.S. housing market are between 6% and 9%. This is similar to the estimates in [Flavin and Yamashita \(2002\)](#), who find a real return to housing of 6.6%, and [Favilukis, Ludvigson and Van Nieuwerburgh \(2010\)](#), who find a real return of 9-10% before netting out depreciation and property taxes.

Column three and four in Table 5 report our estimates for the Singaporean housing market. The direct and price-rent approaches provide similar estimates for the average annual real gross return ( $E[R^G]$ ): 8.5% and 8.8%, respectively. We assume the cost of maintenance and depreciation to be 1.5%, in line with the estimates for the U.S., and the property tax impact to be 0.5%.<sup>21</sup> A conservatively low estimate of the real net returns in the Singapore housing market is therefore between 6.5% and 7%.

The two rightmost columns of Table 5 report the estimates for the housing market in England and Wales. The balance-sheet and the price-rent approaches provide similar estimates for the average annual real gross return ( $E[R^G]$ ): 10.7% and 8.5%, respectively. We maintain the calibration for the cost of maintenance and depreciation at 1.5%. There are no property taxes to be considered in England and Wales. Average real net returns in the U.K. housing market are approximately 7 – 9%.

Overall, the estimates show that real expected returns for housing are between 6% and 9% for all countries in our international panel, and at least 6.5% for England-and-Wales and Singapore. These estimates are in line with the existing literature, and robust to the different methodologies.<sup>22</sup> We note that our estimates for the U.S. and England-and-Wales are consistent with the notion (see [\(Shiller, 2006\)](#)) that average house price growth over extended periods of time is relatively low and the key

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<sup>20</sup>[Malpezzi, Ozanne and Thibodeau \(1987\)](#) provide an overview of the literature on depreciation. For example, [Leigh \(1980\)](#) estimates the annual depreciation rate of housing units in the U.S. to be between 0.36% and 1.36%. Depreciation is also a key calibration parameter for much of a recent literature in macroeconomics that considers households' portfolio and consumption decisions with housing as an additional asset. [Cocco \(2005\)](#) chooses a depreciation rate equal to 1% on an annual basis; [Díaz and Luengo-Prado \(2008\)](#) include an annual depreciation rate of 1.5%. Property taxes in the U.S. are levied at the state level and, while there is variation across states, are generally around 1% of house prices. Property taxes, however, are deductible from federal income tax. We assume that the deductibility reflects a marginal U.S. federal income tax rate of 33%. The net impact is therefore  $(1 - 0.33) * 0.01 = 0.67\%$ .

<sup>21</sup>Singapore levies a 10% annual tax on the estimated rental income of the property. A lower tax rate applies to owner-occupied properties (6%), but we use the more conservative (higher) rate for rental properties. See section 1.2 for details. The tax impact on returns is the tax rate times the average rent-price ratio, estimated at 5%. Hence,  $\tau = 0.1 * 0.05 = 0.5\%$ .

<sup>22</sup>We also note that since most movements in rent-price ratios are driven by movements in house prices and not by movements in rents ([Shiller, 2007](#)), our estimates of returns are relatively unaffected by the time period chosen. For example, since 2013 rent-price ratios in the U.S. have declined to approximately their 2000 levels (see Figure A.2), ending the sample in 2005 would have produced a slightly lower average rent-price ratio. However, focusing on that period would also exclude the house price crash from our estimates of capital gains, thus leading to higher estimated average capital gains. In the overall estimates of expected returns, the higher estimated capital gains would be offset by a lower estimated rent-price ratio.

driver of real housing returns is the high rental yield. Our estimated average capital gains are positive but relatively small (even for Singapore where they are the highest) despite focusing on samples and countries that are often regarded as having experienced major growth in house prices.

### 3.1.2 Real Rental Growth

In order to calibrate the parameter governing rent growth in the Gordon-Growth formula ( $g$ ), we now turn to estimating the average growth rate of rental income, obtained directly from the rental indexes. The national accounts and the rental index provide similar growth rate estimates on the sample where both are available.

The estimated real growth rate of rents is close to zero. For the U.S., our estimate (0.2%) is in line with the estimates of [Campbell et al. \(2009\)](#) that obtain a median growth rate of 0.4% per year. We obtain an identically low estimate (0.2%) of average annual rental growth for Singapore, while the U.K. estimate is somewhat higher at 0.7%. As for the case of real average house price growth, our estimates of small-to-negligible real rent growth are in line with [Shiller \(2006\)](#). In our baseline estimates, we calibrate  $g$  to be 0.2%.

### 3.1.3 The riskiness of housing returns

While for the purpose of our calibration we are directly interested in the expected return  $r$  and the growth rate of rents  $g$ , it is interesting to study how risky housing assets are. How are housing returns correlated with the marginal utility of consumption? Figure 8 plots the growth rates of rents and personal consumption expenditures (PCE) in the U.S. since 1929. In periods of falling PCE, in particular the Great Depression, rents also fell noticeably. The bottom panel shows that there is a (weak) positive relationship between the growth rates of rents and personal consumption expenditures. This suggests that housing rents tend to increase when consumption increases and the marginal utility of consumption is low. Table 6 uses data from [Mack and Martínez-García \(2011\)](#) to report the correlation between annual real house price growth and personal disposable income in a panel of 21 developed and emerging countries. The average correlation is 37%, with a minimum of 5.4% for Luxembourg and a maximum of 63.1% for Spain. Overall, this evidence suggests that housing returns are risky.

### 3.1.4 The long-run valuation puzzle: calibration

We are now fully equipped to calibrate the Gordon Growth model from Section 3.1 using the estimates of  $r$  and  $g$  from Sections 3.1.1 and 3.1.2 respectively. Figures 9 and 10 compare the logarithmic discounts obtained under our baseline calibration ( $r = 6.5\%$  and  $g = 0.2\%$ ) for different leasehold maturities with those observed in the data (estimated with the hedonic model) for the U.K. and Singapore respectively. The 800+ year leaseholds are valued at a 0% discount to freeholds both in the data and in the model. However, the model cannot match the discounts observed for leaseholds with maturities of 250 years or less. For example, for leaseholds with 50-70 years remaining, we observe an average log discount of 38% in the data. The log discount from the model is a mere 2.8%. Intuitively, a model of exponential discounting assigns essentially zero present value to cash flows occurring 100 or more years into the future when discounting at an effective discount rate  $r - g$  of 6% or more.

This intuition is robust to even more conservative calibrations of the parameters  $r$  and  $g$ . In Section 3.1.1 we documented that expected returns of 6.5% and rent growth of 0.2% are already conservative estimates of these parameters. One might conjecture that “super-star” cities like Singapore or London might experience higher rent growth in the future (Gyourko, Mayer and Sinai (2006)). However, it should still be true that rents and consumption be cointegrated in the long run, so that they should share the same long-run growth rate. To allow for this eventuality, we evaluate a “high rent growth rate” scenario by setting  $g = 2\%$ .<sup>23</sup>

Similarly, while the 6.5% average housing return is already at the low end of the estimates reported in Table 5, we provide a “low expected returns” scenario with  $r = 5.5\%$  per year. Figures 12 and 11 compare the discounts obtained in baseline case, the high-rent-growth and low-expected-return scenarios with those estimated in the data for Singapore and the U.K., respectively. Both robustness exercises increase the model implied discounts, but only slightly. Even the calibration that allows for both low returns and high rent growth is far from matching the data and can at best explain only one third of the observed discounts.

While the observed discounts could be matched by an *unrealistic* calibration with  $r - g = 2\%$ , this calibration would then not make sense of the high average return to housing. Therefore, we stress that the simple Gordon-Growth model already highlights the challenge for economic theory posed

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<sup>23</sup>Note that growth rates higher than consumption would have the counterfactual implication that over time a larger and larger fraction of consumption expenditures would be devoted to rents. We also note that the past low growth rate of rents occurred in a period when London and Singapore have already been rising capitals of the world.

by our results. A theory of valuation would have to *jointly* rationalize both a high expected return to housing (6 – 9%) and the low long-run discount rates necessary to match the observed discounts for long-dated leaseholds relative to freeholds. We call this joint problem the "long-run valuation puzzle". Clearly a model of constant discount rates, such as the Gordon-Growth model, cannot solve the long-run valuation puzzle.

### 3.2 Heterogeneity, Frictions and the Collateral Value of Housing

Given the inability of the Gordon-Growth model to match the observed discounts, we now consider whether frictions in the housing market and sources of heterogeneity between leaseholds and freeholds could explain the discounts. We will conclude that none of these frictions are able to explain the significant discounts that we observe between leasehold and freehold properties.

#### 3.2.1 The Collateral Value of Housing

There are two contrasting frictions that could affect our results: a liquidity and a financing friction.

Leaseholds, in particular short dated ones, require lower upfront payment to take ownership of a property (even if only for a limited number of years). This is clearly evidenced by the estimated discounts, which can be quite large. If households have high future income that cannot be immediately monetized, these shorter leaseholds are a more attractive investment than longer leaseholds or freeholds. For example, consider a young couple with high prospective income. They cannot borrow against this income because lenders do not accept "human capital" as a collateral. They should then find a short leasehold, with the possibility of extending the lease in the future as their high income is realized, a more attractive investment option than a long lease or the freehold because the short lease has a lower upfront payment (the price) but provides an entirely similar if limited-in-time ownership of the property. This liquidity effect makes all else equal shorter leaseholds relatively more expensive, thus reducing the discounts compared to a frictionless benchmark. Since this effect worsens the long-run valuation puzzle, we do not assess its quantitative implications.

A second potential source of concern is that properties with short maturity leases are harder to finance (i.e. their collateral value is low) than long maturity ones. In particular, both in Singapore and in the UK it becomes difficult to obtain a mortgage using short maturity leaseholds as collateral. Mortgage lenders in the U.K. typically require 30 years unexpired lease term to remain at the end of

the mortgage ([Council of Mortgage Lenders, 2013](#)). Mortgages normally have maturities between 10 and 30 years with the most common term length being 25 years in the UK market. This means that leasehold purchases have to be financed with shorter duration mortgages once the lease length falls below 55 or 60 years. The loss in "collateral value" for these leaseholds could in part contribute to the large discounts we observe in the data, particularly for leases in the 50 – 70 years of maturity basket.

It is beyond the scope of this paper to provide a full general equilibrium model of housing in the presence of collateral and borrowing constraints. Instead we consider a simple deviation from the Gordon Growth model and check whether the quantitative implication of a reduced form collateral constraint can help explain the observed discounts.

Assume that for the last  $\bar{T}$  years of lease maturity, which we will calibrate to 60 years, the house has lower collateral value. We model this has an effective rent for the last  $\bar{T}$  years that is a fraction  $(1 - \alpha)$  of the original rent. This loss corresponds to the per-period shadow value of liquidity (i.e., the per-period cost to the buyer of having to use own resources to finance the house instead of an external mortgage). Alternatively, we can interpret  $\alpha$  as the total loss in value once the leasehold reaches 60 years of remaining maturity due to the fact that new potential buyers will no longer have access to long-maturity mortgages and might need to make a larger downpayment.

The value of the lease now follows:

$$\begin{aligned}
P_t^T &= \int_t^{t+T} e^{-\rho(s-t)} D_t e^{g(s-t)} (1 - \alpha \mathbf{1}_{\{s>t+T-\bar{T}\}}) ds = \\
&= \int_t^{t+T} e^{-\rho(s-t)} D_t e^{g(s-t)} ds - \alpha \int_{t+T-\bar{T}}^{t+T} e^{-\rho(s-t)} D_t e^{g(s-t)} ds + \\
&\quad + \mathbf{1}_{\{T<\bar{T}\}} \alpha \int_{t+T-\bar{T}}^t e^{-\rho(s-t)} D_t e^{g(s-t)} ds \\
&= \frac{D_t}{\rho - g} \left[ 1 - e^{-(\rho-g)T} - \alpha \left( e^{-(\rho-g)(T-\bar{T})} - e^{-(\rho-g)T} \right) + \mathbf{1}_{\{T<\bar{T}\}} \alpha \left( e^{-(\rho-g)(T-\bar{T})} - 1 \right) \right]. \quad (5)
\end{aligned}$$

Notice that the first multiplicative term in equation (5) is simply the valuation of the freehold under the Gordon-Growth formula  $\left(\frac{D_t}{\rho-g}\right)$ . The first term inside the squared bracket  $\left(1 - e^{-(\rho-g)T}\right)$  is the Gordon-Growth price adjustment for the value of a T-maturity leasehold as shown in equation (4). The second term inside the squared bracket  $\left(\alpha \left(e^{-(\rho-g)(T-\bar{T})} - e^{-(\rho-g)T}\right)\right)$  is the loss in value for the T-maturity leasehold due to the frictions. Notice that this term is zero whenever there are no frictions ( $\alpha = 0$  or  $\bar{T} = 0$ ). The last term inside the squared bracket  $\left(\mathbf{1}_{\{T<\bar{T}\}} \alpha \left(e^{-(\rho-g)(T-\bar{T})} - 1\right)\right)$

captures the notion that if a leasehold has already less than  $\bar{T}$  years left than it would be valued at:

$$P_t^T = \frac{D_t(1-\alpha)}{\rho-g} (1 - e^{-(\rho-g)T}),$$

so that the leasehold is valued as if the rents were only a fraction  $(1-\alpha)$  of the original ones.

The model implied discounts are now:

$$Disc_t^T = e^{-(\rho-g)T} + \alpha \left( e^{-(\rho-g)(T-\bar{T})} - e^{-(\rho-g)T} \right) - \mathbf{1}_{\{T < \bar{T}\}} \alpha \left( e^{-(\rho-g)(T-\bar{T})} - 1 \right).$$

Let us focus on the case in which  $T > \bar{T}$ , i.e. if we are valuing a leasehold with maturity beyond the problematic threshold. Notice the following effects:

1.  $\frac{\partial Disc_t^T}{\partial \alpha} > 0$ , the discount increases the greater the per-period collateral benefit
2.  $\frac{\partial Disc_t^T}{\partial \bar{T}} > 0$ , the discount increases whenever the threshold for financing increases
3.  $\frac{\partial Disc_t^T}{\partial \alpha \partial \bar{T}} < 0$  and  $\lim_{T \rightarrow \infty} \frac{\partial Disc_t^T}{\partial \alpha} = 0$ , the marginal effect of the loss in collateral value on the discount decreases with maturity of the lease and is zero in the limit of very long leases.

In Figures 14 and 13 we calibrate the model for different values of  $\alpha$  (we set  $\bar{T} = 60$ ) and compare the implied discounts to those we obtain in the data. We parametrize  $r$  and  $g$  as in our baseline Gordon Growth estimate at 6.5% and 0.2%, respectively. For the proportional value of the collateralizability of the house ( $\alpha$ ) we explore a range between 5% and 20%, which we believe to be a conservative estimate. The figure shows that even for  $\alpha$  as high as 20%, the model cannot match the empirical discounts at essentially any horizon.

Most importantly even *unrealistically* high assumptions on the loss of collateral value for short duration leaseholds cannot help to explain the discounts for leases of long maturities (for example 150 or 250 years). Intuitively, a lease that has 200 years left today will only incur potential losses of its collateral value 140 years from now, when the lease will have 60 years left. Any losses that occur so far into the future have little impact on present values at conventional discount rates. Formally, this is shown in Point 3 above: the effect of the friction on a lease is decreasing in the maturity of the lease and zero for freeholds.

### 3.2.2 Institutional Differences between Freeholds and Leaseholds

In the UK there are two other possible institutional features that might reduce the value of leaseholds relative to freeholds: ground rents and service charges. However, both of those are far too small in magnitude to explain the estimated difference and we only describe here their qualitative implications for completeness.

A lessee generally has to pay annual ground rent to the freeholder. The rational for the ground rent is that the purchase price of the lease only covers the ownership for the duration of the lease of the property, but not the land the property sits on. The land still belongs to the freeholder who has the right to request that the lessee makes regular payments for the use of the land, the ground rent.

Ground rent payments are generally very small (50-100 pounds per year) for a typical property and in many cases are either zero or a symbolic amount ("a peppercorn"). In fact, all leases extended under the Leasehold Reform Act of 1993 are set as such peppercorn levels. Even in cases where the ground rent is in principle positive, it is often zero in practice, because for the rent to be collected the freeholder has to make a specific written request to the lessee. Oftentimes such requests are not made because the amount collected would be too small to cover the administrative costs. Ground rents are customized on a property by property basis and no centralized database exists. This makes it hard to control for them in the regression analysis. We stress, however, that the amounts involved for almost all properties are so small as not to constitute a problem for our analysis.

