EFFICIENT TUITION FEES, EXAMINATIONS, AND SUBSIDIES*

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this version: 14 January 2004

Abstract.
We assume that students can acquire a wage premium, thanks to studies, and form a rational expectation of their future earnings, which depends on personal "ability". Students receive a private, noisy signal of their ability, and universities can condition admission decisions on the results of noisy tests. We assume first that universities are maximizing social surplus, and contrast the results with those obtained when they are profit maximizers. If capital markets are perfect, and if test results are public knowledge, then, the optimal tuition fee is greater than marginal cost, and there is no sorting on the basis of test scores. Students optimally self-select as a result of pricing only. If capital markets are perfect but asymmetries of information are bilateral, *i.e.*, if universities observe a private signal of each student's ability, or if there are borrowing constraints, then, the optimal policy involves a mix of pricing and pre-entry selection on the basis of test scores. Optimal tuition can then be set below marginal cost, and can even become negative, if the precision of the university's private assessment of students' abilities is high enough.

STILL PRELIMINARY-COMMENTS WELCOME

Keywords: Tuition fees, Examinations, State Subsidies, Higher Education, Incomplete Information.

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1. Introduction

The public universities' state of financial crisis is an acute problem in many countries. Public funds shortages being recurrent, the idea that tuition fees could be increased naturally arises, but is understandably met with fierce resistance of citizens. In the United States, many public schools have faced the hard choice of either cutting educational spending and quality, or increasing price, and tuition has gone up in the recent years (see Winston (1999), and his references). In Europe, the situation is almost tragic. A journalist recently described Britain's universities as "depressingly threadbare, overcrowded and politicized"[1]; it seemed to us —as Frenchmen and university teachers—, that this statement is close to being an excellent description of the current state of our country's (mostly public) universities. To be more precise, we think that in French universities, poverty and bureaucracy are a cause of demoralization. Tony Blair's plans to let tuition fees increase is hugely unpopular[2]. In France, the question is still an absolute political taboo, although many feel that an evolution towards university decentralization and a form of pricing is almost inescapable. Tuition reform has already started in Germany, and is currently discussed in other countries as well. Yet, it seems that the formal economic theory of university pricing, the question of the optimal balance of fees and subsidies has not been studied with enough precision.

The present article proposes an approach of optimal fees, as well as an analysis of university behavior. In doing so, we pay special attention to informational asymmetries. University policies are examined under opposite assumptions; we assume first that universities are non-profit institutions, and contrast the results with those obtained under the assumption that they are rent-seeking, or "for-profit" organizations. The possibility of regulating, or providing incentives to rent-seeking universities, as well as the relevance of normative "marginal cost pricing" theories is also considered in the latter case.

In our model, heterogeneous students can acquire a wage premium, thanks to studies. They form a rational expectation of their future earnings, and apply for higher education on the basis of this forecast. The expected wage premium depends on the university's quality, the number of graduates, and on the student's personal "talent", or "ability". Information is incomplete, in the sense that neither the university, nor the student, can directly observe talent. Students receive a private, noisy signal of their ability, and universities can condition admission decisions on the results of tests, or entrance examinations. The cases in which test results are, and are not publicly disclosed are both analyzed. Higher education is costly, total cost depending on quality, quantity, and the average ability of recruits (the well-known peer effect). A university has the right to set fees, and to set admission standards, in the form of a minimal grade or test score. It also chooses a quality variable and total enrollment. Non-profit universities are assumed to choose their policy in order to maximize social surplus. This provides us with a useful benchmark. The non-profit managers have no concern for equity, or no aversion for inequality, so that our results will depend on efficiency considerations only. In contrast, the rent-seeking university simply

[1] The Economist (2003a)

[2] The Economist (2003b)

maximizes an expression of profit, that is, tuition revenues minus costs, which provides a clear description of the other extreme.

We consider both the case of "perfect capital markets", and the situation in which there are borrowing constraints, with the consequence that talented students from low income families can be barred from higher education by tuition.

Our results are the following. First, if capital markets are perfect, and if test or entrance examination results are public knowledge, and rational students condition their application decisions on private as well as public signals, then, an optimal policy involves a positive fee, and no sorting on the basis of test scores. Students optimally self-select as a result of pricing only. In addition, the optimal tuition fee is greater than marginal cost.

Second, if capital markets are perfect but asymmetries of information are bilateral, in the sense that universities observe a signal of each student's ability which is not disclosed or not taken into account by students, then, the optimal policy of a non-profit university involves a mix of pricing and pre-entry selection on the basis of grades or test scores. The optimal policy can then entail a direct student subsidy: optimal tuition can be set below marginal cost, and can even become negative, if the precision of the university's private assessment of students' abilities is good enough. This result is not due to redistribution or equity motives on the part of the benevolent university; it is driven solely by efficiency considerations.

Third, the rent-seeking university's policies are inefficient: tuition fees tend to be too high and admission standards tend to be too low. An incentive transfer schedule, depending on enrollment and knowledge of the wage distributions, can fully correct the inefficiencies due to rent-seeking or for-profit behavior.

Fourth, when students face borrowing constraints, even if test scores are publicly disclosed, the university's optimal policy must be a combination of pricing and selection, with the possibility that, again, tuition be set below marginal cost, to alleviate the inefficiencies due to the fact that some good students are deterred by price. Again, this result is not due to assumed redistribution motives of the benevolent university manager; it follows from efficiency, surplus maximization considerations only.

To sum up, we find that university pricing is a socially efficient policy, but that it should be mixed with pre-entry selection of students, either because the university has private information on students' abilities, or because, due to financial market imperfections, some students face borrowing constraints, or both. Now, the optimal tuition fee can be optimally set below marginal cost, and will be a decreasing function of the entrance examination's precision as a signal of student ability. The more accurate entrance exams are, the larger the discount on tuition.

To arrive at these results, we had to choose a university objective function. This is problematic, and every choice is open to criticism[3]. Winston (1999) remarkably summarizes the intuitions, and provides a non-technical description, of university economics. We have kept his vision of the "industry" in mind and arrive at results which, we think, do not contradict his observations. Our choice has been to "cut the Gordian knot" and to consider two extreme, probably equally unrealistic cases: the purely benevolent, and the

[3] A few contributions have been devoted to a formal microeconomic analysis of universities, but the topic seems underdeveloped; see, for instance, Garvin (1980), Borooah (1994).

purely greedy, for-profit (or rent-seeking) university managers[4]. The perfect competition case has been studied by Rothschild and White (1995), who emphasize the important idea that students are inputs in the production of their own human capital. In the present contribution, the university is endowed with market power. This could represent a private university with a substantial market share or a leadership position, but also a dominant network of public universities. The industrial organization of the higher education sector remains difficult to model, and in particular, the various forms of price and non-price competition among universities are still very much unexplored[5].

We assume the existence of peer effects, in the sense that the average ability of enrolled students increases the quality of education for a given expenditure, or equivalently, decreases total cost for a given quality objective. The theory of peer group effects, or local interactions in education has been studied, among other contributions, by Arnott and Rowse (1987), de Bartolome (1990), Benabou (1993), and Epple and Romano (1998). There have been many attempts at testing for the presence of peer-group effects in education, from school to college, since the Coleman (1966) report; *e.g.*, among recent contributions, Betts and Morell (1999), Hoxby (2000), Sacerdote (2001), Angrist and Lang (2002), Rothstein (2002), Zimmerman (2003). The common view is that these effects are important in higher education, even if is difficult to estimate their magnitude. All our results would be qualitatively the same if peer group effects were negligible. Yet, our analysis permits one to see how they intervene in the determination of optimal university pricing and enrollment. They would probably become crucial in theories of strategic interactions among universities.

Our representation of university technology comprises a quality variable. Quality can be viewed as an index aggregating particular expenditures and teachers' efforts, including the teacher's endeavor to stimulate student effort. To keep the model relatively simple, we did not introduce a moral hazard (*i.e.,* hidden effort incentives) problem explicitly in the analysis. Recent empirical studies show the importance of education quality on future wages (*e.g.*, Card and Krueger (1992), Angrist and Lavy (1999)), and the effect of teacher incentives on student achievement (*e.g.*, Lavy (2002)).