Similarly, the lessee has to generally pay a service charge to a Management Agency appointed by the freeholder. In apartment buildings sold as leaseholds, the freeholder still manages the common areas of the building and appoints a Management Agency to do so. The service charge is the amount that the lessees pay every year (or as a one-off if major works are carried out) to the freehold's Management Agency to cover the cost of the maintenance of common areas. The quota that each lessee pays depends on his share of the building.

While maintenance costs can be a non-trivial amount, as long as the maintenance is carried out at fair value (the private market cost of the works) service charges are not a problem for our analysis. While of course we cannot rule out that some freeholders attempt to extract monopoly rents via the service charge, there are strong mitigating factors that alleviate this concern. First, in many cases it is actually efficient to have the freeholder manage the property because she will in general own the freehold of many properties (e.g. a landed estate) and can enjoy the resulting economy of scale in

the management of the properties. Second, the lessees can ask for the right to manage (RTM) the property and appoint their own management agency.

### 3.3 Risk, Return, and Discounts in Asset Pricing Models

The Gordon-Growth model and the financing-friction reduced-form model of the previous two sections are deterministic models. We now verify whether the introduction of risk helps to rationalize the long-run valuation puzzle through the lenses of the leading general equilibrium asset pricing models: the external habit formation model of [Campbell and Cochrane \(1999\)](#), the long-run risk model of [Bansal and Yaron \(2004\)](#), and the variable rare disaster model of [Barro \(2006\)](#), [Martin \(2013\)](#) and [Gabaix \(2012\)](#). These models were not originally set up to understand the term structure of discount rates in the housing market – and especially the very far end of the term structure – and therefore our empirical findings are a new testing ground for these theories.

Our results and evaluation of the three theory models considered above complement those in [Binsbergen, Brandt and Kojen \(2012\)](#) who focus on the models' ability to reconcile the expected returns of short-dated dividend strips (up to 3 years) with the equity premium. We therefore only discuss the models briefly here and point out which elements are most connected to the valuation of long-dated claims to housing. In all three models we deviate as little as possible from the calibrations of the stochastic discount factor and cash-flows provided in the original references. Some adjustments to the calibration are necessary to account for the lower growth and somewhat lower risk of rents compared to the equity dividends that were the original focus of these models.

In all three cases we calibrated the rents to have long-run mean growth equal to 0.02% as in our baseline calibration and to be sufficiently risky so that the models match the average expected returns to housing of 6.5%, as in our baseline calibration. Figure 15 and 16, then, show the discounts for long-dated leaseholds relative to freeholds implied by these models together with those observed in the data. In all cases the models are unable to match the discounts and actually tend to produce even smaller discounts than those of the Gordon-Growth model.

In the long-run risk model of [Bansal and Yaron \(2004\)](#) agents, who have preference for early resolution of uncertainty, are concerned about shocks that persistently affect the growth rate of consumption.<sup>24</sup> Therefore, agents dislike claims to very long-term cash flows that are particularly exposed to

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<sup>24</sup>We calibrate the model following the parametrization of [Bansal and Yaron \(2004\)](#). The baseline calibration of the risky

these risks.<sup>25</sup> The model can match the expected return to housing because such an asset is naturally exposed to long-term risks. However, the model also implies that the longer the maturity of a leasehold, the more it loads on long-run risk, the higher the discount rate applied to its valuation. This upward sloping term structure of risk premia contributes to generate low discounts for leaseholds compared to freeholds.

In the external habit model of [Campbell and Cochrane \(1999\)](#) agents care about the surplus consumption relative to a habit level, which itself depends on the history of aggregate consumption.<sup>26</sup> Negative shocks to consumption, with which rents are correlated, induce increases in risk premia because they bring current consumption closer to the habit level. Long-term claims, due to their high duration, are particularly exposed to these shocks and are, consequently, particularly risky. The model, therefore, implies an upward sloping term structure of risk premia that, while it can rationalize the average return to housing, contributes to generate low discounts for leaseholds compared to freeholds.

In the variable disasters model of [Barro \(2006\)](#), [Martin \(2013\)](#) and [Gabaix \(2012\)](#) consumption growth is subject to rare but large negative shocks, the disasters.<sup>27</sup> Agents dislike assets that are exposed to these disasters. While the presence of rare disasters increases the risk premium, it does so uniformly across maturities because claims to future cash flows at all horizons are equally exposed to the underlying disaster risk. Therefore, discount rates will be the same at all horizons and equal to the average return (6.5%). Given that cash flows far into the future are discounted at this relatively high rate, the rare disaster model is not able to match the observed discounts between leaseholds and freeholds.

All three models are subject to a tension between rents that are sufficiently risky to generate a high average expected return to housing and the fact that, as rents become riskier, long-term cash

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asset in that paper implies an expected return of 6.5% a year and, consequently, we maintain the same calibration here. We only modify the average growth rate of cash flows (rents) to match the observed 0.2% annual growth rate of rents as in our baseline calibration.

<sup>25</sup>[Dew-Becker and Giglio \(2013\)](#) shows that half the total price of risk in the long-run risk model comes from fluctuations in consumption with cycles longer than 230 years and three quarters of the risk prices come from fluctuations longer than 75 years. These horizons correspond closely to the maturities of the leaseholds we consider in this paper.

<sup>26</sup>We calibrate the model following the parametrization of [Campbell and Cochrane \(1999\)](#). We impose an average growth rate of rents of 0.2% per year, and a correlation of rent growth and consumption growth of 0.27 to ensure that expected returns on housing are 6.5%.

<sup>27</sup>We calibrate the model in [Gabaix \(2012\)](#) to match the expected return on housing from our baseline calibration (6.5%). This requires modeling housing as a slightly safer claim than equity with respect to disasters with an average resilience of 0.1 instead of 0.09. We also modify the growth rate of dividends (rents) to match the 0.2% annual growth rate of rents, as in our baseline calibration.

flows are discounted at progressively higher rates thus generating smaller discounts for leaseholds with respect to freeholds. Therefore, extending the Gordon-Growth model by taking into account that rents are risky and housing commands a nontrivial risk premium exacerbates the long-run valuation puzzle. While it is beyond the scope of this paper to suggest which modifications could allow these models to better match the data, we illustrate in the next Section which characteristics a model would have to have in order to rationalize the long-run valuation puzzle.

### 3.4 Reduced Form Models of Discount Factors

We finally analyze the features required of a model to rationalize the long-run valuation puzzle. We find our estimates to be consistent with a downward sloping term structure of discount rates. Discount rates have to be sufficiently high in the short to medium run to contribute to high average expected returns on housing, but also sufficiently low in the long run to match the observed discounts applied to long-run cash flows. Two existing classes of models can potentially generate this feature. Models of hyperbolic discounting, along the lines of [Laibson \(1997\)](#) and [Luttmer and Mariotti \(2003\)](#), generate a downward sloping term structure via a declining rate of time preference. The reduced form model of [Lettau and Wachter \(2007\)](#) generates a downward sloping term structure of discount rates for risky assets because claims to long-run cash flows have lower exposure to the unexpected innovation to rents (or dividends), which is the priced shock in the model.<sup>28</sup>

[Lettau and Wachter \(2007\)](#) propose a reduced-form model in which the only priced shock is the unexpected innovation in dividends (rents) and unexpected dividend growth today is negatively correlated with future dividend growth. Therefore, long-term claims to future dividends (rents) are safer than short-term claims because short-term claims do not benefit as much as long-term claims from the future increases in dividend (rent) growth that occur after a negative dividend shock.

Figures 17 and 18 show that the model is able to match the magnitudes of the discounts at different horizons. The intuition is that long-term claims in the model are relatively safe and therefore cash flows arising many years into the future are discounted at low rates of around 2.4% a year. Combined with our baseline calibration of rent growth at 0.2%, the model is therefore able to generate discounts for leaseholds to freeholds as large as those in the data. At the same time, the model is able to match the 6.5% expected return to housing because it implies high short term discount rates, as high as 20%

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<sup>28</sup>The model is then extended in [Lettau and Wachter \(2011\)](#) to also account for the term structure of bonds.

per year for the first few years.

While this model is certainly reduced-form and is not a micro-founded general equilibrium model, its functional form is very informative about the characteristics that any general equilibrium model would have to have in order to match the observed discounts.

We consider next models that feature a variation in the rate of subjective discount (time preference) across horizons. We follow [Laibson \(1997\)](#) and [Luttmer and Mariotti \(2003\)](#) in considering the possibility that agents attach higher discounts to short term cash flows than they do to long term cash flows. Rather than considering the full general equilibrium environment in the original references, we focus on a simple but informative extension of the Gordon Growth model considered in Section 3.1.

We consider a mix of hyperbolic and exponential discounting by assuming that the discount function follows:  $\frac{e^{-\rho s}}{1+\kappa s}$ , where  $\rho > 0$  is the subjective discount rate associated with exponential discounting and  $\kappa > 0$  is the subjective hyperbolic parameter. Intuitively, if  $\kappa = 0$  we recover simple exponential discounting at  $e^{-\rho s}$ , while if  $\rho = 0$  we recover simple hyperbolic discounting at  $\frac{1}{1+\kappa s}$ . This mixed form of discounting tends to behave like hyperbolic discounting in the short run and like exponential discounting in the long run. The relative importance of short-run and long run is the subject of the calibration below.<sup>29</sup> In this case, the T-maturity leasehold is valued at:

$$P_0^T = \int_0^T \frac{e^{-(\rho-g)s}}{1+\kappa s} D_0 ds = D_0 \frac{e^{\frac{\rho-g}{\kappa}} \left( Ei \left( \frac{(T\kappa+1)(g-\rho)}{\kappa} \right) - Ei \left( \frac{g-\rho}{\kappa} \right) \right)}{\kappa},$$

where  $Ei(x)$  is the Exponential Integral discussed in the appendix. The freehold is correspondingly valued at:

$$P_0 = D_0 \frac{e^{\frac{\rho-g}{\kappa}} \Gamma \left( 0, \frac{\rho-g}{\kappa} \right)}{\kappa},$$

where  $\Gamma(x)$  is the Upper Incomplete Gamma function also discussed in the appendix. The discount

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<sup>29</sup>While we adopt here the continuous time formulation of [Luttmer and Mariotti \(2003\)](#), this is also in the spirit of [Laibson \(1997\)](#) where agents aggressively discount the immediate future (the hyperbolic discounting occurs in the short run), but between any two periods in the future they have a constant discount rate (in the long run, the exponential discounting prevails).

between a T-maturity leaseshold and the freehold is now:

$$Disc_0^T = \frac{Ei\left(\frac{(T\kappa+1)(g-\rho)}{\kappa}\right) - Ei\left(\frac{g-\rho}{\kappa}\right)}{\Gamma\left(0, \frac{\rho-g}{\kappa}\right)} - 1.$$

Therefore, this formulation of the discounting problem offers analytical, if not closed form, solutions.<sup>30</sup>

In Figures 20 and 19 we report a calibration of the hyperbolic-exponential model that at the same time (i) matches the observed discounts of leaseholds of different maturities as close as possible and (ii) matches the average return of housing (6.5%). The calibration is obtained setting  $\kappa$  to 10% (the hyperbolic component's discount rate) and  $\rho$  to 1% (the exponential component's discount rate).

Overall, we find that both the [Lettau and Wachter \(2007\)](#) model and the hyperbolic-exponential model imply that a downward sloping term structure of discount rates is necessary in order to jointly rationalize the expected returns to housing and the long-term discount. These positive results, however, have to be interpreted conservatively. We view both models as convenient functional forms to highlight the patterns in the data rather than fully specified general equilibrium models nor do we judge their ability to fit other stylized facts of asset pricing.<sup>31</sup> It remains an open and interesting question for future theoretical work to explore models that could reconcile the long-run valuation puzzle as well as match other stylized facts of asset pricing.<sup>32</sup>.

## 4 Implications of the Findings

### 4.1 Asset Pricing

Our estimates of very long-run discount rates provide a novel testing ground for theoretical asset pricing models, and are complementary to those in [Binsbergen, Brandt and Kojien \(2012\)](#). We provide evidence for very long-run discount rates (80-250 years) for residential housing while they focus on the short-run (1-3 years) discount rates in equity markets. Despite the different asset classes, data,

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<sup>30</sup>Both the exponential-integral and the incomplete-gamma functions are heavily studied functions of complex analysis that can be easily evaluated to arbitrary numerical precision in standard software like Mathematica and Matlab.

<sup>31</sup>For example, a common criticism of the hyperbolic discounting approach in asset pricing is that it only affects risk-free rates but not risk premia ([Luttmer and Mariotti, 2003](#)) and thus implies a downward sloping yield curve for safe bonds.

<sup>32</sup>Intuitive possibilities include models of agents heterogeneity along the lines of [Weitzman \(2001\)](#) or habitat and liquidity theories along the lines of [Amihud and Mendelson \(1991\)](#)

and methodologies we find similar qualitative patterns: short-run discount rates are higher than long-run discount rates.

[Binsbergen, Brandt and Kojen \(2012\)](#) show that leading general equilibrium asset pricing models such as the habit model of [Campbell and Cochrane \(1999\)](#), the long-run risk model of [Bansal and Yaron \(2004\)](#), and the rare disaster model of [Barro \(2006\)](#) and [Gabaix \(2012\)](#) cannot match this pattern.<sup>33</sup> While these models were not specifically set-up to match the horizon-variation in discount rates, the evidence provides a new stringent testing ground for future theoretical advances.

It is important to emphasize that we cannot disentangle the risk-free component from the risk-adjustment component in our estimates of total discount rates. To fix ideas, let us write the total discount rate for a cash flow occurring at time  $T$  as the sum of a risk-free component and a risk adjustment:  $R^T = R_f^T + RP^T$ , where  $R_f^T$  is the risk-free component (or, in other bonds, the yield of a real zero-coupon bond with maturity  $T$ ) and  $RP^T$  is the risk adjustment. Our estimates imply that the total discounts  $R^T$  are declining over the horizon  $T$  and are as low as 2% for horizons of 100 or more years.

Since for residential housing the cash flow is the net rent, it is reasonable to assume that rents very far into the future are at least as risky as a risk-free investments; hence the risk adjustment is positive:  $RP^T > 0$ . In fact, one could argue that since rents and consumption are likely to be cointegrated in the long-run, claims to long-run rents should be as risky as claims to long-run consumption.<sup>34</sup> Intuitively, long-run rents in cities such as London or Singapore could actually be substantially more risky since they load heavily on the performance of the global economy. If this were true then our estimates would imply that agents are not afraid of these long-run risks (i.e. have low aversion to these risks).

If the risk adjustment is positive and potentially non-negligible, then the risk free component has to be very low to match our total discount rate. This implies that agents attach very high present values to payments that occur for sure 100 or more years from now.

To sum up, we conclude that our estimates imply two novel facts. First, agents have low discount rates (in the range of 1%) for risk-free payments occurring far into the future (100 years or more).

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<sup>33</sup>[Lettau and Wachter \(2007\)](#) show in a reduced form models what discount factors are necessary to match the term structure of equities and bonds in the short to medium run.

<sup>34</sup>For example, [Jeske, Krueger and Mitman \(2011\)](#) show that the share of consumption expenditures on housing over total consumption in the U.S has been remarkably constant at 14.1% over the past 40 years.

Second, given plausible estimates of the riskiness of long-run rents (say long-run consumption risk), agents have low risk aversion over risks that materialize in the very long-run.

## 4.2 Environmental Policy

*Any consideration of the costs of meeting climate objectives requires confronting one of the thorniest issues in all climate-change economics: How should we compare present and future costs and benefits? [...] A full appreciation of the economics of climate change cannot proceed without dealing with discounting. ([Nordhaus, 2013](#))*

The above quote from Nordhaus's recent book on the economics of climate change summarizes a long debate on the appropriate discount factors to use in evaluating environmental policies. The economics literature on climate change, starting with the seminal paper of [Nordhaus \(1973\)](#), has pointed out that discounting is of central relevance to the tradeoff between immediate costs, through loss of output, and uncertain benefits that occur very far into the future (see also [Lind, 1982](#); [Cline, 1992](#); [Nordhaus, 1992](#); [Arrow et al., 1996](#); [Weitzman, 1998](#); [Nordhaus, 2001](#); [Groom et al., 2005](#); [Gollier, 2006](#); [Dasgupta, 2007](#); [Nordhaus, 2007](#); [Weitzman, 2007](#); [Gollier and Weitzman, 2010](#); [Goulder and Williams, 2012](#); [Pindyck, 2013](#); [Weitzman, 2013](#)). Estimates of the appropriate discount rate range from the zero discounting of [Stern \(2007\)](#) to as high as 10% per year based on the returns to risky private investments.