Finally, our analysis of optimal pricing is not independent of an economic theory of examination procedures. Important pioneering work on the economic theory of exams is due to Costrell (1994) and Betts (1998). Our philosophy is closer to that of Fernández and Galí (1999), and Fernández (1998), who presents a simplified version of the model analyzed in the former contribution. In her paper, student population is described by a joint distribution of ability and wealth. The problem is to allocate students to high quality, or to low quality schools, knowing that high quality school capacity is fixed and that student ability and quality are complementary inputs in the production of future earnings. For efficiency reasons, high ability types should be allocated to high quality schools. A costly (and socially wasteful) test technology can be used to decide which students will be admitted to high quality schools. Each student can produce a given, deterministic test result at the personal cost of a given amount of effort (which varies with ability). Fernández

[4] In our model, rent-seeking and profit maximization behaviors are formally equivalent.

[5] However, Del Rey (2001) and De Fraja and Iossa (2001), study asymmetric duopoly models; Gary-Bobo and Trannoy (2002) contains an attempt at modelling monopolistic competition.

(1998) then compares a publicly regulated, test-based allocation system with a competitive equilibrium allocation in which schools set prices. Tests and prices are equivalent when students can borrow against future income. We find the same equivalence of tests and fees as screening devices in Section 2 of the present paper, in a setting which is different, but essentially related. She then shows that markets and exams are not equivalent when students cannot borrow. Examination-led allocations dominate market equilibria because, under zero-borrowing, some able-but-poor students take the place of some wealthy-but-less-able students, which improves the overall matching of students to schools, and therefore, aggregate output. The main differences of our work with that of Fernández consist in introducing incomplete information under more radical forms: (students observe noisy signals of their ability, test results are random), in comparing various forms of informational asymmetry, (unilateral and bilateral), and studying variable enrollment and education quality. With the addition of these elements, we show under which circumstances optimal policies should balance selection on the basis of grades and self-selection by means of the price mechanism; we compute the optimal tuition and show how the amount of subsidy (or tuition rebate) is related to the informational properties of the examination technology; we finally sketch the analysis of the role of borrowing constraints in the determination of tuition rebates.

In the following, Section 2 presents basic assumptions and an analysis of our model under complete information assumptions. Section 3 presents the results in the asymmetric information setting. Section 4 is devoted to a variant of the model in which informational asymmetries are bilateral, and Section 5 analyzes the impact of borrowing constraints on the optimal policy.

2. Optimal Tuition Fee and Admission Policy under Complete Information

2.1. The Skill Premium

Our point of departure is a formalization of the skill premium which can be earned by means of higher education. We suppose to simplify the analysis that there exists a university (or college), with a single department, exerting some form of monopoly or market power as a provider of higher education and skilled workers. Let the potential student population be of size N. Each student is characterized by a "talent" or "ability" variable, denoted $\widehat{\theta}$. In the present section, ability is observed by the student, and in the next one, ability is not observed, neither by the university, nor by the student, but the student receives a noisy private signal relative to her (his) own ability. The complete information setting of the present section can be viewed as a limiting case of the more realistic, incomplete information model to be studied later.

The university and public authorities are assumed able to estimate the probability distribution of ability. We suppose that there exist two categories of workers only, the skilled, who are graduates from the university (or college), and the unskilled, who did not study. The unskilled workers' wage rate is a constant w_0 and the university (or college) graduates' wage is w. The future wage w of a skilled worker is random, because it depends on the individual's ability; we suppose that the following relation holds,

$$\ln(w) = \ln(w_0) + \widehat{\Delta}(x, e) + \widehat{\theta}, \tag{1}$$

where $\widehat{\Delta}$, the skill premium, is a function of the number of graduates, denoted x, (it depends in fact on the ratio of skilled over unskilled workers in the economy), and of the quality of studies provided by the university, denoted e.

Students do not exert any personal effort, and all enrolled students receive a diploma, for simplicity. It must however be clear that $\hat{\theta}$ captures the fact that graduates are not all equal, due to differing talents, which are reflected in more or less brilliant grades, as well as more or less lucrative perspectives on the labor market. Therefore, the assumption that all students obtain a degree does not preclude student heterogeneity. We assume that the skill premium is decreasing with respect to x, and increasing with respect to e (and continuously differentiable with respect to both variables); these are reasonable assumptions, compatible with a general equilibrium model in which skilled and unskilled labor are used as inputs in the production process.

With the help of these assumptions, we can derive the demand for higher education, that is, determine the number of candidates for registration. University policy is characterized by three variables: the number of graduates x (equal to the number of enrolled students in equilibrium), the quality of studies e, and the tuition fee, denoted p. Potential students decide to apply for registration in view of these three variables. An important, and probably strong —although very common— assumption is that all agents observe and (or) form correct expectations about the skill premium $\widehat{\Delta}$ (for a discussion of this approach, e.g. Manski (1993)).

The student's preferences are assumed to be represented by the same inter-temporal, infinite horizon, additively separable utility function. Higher education takes place in the first period of the student's life-cycle, at time $t = 0$. Utility is assumed to be quasi-linear with respect to consumption at $t = 0$. Formally, for a consumption profile $c = (c_0, c_1, c_2, ...)$ we define utility $u(c)$ as follows,

$$u(c) = c_0 + \sum_{t=1}^{\infty} \alpha^t \ln(c_t), \tag{2}$$

where

$$\alpha = \frac{1}{1+r} \tag{3}$$

is the discount factor associated with a psychological interest rate $r > 0$, used by agents to discount future utility.

2.2. Number of Candidates for Registration (Demand)

To simplify the analysis, we assume that a worker's wage is constant during her (his) entire working life. Agents do not save, and consume their wages, which are expressed in real terms. With the help of these assumptions, an agent who doesn't study remains an unskilled worker for life; his or her utility can be written, using (2) and (3),

$$u_0 = w_0 + \frac{\ln(w_0)}{r}. \tag{4}$$

Using (1), (2) and (3), if the agent chooses to study, her expected utility, conditional on ability $\widehat{\theta}$ can be written,

$$u_\theta = -p + \frac{\ln(w_0)}{r} + \frac{\widehat{\Delta}}{r} + \frac{\widehat{\theta}}{r}, \tag{5}$$

since the student is not working at time 0, and pays p as a tuition fee. Introducing a capitalized skill premium, as well as a capitalized value of ability,

$$\Delta(x,e) = \frac{\widehat{\Delta}(x,e)}{r}, \qquad \theta = \frac{\widehat{\theta}}{r}, \tag{6}$$

we get the following expression: $u_\theta = -p + \ln(w_0)/r + \Delta + \theta$. An individual with ability θ chooses to study if and only if $u_\theta \geq u_0$, that is, if and only if,

$$\theta \geq y \equiv p + w_0 - \Delta, \tag{7}$$

where y is an ability threshold above which agents do apply for registration at the university. This threshold depends on p and Δ and thus on the values of (e,p,x) (the quality of studies, tuition fee and total number of graduates), since Δ is a function of (x,e).

Let us denote by F the cumulative probability distribution of θ. In the following, we assume that F is continuously differentiable with a density denoted f. With the help of these notations, the demand facing the university is $N\,(1 - F(y))$.

Now, we want to allow for the possibility of student selection by the university. Let us assume for the time being that ability is observable, and that the university chooses an ability threshold, denoted z, below which student applications are rejected, that is, only the $\theta \geq z$ are accepted. Thus, "effective" demand, which is the combined product of screening and pricing, depends on p and z. Formally, let first

$$t = max\{y, z\}. \tag{8a}$$

Effective demand, denoted q, is a function of t defined as

$$q(t) = N(1 - F(t)). \tag{8b}$$

2.3. University Teaching Technology and Peer Group Effects

Our model will be fully specified if we describe higher education costs. We would like to capture the idea that the often discussed "peer group effects" can be present and affect the quality of education for a given cost, or equivalently, affect the cost of providing a given quality of education. To do so, we assume that there exists a continuously differentiable university cost function, denoted C, depending on chosen education quality e, on the number of enrolled students x, and on average student ability, denoted v. In other words, total university cost is given by $C(x,e,v)$, where v is the expected value of ability, knowing that abilities $\theta \geq t$ are enrolled by the university. We provide a formal expression for v

below. It is natural to assume that C is increasing with respect to x and e, and non-increasing with respect to v. This assumption, which is not the most general way of modelling the fact that students are inputs in the production process of their own human capital (e.g., Rothschild and White (1995)), nevertheless captures the essential idea behind peer effects.