The debate has tried to infer discount rates from the realized returns of traded assets such as private capital, equity, bonds, and real estate. These estimates of average returns, however, reveal only the average yearly returns on these assets. For example our estimates in Figure 5 find that the average real returns to residential housing are on the order of 6-8%. However, the crucial estimates to evaluate climate-policy are the discount rates for cash-flows very far into the future. Of course in models with constant discount rates, such as the Gordon Growth model, the average return provides all the necessary information. We have, however, shown in Section 3 that models of constant discount rates cannot be reconciled with the data.

Our contribution to this debate is to provide the first direct estimates of discount rates at horizons that are relevant for climate-change policies (e.g. 70-100 years and beyond). Our findings that long-run discount rates are low, much lower than average returns, can potentially suggest a radical reassessment of the value of climate policies.

It is important to recall from Section 4.1 that our estimates combine two different elements: risk-free discounts and risk adjustments; both are estimated to be low. As noted in Barro (2013) the risk-less and risky attitude towards long-term discounting have opposite prediction for climate policies depending on their risk profile. Our results imply that agents in the long-run have low risk-free discount rates and are relatively less concerned towards long-run risks. This implies that households are willing to invest in policies that reduce with certainty the adverse effects of climate change, even if the benefits will only arise far in the future. However, agents appear relatively unwilling to invest in policies that only reduce the risk of even potentially large environmental disasters in the far future.

Our estimates of very long-run discount rates can also inform other cost-benefit analyses regularly undertaken by governments (Feldstein, 1964; Layard and Glaister, 1994; Stiglitz, 1994; Arrow et al., 2013). Examples include the evaluation of long-term infrastructure projects (e.g. the Hoover dam, airports) and decisions between production techniques that require large upfront investments but have low costs of operation versus technologies with the opposite expenditure patterns (e.g. nuclear power versus conventional power). Similarly, Auerbach, Gokhale and Kotlikoff (1994) have pointed out that an analysis of optimal fiscal policy requires taking a stance on long-run discount rates to evaluate the present value of leaving large debts to future generations.

### 4.3 Real Estate

Our results are also of direct relevance for real estate economics and the ongoing effort to understand house prices. We add to the recent research effort to understand the return properties of real estate (Flavin and Yamashita, 2002; Piazzesi, Schneider and Tuzel, 2007; Favilukis, Ludvigson and Van Nieuwerburgh, 2010) by focusing on a previously unexplored aspect of real estate: the term structure of house prices. Our findings that the term structure for housing discount rates is downward sloping and has strong time series variation poses yet unexplored questions for modeling house prices, construction and investment.

### 4.4 Rational Bubbles

Our estimates of long-run discount rates can also be used to *directly* test for the presence of infinitely-lived rational bubbles. The existence of bubbles is one of the most fundamental, oldest, and most difficult questions in economics. In their recent survey of the literature on bubbles, Brunnermeier

and Oehmke (2013) emphasize that “identifying bubbles in the data is a challenging task. The reason is that in order to identify a bubble, one needs to know an asset’s fundamental value, which is usually difficult to measure.” We show below that this is not the case for our tests: our tests are *model independent*.

The classic infinitely-lived rational bubble models of Blanchard and Watson (1982) and Froot and Obstfeld (1991) feature a failure of the no-bubble condition, which is routinely imposed in most economic models. The no-bubble condition requires that the present value of a payment occurring in the limit as the horizon goes to infinity is zero:

$$\lim_{T \rightarrow \infty} E_t[\xi_{t,T} P_T] = 0,$$

where  $\xi_{t,T}$  is a model-implied discount factor between date  $t$  and  $T$  and  $P_T$  the price of the asset at time  $T$ . Our data is uniquely suited to test this condition because we can estimate the present value of a claim to rents occurring at very long horizons, for example  $T = 999$  years. More formally:

$$P_t - P_t^T \approx \lim_{T \rightarrow \infty} E_t[\xi_{t,T} P_T], \quad \text{for large } T.$$

Intuitively, the difference in value between a freehold ( $P_t$ ) and a 999-maturity leasehold ( $P_t^{999}$ ) is the present value of the claim to rents starting 999 years from today and extending to the infinite future (i.e. the present value of a freehold 999 years from now,  $E_t[\xi_{t,999} P_{999}]$ ). Therefore we can test whether the no-bubble condition holds, on average, by testing whether the discount of very long leases to freeholds is zero. We correspondingly formulate our null hypothesis of no-bubbles as:  $Disc^T = 0$  for  $T > 800$  years.

The estimates of extreme long-run discounts for Singapore and the UK are reported in Figures 5 and 2. In all cases the point estimates of the discounts are negligible and not statistically significant for  $T$  sufficiently large, 800 or more years. We conclude that there is no evidence in our data supporting the presence of infinitely-lived rational bubbles.<sup>35</sup> As an even more stringent test, Figure 21 shows that there is no evidence of a bubble at any point in time between 1995 and 2013 in Singapore. It does so by showing that the price of freeholds  $P_t$  and those of 999-year leasehold  $P_t^{999}$  are essentially

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<sup>35</sup>While the literature has already put forward theoretical arguments for the fragility of the existence of infinitely-lived rational bubbles (for example Tirole (1985)), our tests provide direct empirical evidence for the absence of such bubbles.

identical, and certainly within the 1% confidence interval of each other, at all points in time. This test is of particular interest because it shows the absence of an infinitely-lived rational bubble even at the peak of the housing market in 2013 after years of strong house price growth, when many commentators were hinting at the presence of a large bubble.

The strength of directly testing the no-bubble condition is that all models that assume the absence of infinitely-lived rational bubbles have the same implication: that the fundamental present value of a payment occurring in infinite time ( $\lim_{T \rightarrow \infty} E_t[\xi_{t,T} P_T]$ ) is equal to zero. We do not need to specify a model (a choice of  $\xi_{t,T}$  and of a stochastic process for rents) in order to obtain a fundamental value to compare to the valuation in the data. All no-bubble models imply that such fundamental value is zero. Our direct testing methodology is made possible by the uniqueness of our data that allows us to identify the terminal no-bubble condition. Such tests have been elusive because we do not normally observe traded claims to payments that only occur extremely far into the future. Our direct tests contrasts sharply with a large previous literature (for example: [Flood and Garber \(1980\)](#); [Evans \(1991\)](#); [Diba and Grossman \(1988b,a\)](#); [West \(1987\)](#)) that had to either deal with the thorny problem of establishing fundamental values or find indirect ways to test for bubbles.<sup>36</sup>

We note, however, that our bubble tests should not be interpreted as providing evidence for the absence of *all types* of bubbles. We provide evidence against a specific, in the theoretical literature very common, type of bubble: the infinitely-lived rational bubble. Our tests are uninformative with respect to the presence of finitely-lived bubbles of the kind described for example in [Abreu and Brunnermeier \(2003\)](#) and [DeMarzo, Kaniel and Kremer \(2008\)](#).

## 5 Conclusions

We provide novel estimates of very long-run discount rates by exploring unique features of the U.K. and Singapore housing markets where properties trade as either freeholds (infinite maturity ownership) or leaseholds of various maturities. We find that low long-run discount rates, much lower than routinely assumed by economic theory, are necessary in order to explain both the relatively high expected return to housing and the observed discounts between long-run leaseholds and freeholds.

Our results provide new insight on the term structure of house prices, a new testing ground for

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<sup>36</sup>See [Flood and Hodrick \(1990\)](#) for a review of the deep econometric problems that had a chilling effect on the empirical literature attempting to test for the presence of bubbles.

theoretical asset pricing models, and a direct estimate of the long-run discount rates that are crucial to evaluate environmental policies and other immediate actions that only have payoffs very far into the future.

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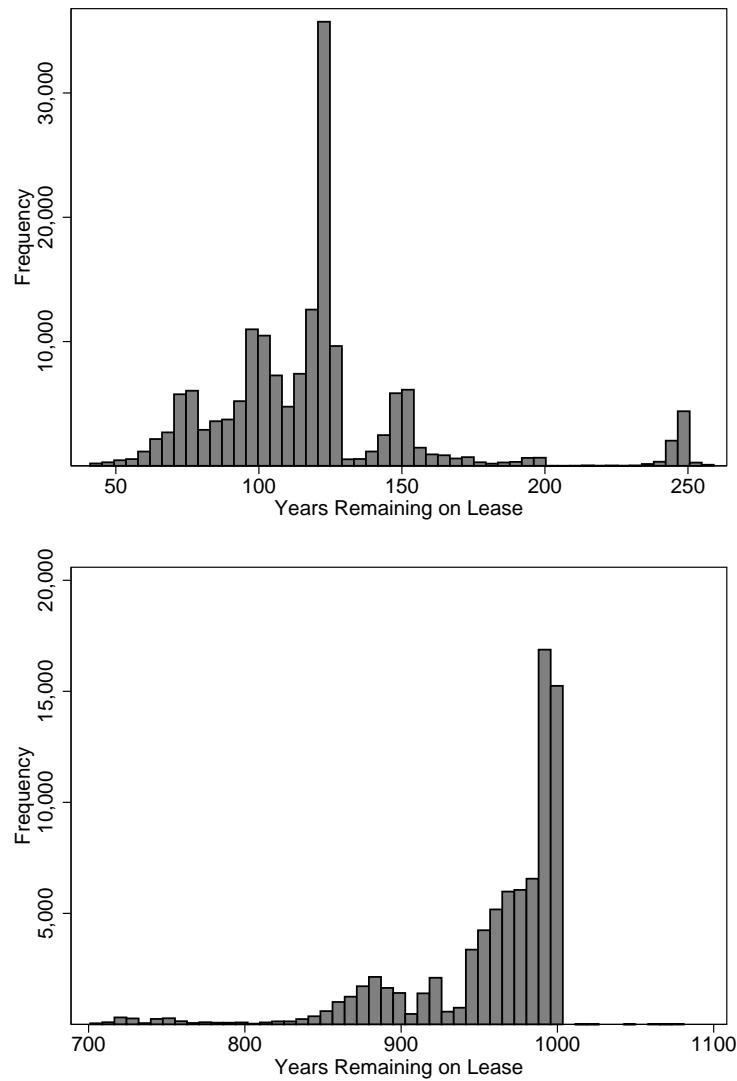
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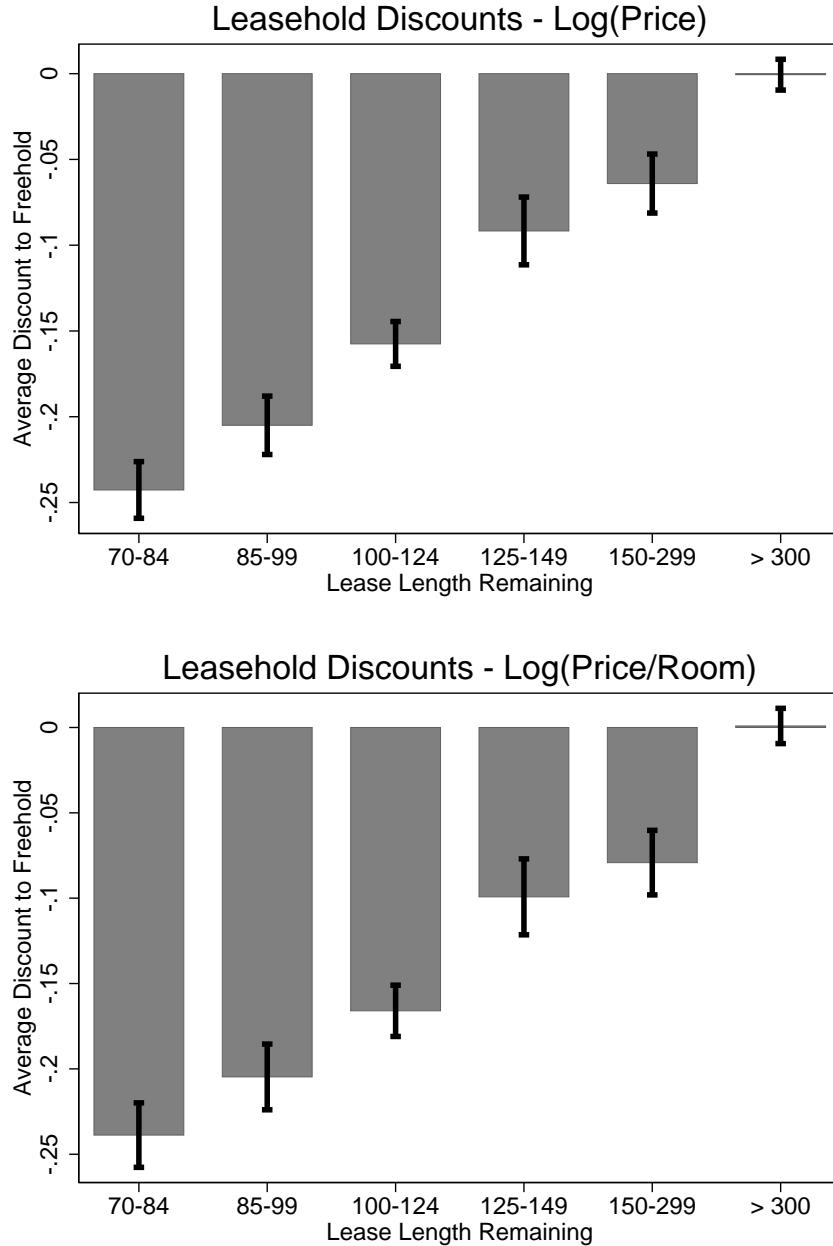
## Figures

**Figure 1:** Distribution of Remaining Lease Lengths at Sale (U.K.)



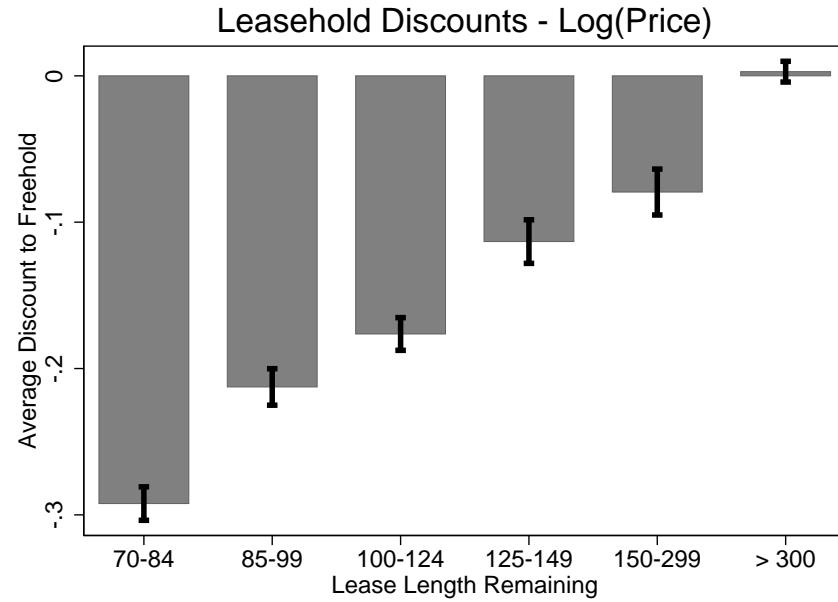
**Note:** This figure shows the distribution of years remaining on the lease for the leasehold transactions in our U.K. transaction sample.