At this stage, it will probably clarify the discussion if we show that the model described above is formally equivalent to one in which the the peer group effect directly affects the skill premium through average student ability v. To see this, assume that the job market values worker quality, denoted e, and that worker quality is produced with the help of teacher effort η and peer effects so that $e = \phi(\eta, v)$, where ϕ is a kind of production function, which is increasing with respect to η for all v. Now, if the teaching technology is represented by a cost function depending on teacher effort and the number of students only, that is, if $C = \tilde{C}(x, \eta)$, and if the skill premium depends on quality and the number of graduates only, that is, $\Delta = \Delta(x, e)$, then, the latter can be expressed as a function of effort η as follows,

$$\tilde{\Delta}(x, \eta, v) \equiv \Delta(x, \phi(\eta, v)).$$

This shows reasonable conditions under which the approach in which the skill premium would be a function of average peer ability is formally equivalent to one in which average peer ability is an argument of the university cost function. It happens that the latter approach is much simpler than the former from the technical point of view, as will be seen below.

2.4. The Philanthropic View and Social Optimum

We define a higher education institution as "philanthropic" when its objective is to maximize the social value, or social surplus, of its education activity. It is of course a strong assumption to assume that a university is philanthropic, but this approach will provide us with a very clear benchmark, equivalent to the idea of Pareto optimum. Since utilities are quasi-linear, social surplus maximization yields efficient policies and allocations. In the following, we will of course contrast this view with other, less optimistic theories of the university objective, that will be called "cynic views".

It is now possible to compute the optimal policy of a philanthropic university. The university will be financed by subsidies (or by donations), in the case of a deficit. The required amount of public resources (or donations) is simply,

$$D = C(x, e) - px. \tag{9}$$

The amount D will be subtracted from social surplus to balance the university budget at time 0. We therefore suppose that the share of total cost that the students of a given cohort do not pay for in the form of tuition fees will be paid in the form of (lump-sum) taxes, or contributions, made by the same or other other agents, such as alumni donations, etc. There are of course more subtle relations between public pricing, public subsidies voluntary contributions, and the tax system, involving political economy and redistribution problems which will not be studied in the present analysis. In particular, we assume that the social justice problems are solved by means of other redistributive tools, in the hands of independent public authorities. More precisely, we assume that equality of opportunity problems are solved in the sense that no student with the necessary talents is barred from studying because of a financial constraint. We are perfectly aware of the fact that this picture is a bit too rosy, and that more sophisticated modelling work on the case in which imperfections of credit markets and family backgrounds differences create unequal opportunities is needed. We address the borrowing constraints question in the final section of this paper. In the present section and the following one, our analysis aims at showing the simple structure of the optimal higher education pricing problem in a pure efficiency case, which can again be seen as a benchmark. Under the assumptions made above, the philanthropic objective (social surplus) can be defined as follows,

$$W = xE\left(u_\theta \mid \theta \geq y,\, \theta \geq z\right) + (N - x)u_0 + px - C(x, e, v), \tag{10}$$

and can be interpreted as the total sum of the x graduates' expected utilities, and of the remaining $(N - x)$ unskilled workers. If the expressions of utilities u_θ and u_0 given by (4)-(6) are substituted in W, we get, after some straightforward simplifications,

$$W = x\left(\Delta(x, e) - w_0 + v(t)\right) - C(x, e, v(t)) + Nu_0, \tag{11}$$

where, by definition,

$$v(t) = E(\theta \mid \theta \geq t) = \frac{\int_t^\infty \theta f(\theta) d\theta}{1 - F(t)}, \tag{12}$$

is the mean ability of an individual, knowing that this individual is a student, and $t = max(y, z)$. Thus, W is the difference between total student or skilled worker productivity, $x(\Delta + v)$, and total opportunity costs xw_0 plus total direct costs C of higher education.

To determine the optimum, the social surplus (11) must be maximized under the constraints that the number of enrolled students cannot be higher than the number of screened applications, that is, $x \leq q(t)$ with $t = max(y, z)$ and $y = p + w_0 - \Delta$.