**Figure 2:** Price Discount by Remaining Lease Length (U.K.) – Houses with hedonics



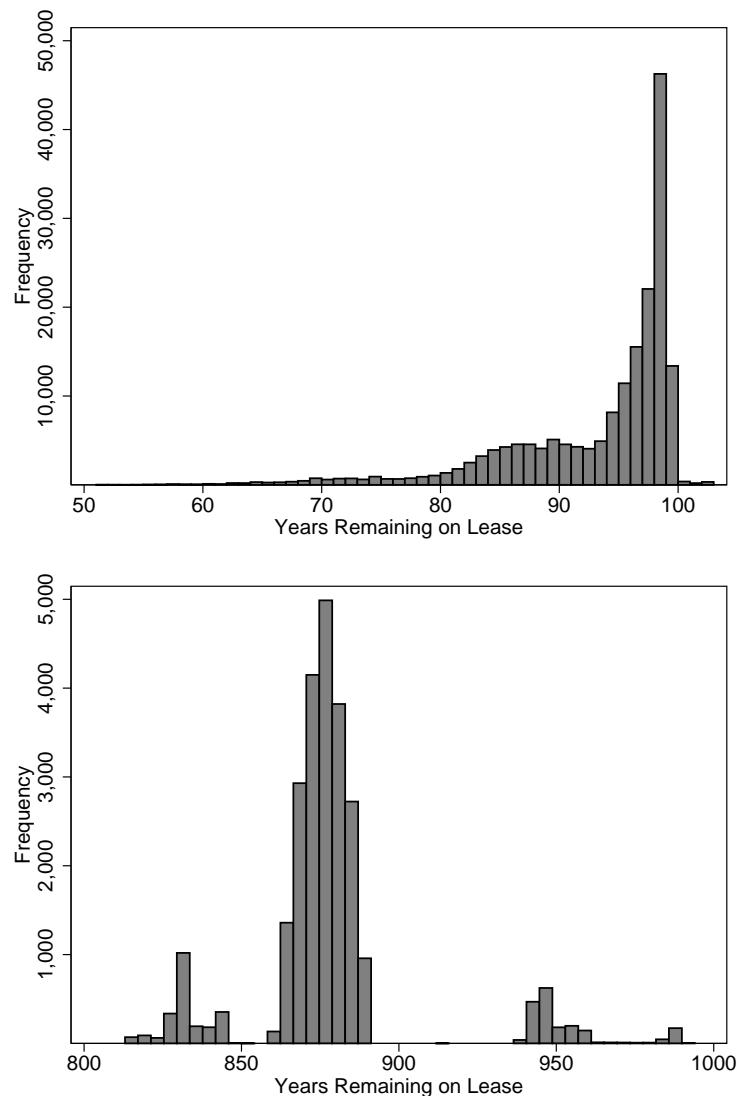
**Note:** This figure shows  $\beta_i$  coefficients from regression (1). To convert into percentage discounts for leasehold properties of a certain maturity, construct  $e^\beta - 1$ . In the top panel the dependent variable is the log price paid for properties in the U.K. between 2009 and 2013, corresponding to column (3) in Table 2, in the bottom panel it is the log price per room, corresponding to column (6) in Table 2. We only include properties which we could match to property listings with information on the number of bedrooms and bathrooms. We include postal district by property type by transaction month by number of bedrooms fixed effects. We also control for the number of bathrooms and the total number of rooms, as well as whether the property is a new construction. The bars indicate the 95% confidence interval of the estimate using standard errors clustered at the level of the fixed effects.

**Figure 3:** Price Discount by Remaining Lease Length (U.K.) – All houses



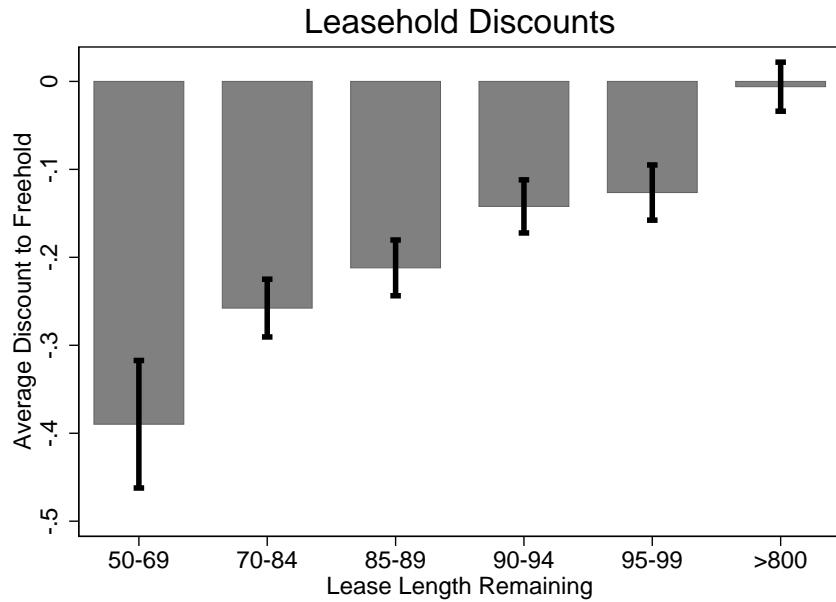
**Note:** This figure shows  $\beta_i$  coefficients from regression (1). To convert into percentage discounts for leasehold properties of a certain maturity, construct  $e^\beta - 1$ . The dependent variable is the log price paid for properties in the U.K. between 2009 and 2013. All properties for which we could not observe the number of bedrooms or bathrooms are assigned the same indicator variable for house characteristics. Relative to Figure 2 this allows us to also use transactions which we could not match to “for sale” listings. We include postal district by property type by transaction month by number of bedrooms fixed effects. We also control for the number of bathrooms and the total number of rooms, as well as whether the property is a new construction. The bars indicate the 95% confidence interval of the estimate using standard errors clustered at the level of the fixed effects.

**Figure 4:** Distribution of Remaining Lease Lengths at Sale (Singapore)



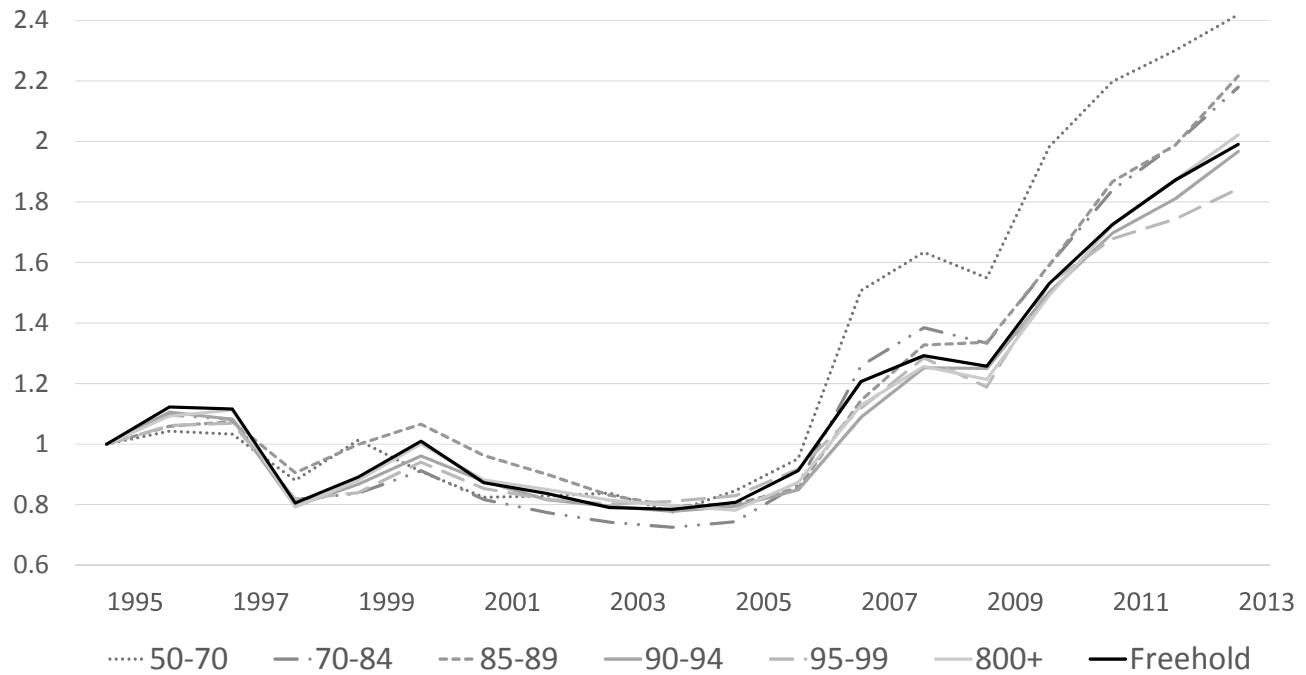
**Note:** This figure shows the distribution of years remaining on the lease for the leasehold transactions in our Singapore transaction sample.

**Figure 5:** Price Discount by Remaining Lease Length (Singapore)



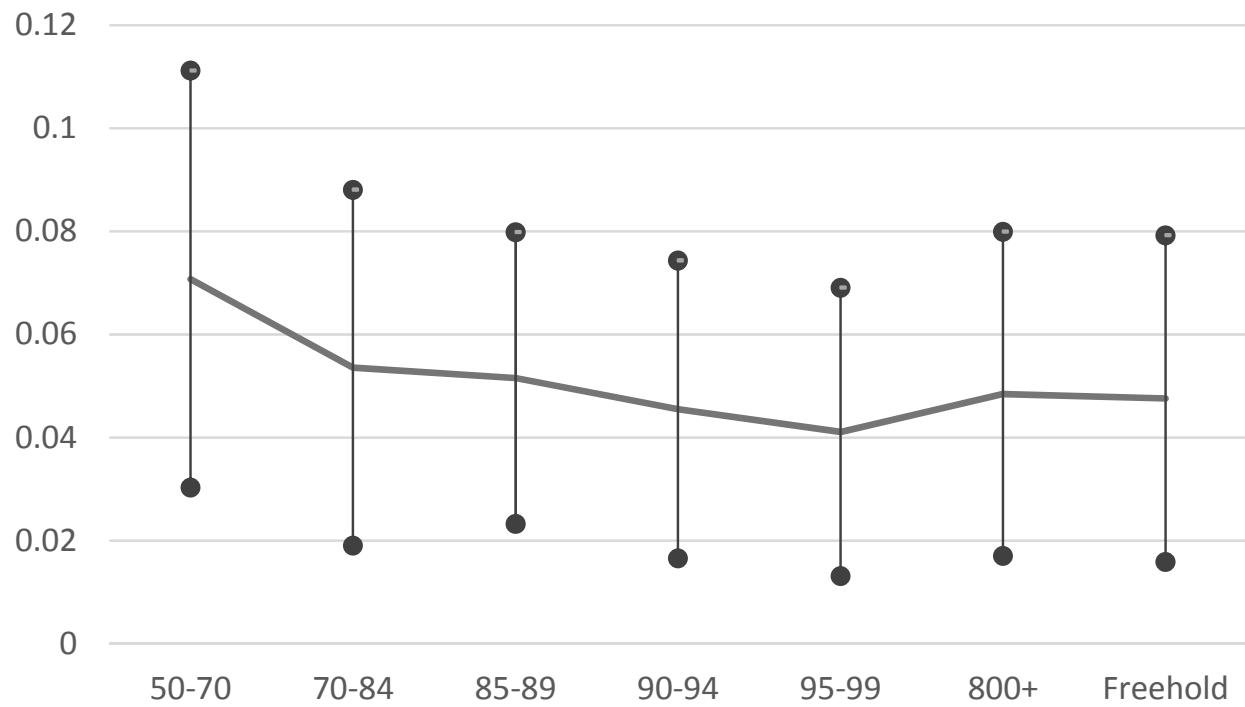
**Note:** This figure shows  $\beta_i$  coefficients from regression (2). To convert into percentage discounts for leasehold properties of a certain maturity, construct  $e^\beta - 1$ . The dependent variable is the log price per square foot paid for properties sold by private parties in Singapore between 1995 and 2013. We include fixed effect at the 5-digit postcode by property type (apartment, condominium, detached house, executive condominium, semi-detached house and terrace house) by title type (Strata or Land) by transaction month. We control for the age of the property (by including a dummy variable for every possible age in years), the size of the property (by including a dummy for each of 40 equally sized groups capturing property size) and the total number of units in the property. The bars indicate the 95% confidence interval of the estimate using standard errors clustered at the level of the fixed effect.

**Figure 6:** Price Index by Lease Type - Singapore



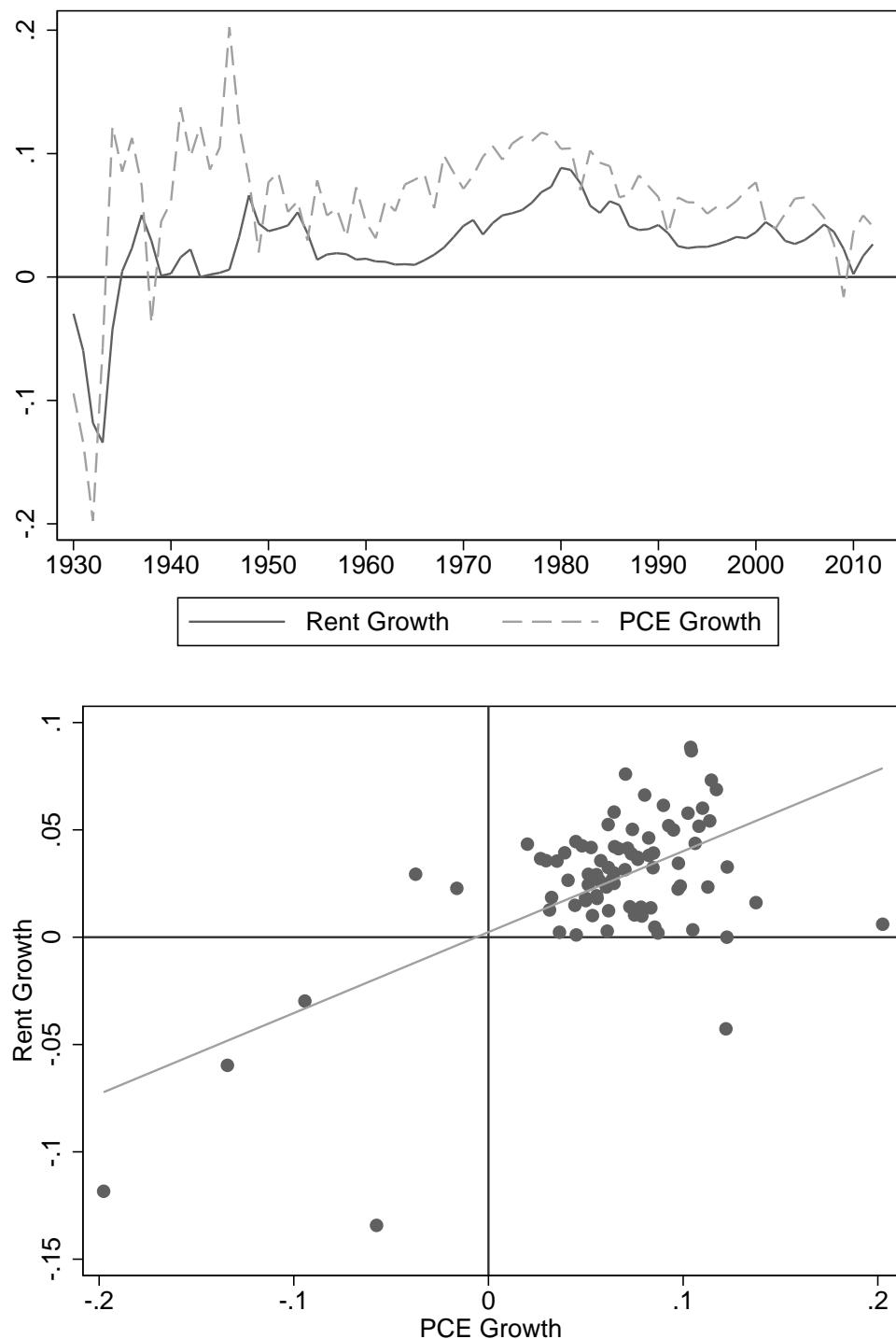
**Note:** This figure shows hedonic price indices for different lease length properties in Singapore, using the regression estimates from regression (3). We include 4-digit postcode by property type by title type fixed effects. We also control for the age of the property (by including a dummy variable for every possible age in years), the size of the property (by including a dummy for each of 40 equally sized groups capturing property size) and the total number of units in the property.