The philanthropic faculty will always be able to close the gap between x and $q(t)$ with the help of an increase of the tuition fee p or of the pass-threshold z if $x < q(t)$, because q is a decreasing function of t. Thus $x = q(t)$ at the optimum. To see this, suppose that $z > y$ and that $x < q(z)$. Then, locally, $dW/dz = (x - C_v)v'(t)$, where C_v is the partial derivative of C with respect to v. But, by assumption, $C_v < 0$ and

$$v'(t) = \frac{f(t)}{1 - F(t)}\left(v(t) - t\right) \geq 0, \tag{13}$$

because $v(t) = E(\theta \mid \theta \geq t) \geq t$, with a strict inequality if t is not the highest value in the support of f. It follows that $dW/dz > 0$ and the university should increase z. Suppose now that $y > z$ and $x < q(y)$. Then locally, for the same reasons as above, $dW/dp = dW/dy = (x - C_v)v'(y) > 0$, and the university should increase p. As a consequence, it is possible to set $x = q(t)$, and to invert the effective demand curve. We then obtain the expression of an ability-threshold t^*, or marginal ability, which is a function of x. Inverting (8b), we get,

$$t = t^*(x) \equiv F^{-1}\left(1 - \frac{x}{N}\right), \tag{14}$$

giving the value of t for which exactly x individuals with abilities greater than t apply for registration at the university.

It is then useful to remark that,

$$xv\left(t^*(x)\right) = x\frac{\int_{t^*(x)}^{\infty}\theta f(\theta)d\theta}{x/N} = N\int_{t^*(x)}^{\infty}\theta f(\theta)d\theta, \tag{15}$$

and for future reference, using Leibniz's rule, we get

$$\frac{d}{dx}\left[xv\left(t^*(x)\right)\right] = t^*(x). \tag{16a}$$

and

$$\frac{dv(t^*(x))}{dx} = \left(\frac{1}{x}\right)\left[t^*(x) - v(t^*(x))\right] < 0 \tag{16b}$$

In order to derive the necessary conditions for an optimum, we now maximize the following function with respect to (x, e),

$$W(x,e) = x\left(\Delta(x,e) - w_0\right) + xv(t^*(x)) - C[x, e, v(t^*(x))] + Nu_0, \tag{17}$$

Let us denote C_x, Δ_x, C_e, Δ_e the partial derivatives of C and Δ with respect to x and e respectively. The necessary conditions for optimality are,

$$x\Delta_x + \Delta - w_0 + t^* = C_x + C_v\left(\frac{1}{x}\right)(t^* - v(t^*)), \tag{18}$$

$$x\Delta_e = C_e. \tag{19}$$

Conditions (18) and (19) are easily interpreted. Equation (18) says that the marginal social value of a graduate must be equal to its marginal cost, while equation (19) says that the marginal social value of quality $x\Delta_e$ must be equal to its marginal cost C_e, at the optimum. The marginal value of a graduate is the sum of two terms: the graduate's marginal ability t^*, and the marginal "social revenue" of the skills produced by the university, that is, $x\Delta_x + \Delta - w_0$. A negative term $x\Delta_x$ appears in the latter expression (since $\Delta_x < 0$ by assumption), and expresses the fact that an additional skilled worker lowers the wage of all the other graduates on the labor market. At the optimum, the university must take this effect into account and should not flood the market with too many skilled workers. The marginal cost is itself the sum of two terms: the first is the direct marginal cost C_x, the second, which is also positive (being the product of two negative terms) is the marginal "peer effect". Increasing x by one unit reduces the average quality of students as shown by (16b); hence, it reduces the peer group effect, which increases the cost by $(dv/dt)(dt^*/dx)C_v$.

The optimal allocation is basically expressed in terms of x, e and t. The remaining part of the analysis is mainly a problem of implementation. Some form of student screening takes place at the optimum, since the optimal t is not equal to its smallest possible value; then, do we wish the optimal screening level to be implemented by means of a "merit list" (i.e., by the entrance admission-threshold z) or by "means of money" (i.e., the tuition p)? An increase of the tuition fee p will increase the average quality v of students equally well as an increase of the admission threshold z, these tools are perfect substitutes. This is true if students and the university perfectly observe the ability levels. We will see that this conclusion does not hold fully in an incomplete information version of the model, although money and merit may remain imperfect substitutes.