**Figure 7: Average Annualized Capital Gain - Singapore**



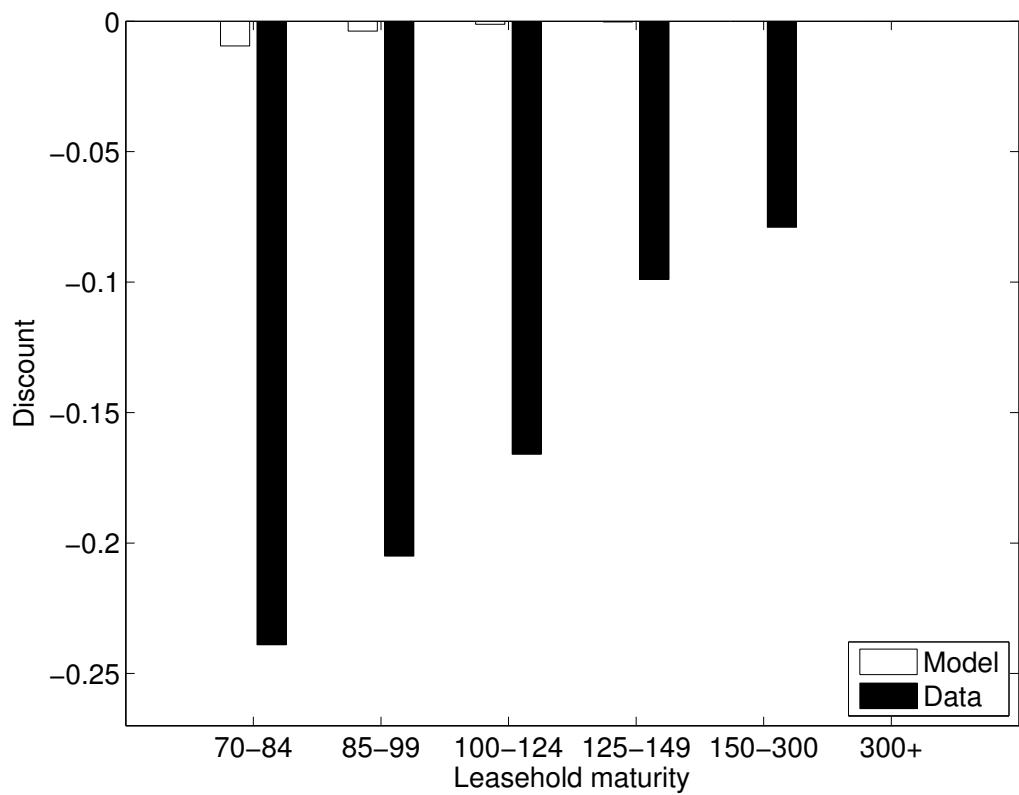
**Note:** This figure shows the average annualized capital gains for leaseholds of different maturities, using estimates from regression 3. We plot point estimates and 1 standard deviation error bounds.

**Figure 8:** Rent Growth vs. PCE Growth in U.S.



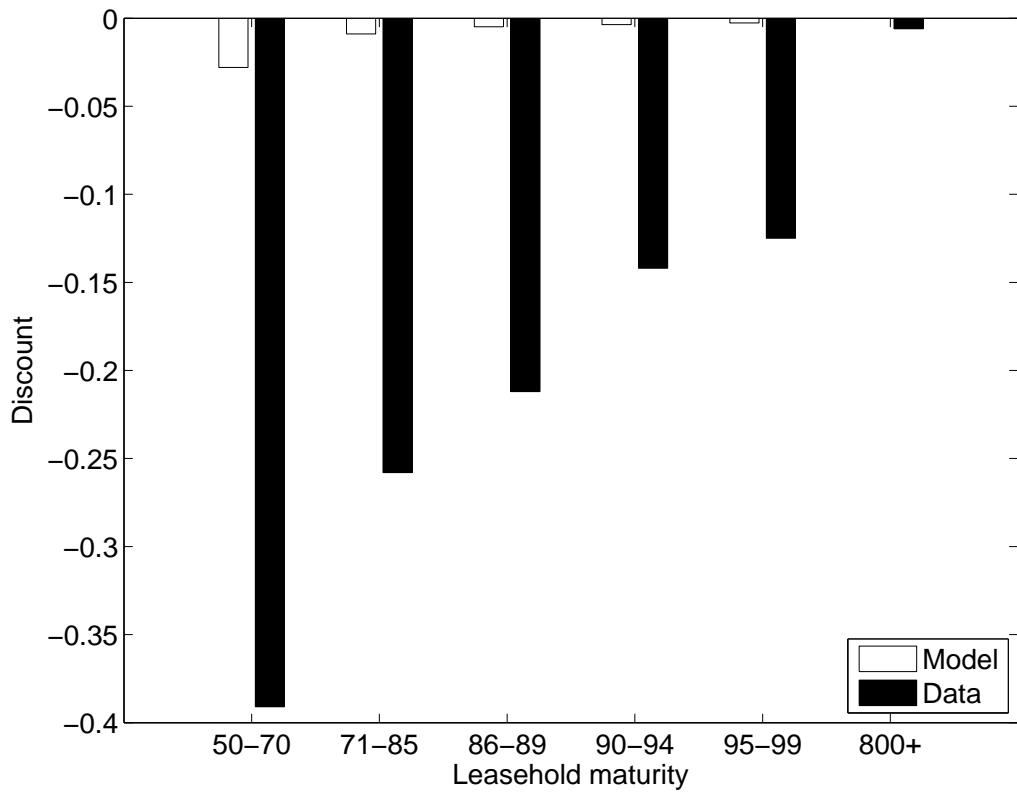
**Note:** The figure shows the annual growth rates of the “Consumer Price Index for All Urban Consumers: Rent of primary residence” (FRED ID: CUUR0000SEHA) and “Personal Consumption Expenditure” (FRED ID: PCDGA) since 1929.

**Figure 9:** Gordon-Growth Discounts vs. Data - U.K.



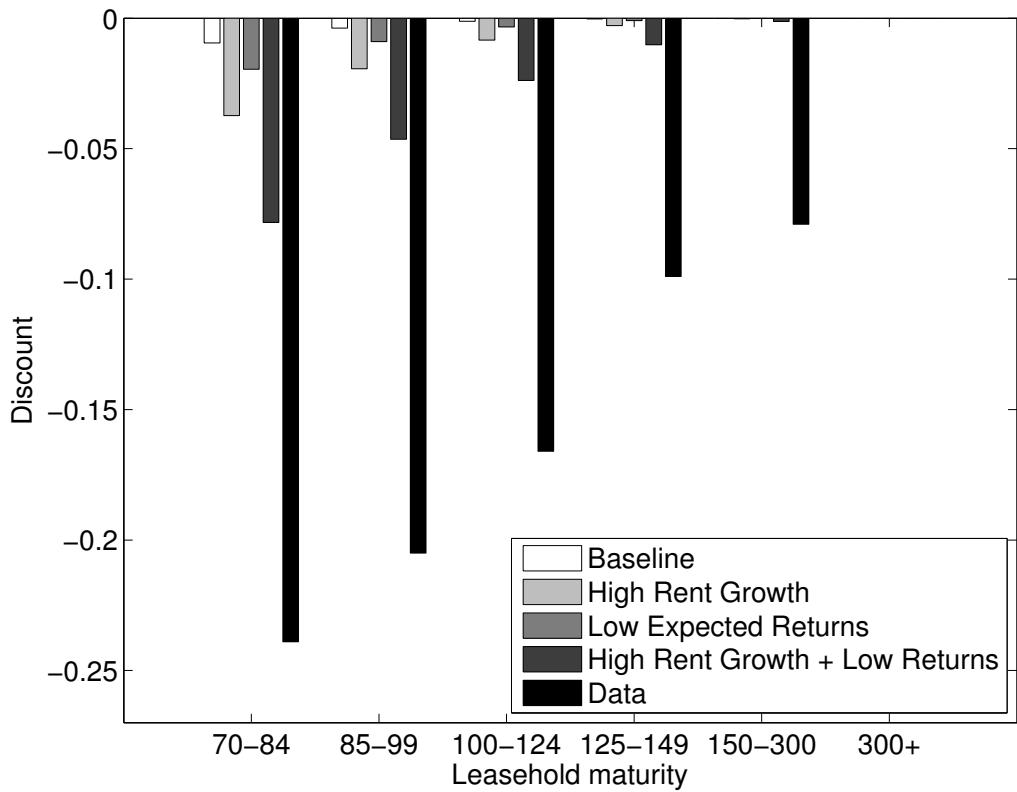
**Note:** The figure shows the discounts for leaseholds observed in the U.K. and the discounts predicted by a standard parameterization Gordon-Growth model with  $r = 6.5\%$  and  $g = 0.2\%$ .

**Figure 10:** Gordon-Growth Discounts vs. Data - Singapore



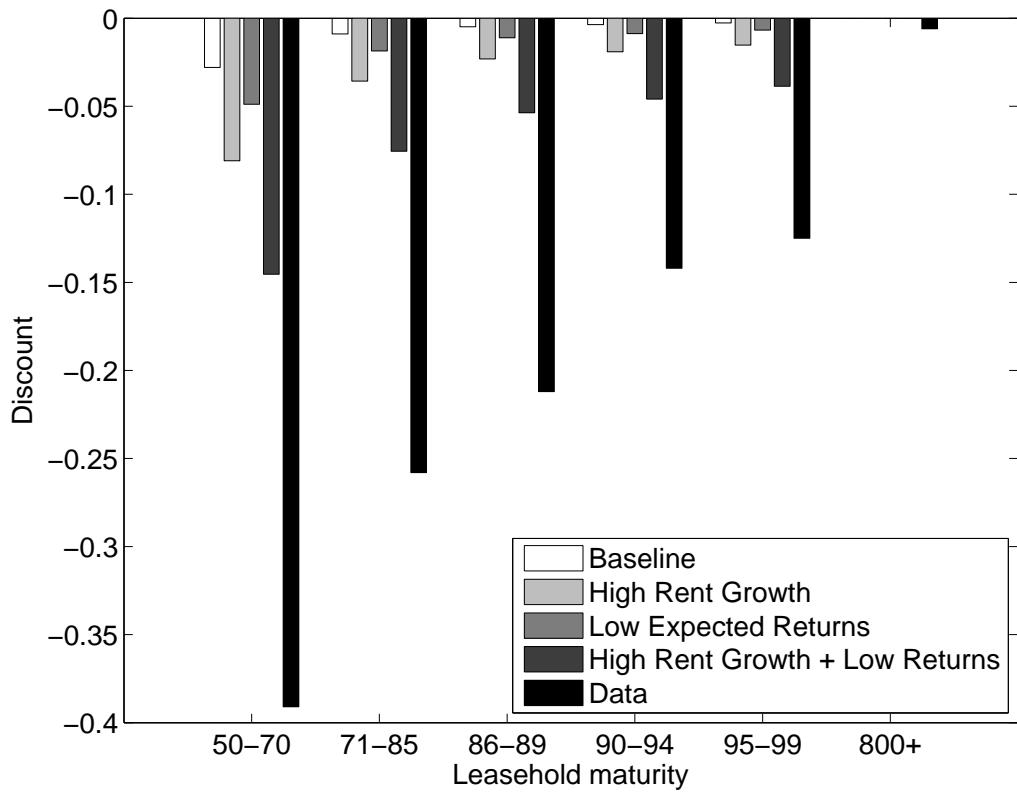
**Note:** The figure shows the discounts for leaseholds observed in Singapore and the discounts predicted by a standard parameterization Gordon-Growth model with  $r = 6.5\%$  and  $g = 0.2\%$ .

**Figure 11:** Gordon-Growth Discounts vs. Data - UK



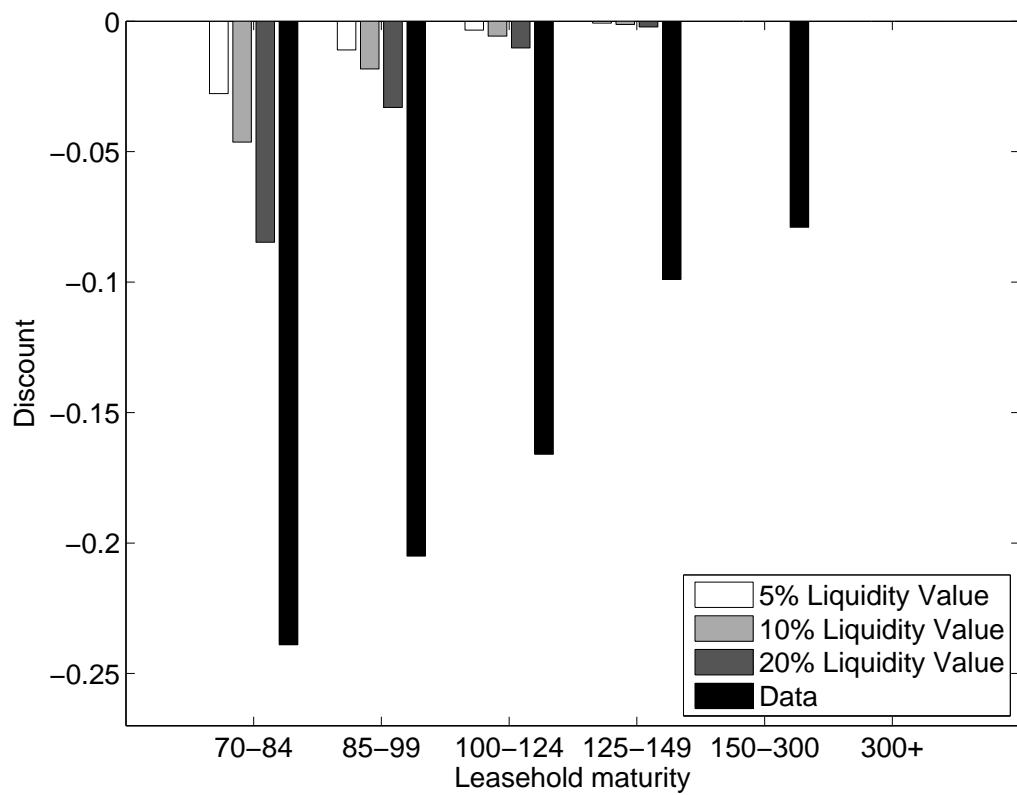
**Note:** The figure shows the discounts for leaseholds observed in the UK and the discounts predicted by a number of parameterizations of the Gordon-Growth model. The baseline calibration has  $r = 6.5\%$  and  $g = 0.2\%$ . A “low expected return” calibration takes  $r = 5.5\%$ , while a “high rent growth” calibration takes  $g = 2\%$ .

**Figure 12:** Gordon-Growth Discounts vs. Data - Singapore



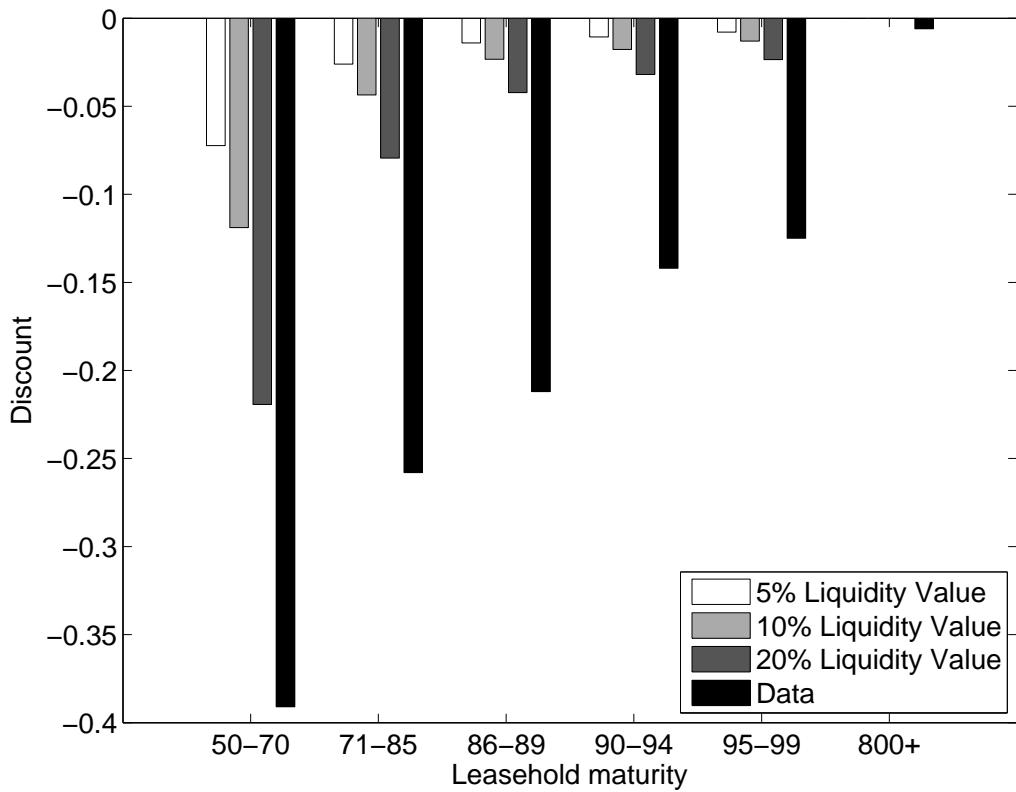
**Note:** The figure shows the discounts for leaseholds observed in Singapore and the discounts predicted by a number of parameterizations of the Gordon-Growth model. The baseline calibration has  $r = 6.5\%$  and  $g = 0.2\%$ . A “low expected return” calibration takes  $r = 5.5\%$ , while a “high rent growth” calibration takes  $g = 2\%$ .

**Figure 13:** Liquidity Discounts vs. Data - U.K.



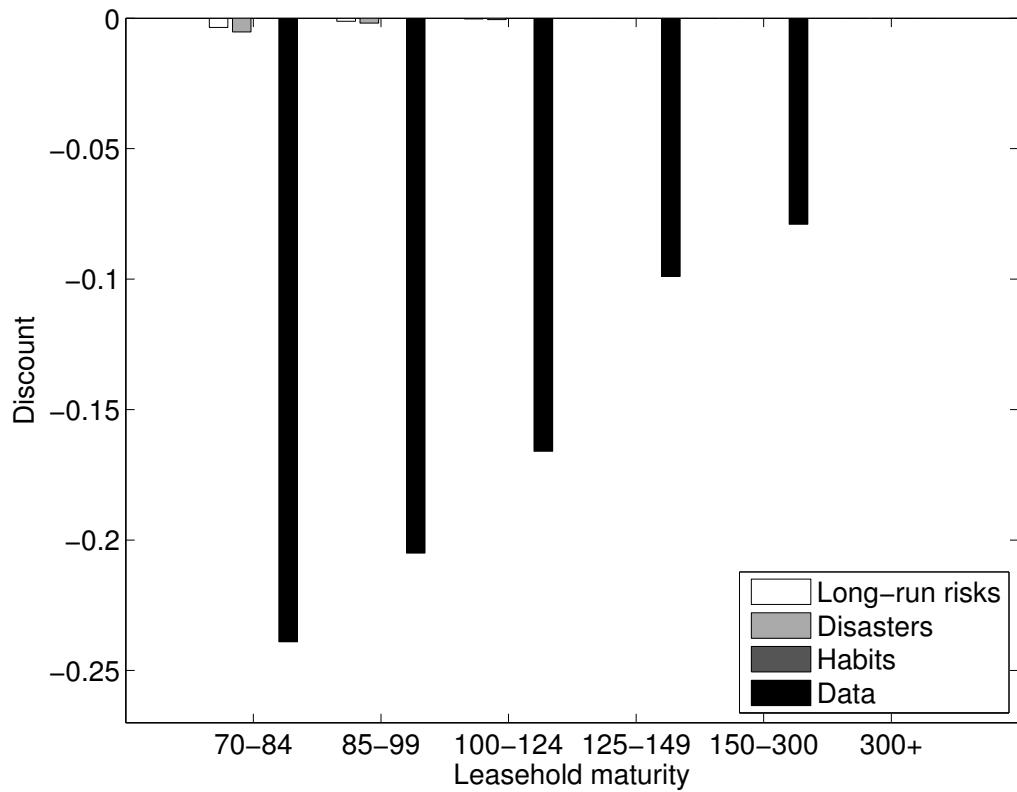
**Note:** The figure shows the discounts for leaseholds observed in the U.K. and the discounts predicted by a parameterizations of the liquidity discounting model in section 3.2.1 using  $r = 6.5\%$  and  $g = 0.2\%$ .

**Figure 14:** Liquidity Discounts vs. Data - Singapore



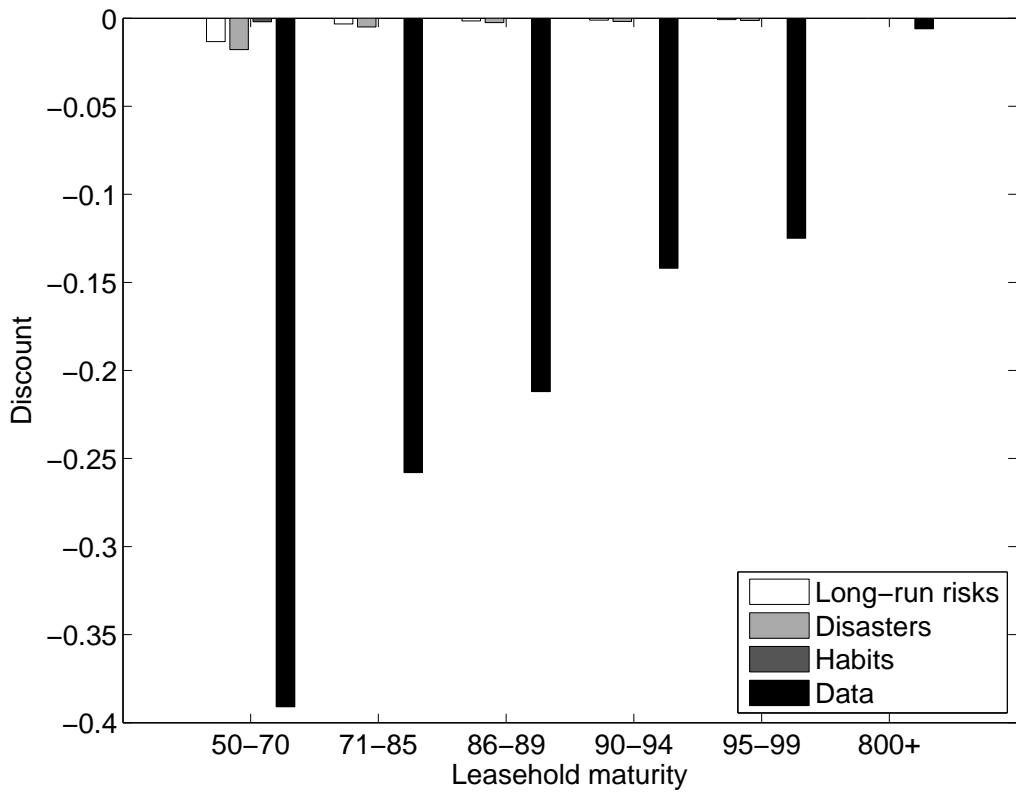
**Note:** The figure shows the discounts for leaseholds observed in Singapore and the discounts predicted by a parameterizations of the liquidity discounting model in section 3.2.1 using  $r = 6.5\%$  and  $g = 0.2\%$ .

**Figure 15:** Asset Pricing Model Discounts vs. Data - U.K.



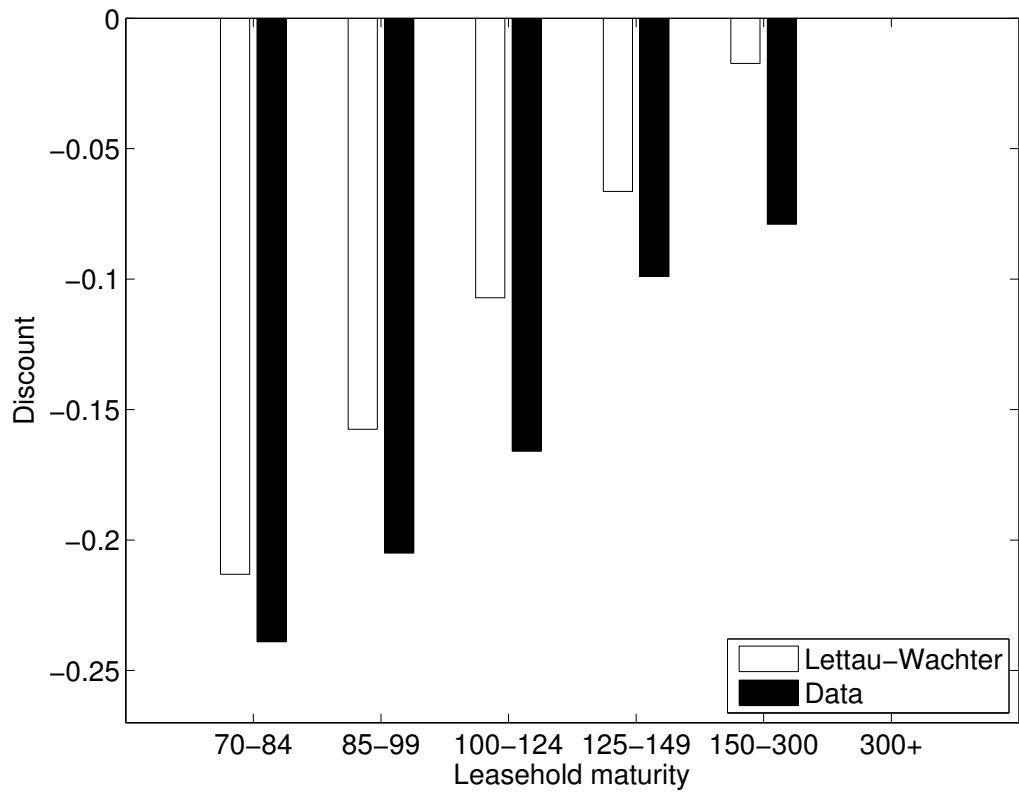
**Note:** The figure shows the discounts for leaseholds observed in the U.K. and the discounts predicted by the long-run risk model, the variable rare-disaster model, and the habit-formation model. The calibrations impose that housing has expected return of 6.5% and growth rate of 0.2%.

**Figure 16:** Asset Pricing Model Discounts vs. Data - Singapore



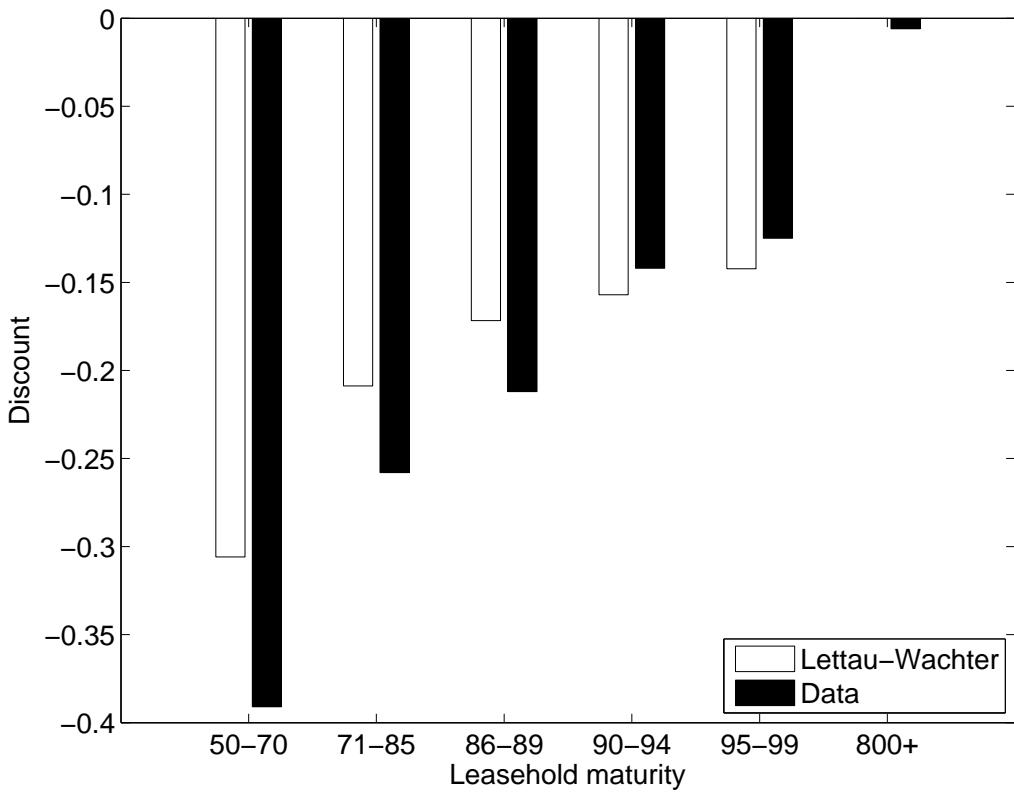
**Note:** The figure shows the discounts for leaseholds observed in Singapore and the discounts predicted by the long-run risk model, the variable rare-disaster model, and the habit-formation model. The calibrations impose that housing has expected return of 6.5% and growth rate of 0.2%.

**Figure 17:** Lettau and Wachter Discounts vs. Data - U.K.



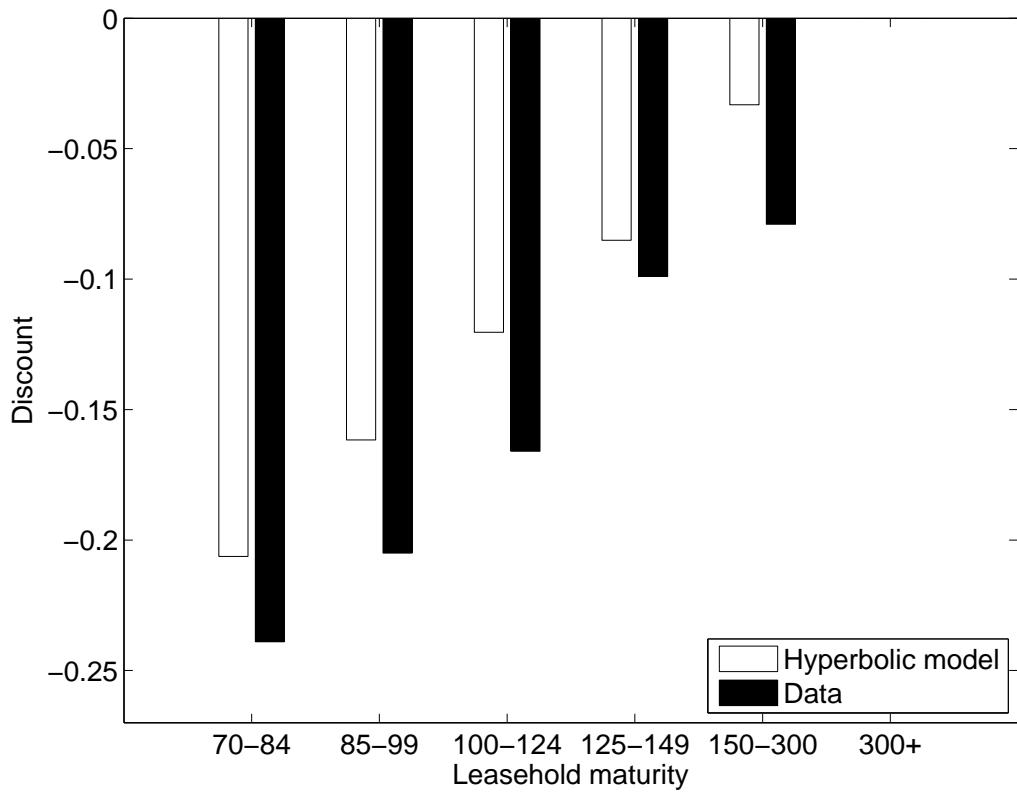
**Note:** The figure shows the discounts for leaseholds observed in the U.K. and the discounts predicted by the Lettau-Wachter model. The calibrations impose that housing has expected return of 6.5% and growth rate of rents of 0.2%.

**Figure 18:** Lettau and Wachter Discounts vs. Data - Singapore



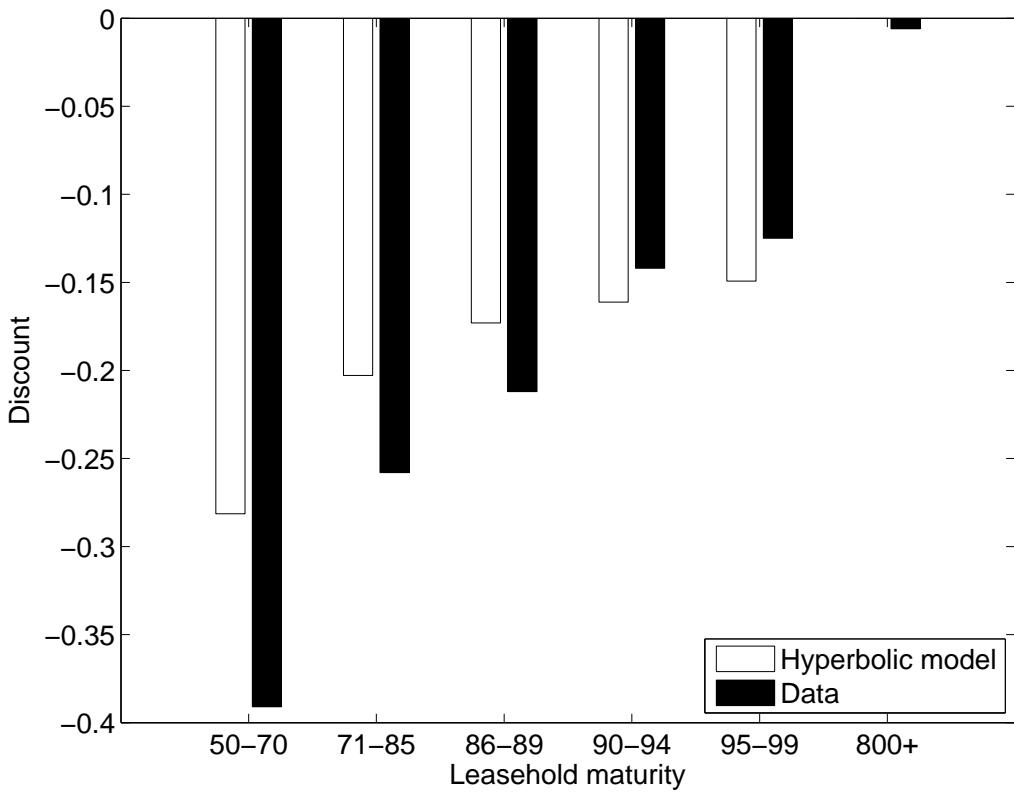
**Note:** The figure shows the discounts for leaseholds observed in Singapore and the discounts predicted by the Lettau-Wachter model. The calibrations impose that housing has expected return of 6.5% and growth rate of rents of 0.2%.

**Figure 19:** Hyperbolic Discounting Model Discounts vs. Data - U.K.



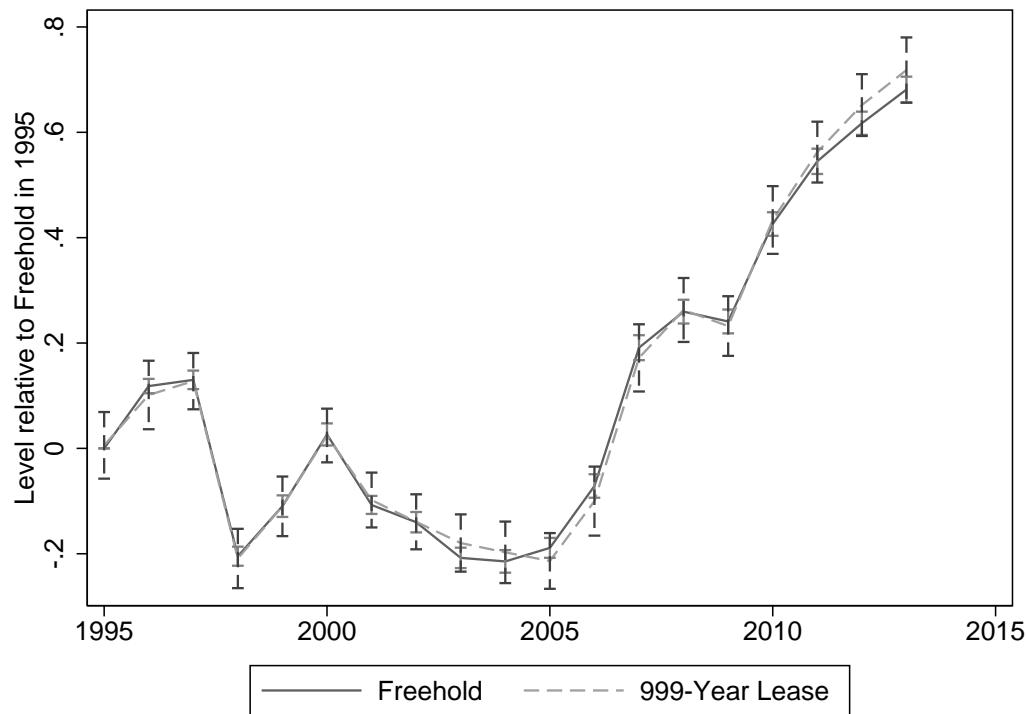
**Note:** The figure shows the discounts for leaseholds observed in U.K. and the discounts predicted by a parameterizations of the hyperbolic discounting model in section 3.4 using  $r = 6.5\%$  and  $g = 0.2\%$ ,  $\kappa = 10\%$  and  $\rho = 1\%$ .

**Figure 20:** Hyperbolic Discounting Model Discounts vs. Data - Singapore



**Note:** The figure shows the discounts for leaseholds observed in Singapore and the discounts predicted by a parameterizations of the hyperbolic discounting model in section 3.4 using  $r = 6.5\%$  and  $g = 0.2\%$ ,  $\kappa = 10\%$  and  $\rho = 1\%$ .

**Figure 21:** Time Series of 999-Year Leases and Freeholds



**Note:** The figure shows the a time series of the price level of 999-Year leaseholds and freeholds in Singapore between 1995 and 2013. Estimates are obtained from a regression of  $\log(\text{price}/\text{sqft})$  on 5-digit postcode by property type by title type fixed effects, the same control variables as Table 4 and a separate dummy for each year by lease type (Freehold, 99-Year Lease, 999-Year Lease). All price levels are relative to freeholds in 1995. The bars indicate the 95% confidence interval of the estimate using standard errors clustered at the level of the fixed effect.

## Tables

**Table 1:** Data Sample - UK

Year	N	Price ('000)	Beds	Baths	Total Rooms	Share Leaseholds
2009	192,949	212.2	2.97	1.48	6.09	11.2%
2010	200,644	233.4	2.98	1.49	6.14	11.6%
2011	189,958	229.0	2.98	1.58	6.10	11.2%
2012	185,847	238.0	2.99	1.56	6.10	11.3%
2013	85,204	237.1	2.97	1.50	6.05	10.7%

**Note:** This table shows the sample and key summary statistics by year for our U.K. transaction sample. Price is reported in thousands of Pound Sterling.

**Table 2:** Impact of Lease Type on Price - United Kingdom

	LOG(PRICE)			LOG(PRICE / ROOM)		
	(1)	(2)	(3)	(4)	(5)	(6)
<b>Lease Length Remaining</b>						
70-84 Years	-0.255*** (0.006)	-0.258*** (0.007)	-0.243*** (0.008)	-0.249*** (0.007)	-0.249*** (0.008)	-0.239*** (0.010)
85-99 Years	-0.216*** (0.008)	-0.214*** (0.008)	-0.205*** (0.009)	-0.214*** (0.008)	-0.212*** (0.008)	-0.205*** (0.010)
100-124 Years	-0.151*** (0.007)	-0.159*** (0.006)	-0.158*** (0.007)	-0.157*** (0.008)	-0.164*** (0.007)	-0.166*** (0.008)
125-149 Years	-0.093*** (0.009)	-0.098*** (0.008)	-0.092*** (0.010)	-0.101*** (0.010)	-0.106*** (0.010)	-0.099*** (0.011)
150-299 Years	-0.059*** (0.009)	-0.061*** (0.008)	-0.064*** (0.009)	-0.076*** (0.010)	-0.078*** (0.009)	-0.079*** (0.010)
300+ Years	-0.011** (0.005)	-0.009** (0.004)	-0.001 (0.005)	-0.009* (0.005)	-0.007 (0.005)	0.001 (0.005)
Fixed Effects	District × Prop Type × Quarter	5-digit PC × Prop Type × Month	5-digit PC × Prop Type × Month × Beds	District × Prop Type × Quarter	5-digit PC × Prop Type × Month	5-digit PC × Prop Type × Month × Beds
Controls	✓	✓	✓	✓	✓	✓
R-squared	0.746	0.743	0.751	0.651	0.646	0.657
N	851,483	851,483	851,483	764,524	764,524	764,524

**Note:** This table shows results from regression (1). To convert into percentage discounts for leasehold properties, construct  $e^\beta - 1$ . The dependent variables are the price (columns 1-3) and the price per room (columns 4-6) for properties sold in England and Wales between 2009 and 2013. We include postal district by property type by transaction time fixed effects. In columns (1) and (4) the transaction time is the transaction quarter, in the other columns the transaction month. In columns (3) and (6) we also interact the fixed effects with the number of beds in the property. We also control for the number of bedrooms, bathrooms and the total number of rooms, as well as whether the property is a new construction. Standard errors are clustered at the level of the fixed effect. Significance Levels: \* ( $p<0.10$ ), \*\* ( $p<0.05$ ), \*\*\* ( $p<0.01$ ).

**Table 3:** Data Sample - Singapore

Year	N	Price ('000)	Size (sqft)	Age	99-Year Lease	999-Year Lease
1995	12,412	1,149	1,758	3.42	34%	9%
1996	18,434	1,269	1,676	2.44	37%	14%
1997	12,534	1,179	1,709	2.49	53%	7%
1998	13,095	806	1,689	2.14	64%	5%
1999	23,500	991	1,827	2.92	42%	8%
2000	12,615	1,188	1,925	4.05	43%	8%
2001	11,577	883	1,732	3.35	57%	4%
2002	17,618	853	1,631	2.83	52%	6%
2003	9,807	826	1,656	4.23	50%	6%
2004	11,231	894	1,701	4.64	42%	6%
2005	16,771	1,037	1,848	5.09	37%	6%
2006	24,261	1,276	1,845	5.44	35%	6%
2007	39,203	1,625	1,719	5.14	40%	8%
2008	13,919	1,357	1,598	5.67	45%	7%
2009	32,967	1,362	1,550	4.87	43%	8%
2010	34,481	1,586	1,490	5.58	48%	6%
2011	25,236	1,475	1,341	4.54	58%	4%
2012	36,652	1,453	1,268	4.27	63%	4%
2013	15,215	1,539	1,248	3.57	69%	4%

**Note:** This table shows the sample and key summary statistics by year for our Singapore transaction sample. Price is reported in thousands of Singapore Dollars.

**Table 4:** Impact of Lease Type on Price per Square Foot - Singapore

	(1)	(2)	(3)	(4)	(5)
<b>Lease Length Remaining</b>					
50-70 Years	-0.392*** (0.024)	-0.384*** (0.028)	-0.391*** (0.037)	-0.446*** (0.041)	-0.441*** (0.042)
70-85 Years	-0.259*** (0.011)	-0.266*** (0.014)	-0.258*** (0.017)	-0.468*** (0.034)	-0.265*** (0.022)
85-89 Years	-0.196*** (0.011)	-0.213*** (0.014)	-0.212*** (0.016)	-0.100*** (0.032)	-0.219*** (0.021)
90-94 Years	-0.143*** (0.012)	-0.146*** (0.014)	-0.142*** (0.015)	-0.172*** (0.028)	-0.146*** (0.021)
95-99 Years	-0.136*** (0.013)	-0.122*** (0.014)	-0.125*** (0.016)	-0.129*** (0.028)	-0.128*** (0.020)
> 800 Years	0.002 (0.011)	-0.008 (0.013)	-0.006 (0.014)	0.023 (0.032)	-0.000 (0.019)
Fixed Effects	5-digit PC × Title Type × Quarter	5-digit PC × Prop Type × Title Type × Quarter	5-digit PC × Prop Type × Title Type × Month	5-digit PC × Prop Type × Title Type × Month	5-digit PC × Prop Type × Title Type × Month
Controls	✓	✓	✓	✓	✓
Restrictions	.	.	.	New Only	Private Buyer
R-squared	0.967	0.968	0.972	0.976	0.969
N	378,768	378,768	378,768	223,810	220,044

**Note:** This table shows results from regression (2). To convert into percentage discounts for leasehold properties, construct  $e^\beta - 1$ . The dependent variable is the price per square foot paid for properties sold by private parties in Singapore between 1995 and 2013. We include fixed effect at the 5-digit postcode by property type (apartment, condominium, detached house, executive condominium, semi-detached house and terrace house) by title type (Strata or Land) by transaction date. In columns (1) and (2), the transaction date interaction is for the transaction quarter, in column (3) - (5) the transaction month. We control for the age of the property (by including a dummy variable for every possible age in years), the size of the property (by including a dummy for each of 40 equally sized groups capturing property size), and the total number of units in the property. In column (4) we only focus on properties that were built within the last 3 years of our transaction date; in column (5) we only focus on properties that were bought by a private individual (and not the HDB). Standard errors are clustered at the level of the fixed effect. Significance Levels: \* ( $p < 0.10$ ), \*\* ( $p < 0.05$ ), \*\*\* ( $p < 0.01$ ).

**Table 5:** Expected Returns and Rental Growth

	United States		Singapore		United Kingdom	
	Balance Sheet	Price/Rent	Direct	Price/Rent	Balance Sheet	Price/Rent
<b>Gross Return</b>	10.3%	10.7%	8.7%	9.0%	12.5%	10.9%
<i>Rental Yield</i>	8.3%	9.8%	3.9%	4.7%	9.7%	6.9%
<i>Capital Gain</i>	2.0%	0.8%	4.9%	4.3%	2.8%	4%
Depreciation	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%
Taxes	0.67%	0.67%	0.5%	0.5%	0%	0%
<b>Net Return</b>	<b>8.13%</b>	<b>8.53%</b>	<b>6.7%</b>	<b>7%</b>	<b>11%</b>	<b>9.4%</b>
Sample	1953-2012	1988-2012	2004-2012	1990-2012	1989-2012	1996-2012
Real Rent Growth		0.1%		0.2%		0.7%
Sample		1988-2012		1990-2012		1996-2012

**Note:** This table shows our estimates for net return to housing and real rent growth in the U.S., the U.K. and in Singapore. See appendix [A.2](#) for details.

**Table 6:** Time Series Properties of Real House Price Growth

	Real HP Growth		Real PDI Growth		
	Mean	Std. Dev.	Mean	Std. Dev.	Correlation
Australia	3.20%	6.89%	1.43%	2.77%	0.093
Belgium	2.80%	5.87%	1.17%	2.27%	0.436
Canada	2.51%	7.63%	1.37%	2.10%	0.489
Switzerland	0.94%	4.73%	1.12%	1.63%	0.445
Germany	-0.29%	2.31%	1.27%	1.70%	0.288
Denmark	1.57%	8.99%	1.09%	2.29%	0.211
Spain	2.05%	8.26%	0.83%	2.46%	0.631
Finland	2.04%	8.19%	2.07%	3.21%	0.482
France	2.52%	5.23%	1.22%	1.58%	0.358
U.K.	3.53%	8.54%	2.20%	2.74%	0.355
Ireland	3.70%	9.73%	1.83%	3.59%	0.529
Italy	0.60%	8.28%	0.82%	2.44%	0.325
Japan	-0.24%	4.28%	1.55%	1.40%	0.587
S. Korea	0.59%	7.70%	3.95%	4.58%	0.235
Luxembourg	3.94%	6.68%	2.84%	3.75%	0.054
Netherlands	2.32%	9.43%	0.48%	3.25%	0.472
Norway	2.76%	7.23%	2.22%	2.52%	0.064
New Zealand	2.20%	7.73%	0.98%	3.45%	0.530
Sweden	1.50%	7.27%	1.34%	2.28%	0.431
U.S.	1.13%	3.89%	1.60%	1.56%	0.371
S. Africa	0.88%	9.65%	0.53%	3.05%	0.373

**Note:** This table shows time series properties of quarterly frequency annual growth rats of real house prices and personal disposable income, as collected by [Mack and Martínez-García \(2011\)](#). Columns (1) and (2) show the mean and standard deviation of real house price growth. Columns (3) and (4) the mean and standard deviation of real personal disposables income growth. Column (5) shows the correlation of real house price growth with real personal disposible income growth.

# Appendix

## A.1 Institutional Details

### A.1.1 United Kingdom

Over time, a number of laws have regulated the rights of leaseholders to extend their lease terms. There are three key Acts of Parliament that regulate this process. The 1967 Leasehold Reform Act enables tenants of houses (not flats) held on long leases to acquire either the freehold (a process called "enfranchisement") or an extended lease term. The 1993 Leasehold Reform, Housing and Urban Development Act conferred rights to collective enfranchisement and lease extension on groups of flat owners in the same building who have been in occupation for a number of years. The Commonhold and Leasehold Reform Act of 2002 extended the right to lease extensions to individuals who have owned (but not necessarily occupied) flats for at least two years.

The above laws codify the bargaining process for a lease extension in the following way. First, the leaseholder files a proposal for extension, with an offered premium. The freeholder reverts with a counteroffer, and the two parties can then bargain on the final price of the extension. If the two parties cannot agree on a price, the leaseholder can ask a special tribunal, the Leasehold Valuation Tribunal, to assess the "fair value" of the extension. The "fair value" that the law refers to is intended to be the market value: the amount that the property "might be expected to realize if sold on the open market by a willing seller to a willing buyer". Essentially, the law guarantees that the leaseholder is able to remain in the property for a longer term if she is willing to pay the market value for the extension. This removes some of the bargaining frictions that can be associated with the cost of moving and that result in a potential hold-up problem by the freeholder. Using an analogy to the bond market, this is equivalent to saying that the short-term investor can roll over her investment at what will be the prevailing interest rate without paying major transaction costs when doing so. Therefore, the reduction of this friction suggests that the price paid for a leasehold will more closely reflect the value of the rental income that accrues over the term of the leasehold.

Her Majesty Revenue and Customs (HMRC), the tax authority for England and Wales, gives equal treatment to the price paid for any term of leasehold or for a freehold when levying Stamp Duty Land Tax (SDLT) on residential property transactions.<sup>37</sup> The HMRC does not levy property taxes on actual ownership, it only taxes transactions (changes in ownership).

### A.1.2 Singapore

Residential properties in Singapore can be classified into land titles or strata titles. Land title properties occupy land that is exclusive to the owner (like a detached house), whereas a strata title comprises units in cluster housing (flats or apartments) or in condominium developments. Owners of strata properties enjoy exclusive title only to the airspace of their individual unit. The land that the

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<sup>37</sup>The first £125,000 are exempt from stamp duty, with rates rising progressively thereafter. For more details see: <http://www.hmrc.gov.uk/sdlt/calculate/leasehold.htm>.

development is built on is shared by all the owners of the project, based on the share of the strata title unit owned by each owner. Owners are free to sell their individual unit. In order to sell the land, they will have to go via an “en bloc” sale, which requires a minimum of 80% of the owners’ consent.<sup>38</sup>

A large fraction of the Singaporean housing stock consists of Housing and Development Board (HDB) properties, mostly in the form of flats. The HDB flats are part of a state-subsidized homeownership program and leases are often granted at below market values. We exclude these properties from our analysis and focus instead on the private market.

Finally, property taxes are independent of the form and duration of ownership. Property taxes are levied on the *Annual Value* (AV), the tax-authority assessed 1-year rental income of the property. For rental properties, the tax rate is set at 10% of AV; for owner-occupied properties, it rises from 0% on the first \$6,000 to a marginal rate of 6% for AVs exceeding \$65,000.<sup>39</sup> The rental income, and therefore the Annual Value, of a property is unaffected by the length of the lease under which the property is owned. Property transactions are also subject to stamp duty irrespective of the form and duration of ownership.<sup>40</sup>

## A.2 Unconditional Expected Returns

This appendix describes the methodology and data used to compute average real returns and rent growth for residential properties in Section 3.1.1. We report the details of the calculations in an online appendix.

**The balance-sheet approach** Following Favilukis, Ludvigson and Van Nieuwerburgh (2010), this approach uses information about the value of the stock of residential real estate to estimate the value (price) of housing and total household expenditure on housing as a measure of the value of rents in each period. Since we are only interested in the return to a representative property, we need to control for changes in the total housing stock. We proxy for the change in the stock by population growth, assuming that at least over long periods the per capita stock of housing is constant. We derive the gross return to housing in each period as:

$$R_{t+1}^G = \frac{V_{t+1}^H + CE_t^H}{V_t^H} \frac{\pi_t}{\pi_{t+1}} \frac{L_t}{L_{t+1}},$$

where  $V^H$  is the value of the housing stock,  $CE^H$  is the household expenditure on housing,  $\pi$  is the CPI price level index, and  $L$  is population.

- For the U.S. we follow Favilukis, Ludvigson and Van Nieuwerburgh (2010) and use data from

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<sup>38</sup>80% consent is necessary if the development is at least 10 years old and 90% consent is necessary if the development is less than 10 years old.

<sup>39</sup>Starting from January 1, 2014, property taxes were made more progressive. For details see: <http://www.iras.gov.sg/irasHome/page04.aspx?id=2094>.

<sup>40</sup>Stamp duties are transaction taxes, and are assessed on the purchase value of the property. The first \$180,000 are assessed at 1%, the next \$180,000 at 2% and each additional increase in the sales prices at 3%. See <http://www.iras.gov.sg/irasHome/page04.aspx?id=8748fordetails>.

the Flow of Funds (obtained from the Federal Reserve Board and the Federal Reserve Bank of St. Louis). For the value of the housing stock we sum the value of two series: owner-occupied real estate and tenant-occupied real estate (FL155035005, FL115035023) from the Flow of Funds. From the Federal Reserve Bank of St. Louis we obtain: (i) household expenditure on housing in each period, series number DHUTRC1A027NBEA of the National Income and Product Accounts (personal consumption expenditures - services: housing and utilities); (ii) Population estimates (POP); and (iii) the Consumer Price Index (USACPIBLS).

- For the U.K., using the same procedure, we combine the value of the total stock of housing (series CGRI) and the total expenditures on housing (series ADIZ) from the National Accounts (available from the Office of National Statistics). From the same source, we obtain the CPI (series D7BT). We adjust for the change in the stock of housing using the population growth series from ONS for England and Wales.
- We do not use the balance-sheet method for Singapore, where we have a more direct approach (see below) to construct housing returns.

**The price-rent approach** This approach constructs a time series of returns by combining information from a house price index, a rent index, and an estimate of the price-to-rent ratio in a baseline year. Without loss of generality suppose we have the rent-to-price ration at time  $t = 0$ . We can derive the time series of the rent-to-price ratio as:

$$\frac{P_t}{D_{t+1}} = \frac{P_t}{P_{t-1}} \frac{D_t}{D_{t+1}} \frac{P_{t-1}}{D_t}, \quad \frac{P_0}{D_1} \text{ given.}$$

where  $P$  is the price index and  $D$  the rental index. Notice that, given a baseline year  $\frac{P_0}{D_1}$ , only information about the growth rates in prices and rents are necessary for the calculations.

We then compute real returns using the formula:

$$R_{t+1}^G = \left( \frac{D_{t+1}}{P_t} + \frac{P_{t+1}}{P_t} \right) \frac{\pi_t}{\pi_{t+1}}.$$

- For the U.S. we follow [Favilukis, Ludvigson and Van Nieuwerburgh \(2010\)](#) and use the Case-Shiller 10-city house price index (series SPCS10RSA from the Federal Reserve Bank of St. Louis), and compute rent growth using the BLS shelter index (the component of CPI related to shelter, item CUSR0000SAH1 from the Federal Reserve Bank of St. Louis). However, differently from [Favilukis, Ludvigson and Van Nieuwerburgh \(2010\)](#), we choose 2012 as a baseline year for the rent-price ratio, which is estimated at 0.1, because of the availability of high quality data for that year. We obtained two independent estimates for the rent-price ratio in the base year of 2012. The first estimate is the price-rent ration implied by the balance-sheet approach. The second estimate is a direct estimate obtained using data by the real estate portal Trulia. Figure A.1 shows the distribution of rent-price ratios across the 100 largest MSAs provided by Trulia.<sup>41</sup>

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<sup>41</sup>We thank Jed Kolko and Trulia for providing these data. Trulia observes a large set of both for-sale and for-rent listings.

Both independent estimates imply a rent-price ratio of 10% in 2012. Figure A.2 suggests that these rent-price ratios are close to their long-run average.

- For Singapore we obtain a time series of price and rental indexes for the whole island from the Urban Redevelopment Authority (the official housing arm of the government). We have three independent estimates of the rent-price ratio in the baseline year of 2012: 3.5% using the “direct method” (see below), 4% from Real Estate Exchange and DTZ Research, and 3.5% from squarefoot.com.sg. We use the more conservative 3.5% estimate in our analysis.
- For England and Wales we use the house price index from the UK Land Registry to compute price appreciation and we use the CPI component “Actual rents for housing” (series D7CE) from the Office of National Statistics as a rental index. As a baseline we used the 6% rent-price ratio in 2012 obtained from the balance-sheet approach.

**The direct approach** In the case of Singapore we have data on the median house price and median rent, obtained from the Urban Redevelopment Authority. The data are reported for the three regions of the island (Core Central Region, Rest of Central Region, Outside Central Region), and we take an average of the return across the three to compute our return series (though note that average returns are essentially identical across regions). We stress that these are not indexes but actual dollar values. Therefore, we compute real returns directly as:

$$R_{t+1}^G = \frac{P_{t+1} + D_{t+1}}{P_t} \frac{\pi_t}{\pi_{t+1}}.$$

### A.3 Hyperbolic-Exponential Discounting

We include here details for the derivations in Section 3.4 of the paper. First, let us focus on a model of pure hyperbolic discounting. In continuous time, the hyperbolic discount function is simply  $\frac{1}{1+\kappa s}$ , where  $\kappa > 0$  is the subjective hyperbolic parameter. To gather intuition, assume that rents were constant at  $D$ . Let us value the  $T$  lease contract. For simplicity consider the  $t = 0$  starting condition.

$$P_t^T = \int_0^T \frac{1}{1 + \kappa s} D ds = D \frac{\ln(1 + \kappa T)}{\kappa}.$$

The obvious problem with this type of discounting when applied to longer term assets is that the valuation of claims diverges (even without dividend growth) as the horizon  $T$  increases ( $T \rightarrow \infty$ ).

In the paper, therefore, we augmented the hyperbolic discount function to include an exponential term:  $\frac{e^{-\rho s}}{1 + \kappa s}$ , where  $\rho > 0$  is the subjective discount rate associated with exponential discounting. The corresponding valuations of leaseholds and freeholds reported in the paper can be derived by substitution using the Exponential Integral function  $Ei(x)$ , defined as:

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The rent-price ratio is constructed using a MSA-level hedonic regression of log(price) on property attributes, zip code fixed effects, and a dummy for whether the unit is for sale or for rent. The rent-to-price ratio is constructed by inverting the exponent of the coefficient on this dummy variable.

$$Ei(x) \equiv - \int_{-x}^{\infty} \frac{e^{-t}}{t} dt,$$

and the Upper Incomplete Gamma function defined as:

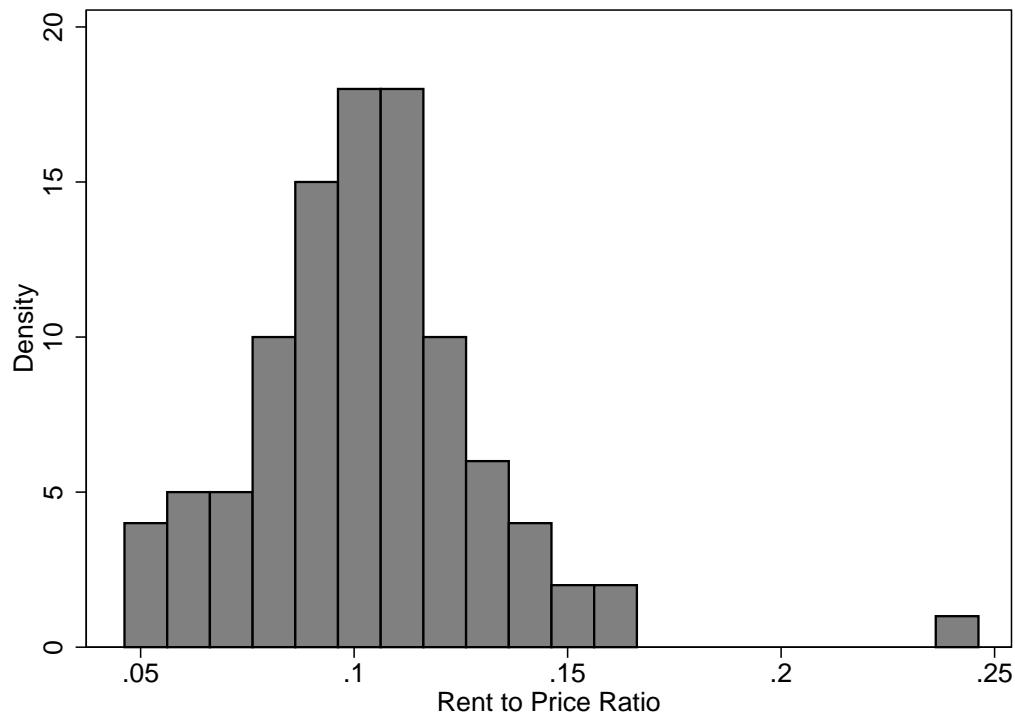
$$\Gamma(0, x) \equiv \int_x^{\infty} \frac{e^{-t}}{t} dt.$$

## Appendix References

**Favilukis, Jack, Sydney C Ludvigson, and Stijn Van Nieuwerburgh.** 2010. "The macroeconomic effects of housing wealth, housing finance, and limited risk-sharing in general equilibrium." National Bureau of Economic Research.

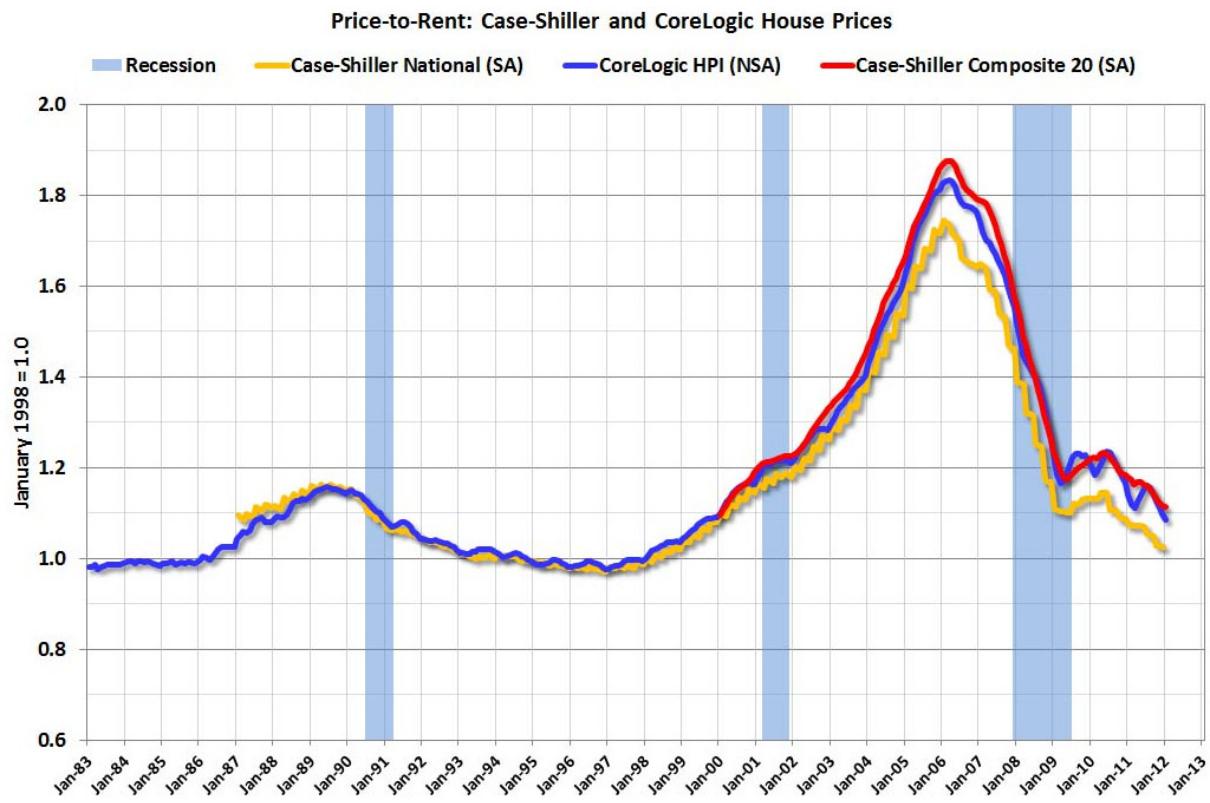
## Appendix Figures

**Figure A.1:** Cross-Sectional Distribution of Price-Rent Ratio in the U.S.



**Note:** The figure shows the distribution of the rent-to-price ratio for the 100 largest MSAs in the U.S. in September 2013 as constructed by Trulia, which observes a large set of both for-sale and for-rent listings. It is constructed using a metro-level hedonic regression of  $\ln(\text{price})$  on property attributes, zipcode fixed effects, and a dummy for whether the unit is for sale or for rent. The rent-to-price ratio is constructed by inverting the exponent of the coefficient on this dummy variable.

**Figure A.2:** Price-Rent Ratio Timeseries in the U.S.



**Note:** The figure shows the time series of the price-rent ratio in the U.S. as constructed by <http://www.calculatedriskblog.com/>.