In welfare terms, given our utilitarian formulation of the social surplus, and the quasi-linearity of preferences, the tuition p has no effect on welfare, apart from its role in determining the ability of the marginal enrolled student t^*, that is, its role as a screening tool. The tuition has redistributive effects, but they do not matter under quasi-linearity. Again, this conclusion would not hold in a world in which informational and financial imperfections play an important role, as will be seen below.

Formally, equations (18) and (19) determine the optimal values (x^*, e^*), from which we derive $t^{**} \equiv t^*(x^*)$, the optimal ability of the marginal student. In addition, we know that $t^{**} = max(y^*, z^*)$ for some pair (y^*, z^*). The relation of y^* with the optimal tuition p^* is derived from (7) above, and given by the formula,

$$p^* = y^* + \Delta(x^*, e^*) - w_0. \tag{20}$$

Implementation of the optimum by means of tuition means choosing $y^* = t^{**} > z^*$ and choosing the tuition p^* so that (20) holds with $y^* \equiv t^*(x^*)$. Implementation of the optimum by "merit" is tantamount to choosing $z^* = t^{**} > y^*$ and choosing the tuition p^* so that (20) holds.

It follows that there are two possible pricing regimes in our complete information model: one in which tuition matters because it determines demand (locally) and one in which sorting according to ability makes tuition ineffective as a screening device. In the second regime, one can say that the usual interpretation of some observed facts holds: tuition is set at a deliberately low level to create an excess demand, which aims at facilitating the selection of good students by the university. We are now equipped to clarify the meaning of the often discussed application of "marginal cost pricing" to universities.

2.5. "Marginal Cost" Pricing and Optimality

At this stage, it should be noted that implementation by means of a tuition fee does not require any observation of the admitted student's abilities: it would therefore also work under conditions of asymmetric information, when ability θ is a private information of the applicants. Under these informational conditions, the only thing that the university authorities need to know is, as usual, the distribution of abilities F. The university can then implement an optimum by setting p^* appropriately (so that $t^{**} = y^*$ of course), and students will self-select according to the optimal screening rule, since only abilities greater than t^{**} will apply. In this case, introducing an entrance examination procedure would produce information on abilities, but would presumably be costly. Examination costs can then be saved by the philanthropic university manager, insofar as redistribution effects do not matter, because pricing alone does the entire job of implementing the optimum.

Consider now the optimal tuition in this case. Using (18), (20), and $y^* = t^*(x^*)$, we easily derive,

$$p^* = C_x + C_v \hat{v}_x - x^* \Delta_x. \tag{21a}$$

where, using (16b),

$$\hat{v}_x \equiv \frac{t^*(x) - v(t^*(x))}{x} < 0. \tag{21b}$$

Since $x\Delta_x \leq 0$, $C_x > 0$, $C_v < 0$ and $\hat{v}_x < 0$, we find that p is unambiguously positive. Thus, we can state the following.

Result 1. *If abilities are private information of the students applying for higher education, tuition alone can be used to screen applicants and to implement a social optimum. In this case, at any interior optimum, tuition is greater than the university's marginal cost of a student.*

When abilities are not observable, tuition fees are used to discourage the students whose abilities lie below the socially optimal threshold. These fees are not just a token, since they must then cover the marginal cost of education (including the marginal peer effect). Define now, the university fixed cost

$$K(e, v) = \lim_{x \to 0^+} C(x, e, v), \tag{22}$$

which depends on the chosen quality of studies e, and possibly on the average ability of recruits v, and assume that marginal cost C_x is increasing with respect to x. From (21a), multiplying by x and subtracting C, we get,

$$px - C = xC_x + xC_v\hat{v}_x - x^2\Delta_x - C.$$

Define the university rent as $R = px - C$ (this does not mean that the university will earn this "rent" at the optimum, because the social planner can tax it). Then, from the above expression, we get,

$$R + K = -x^2\Delta_x + xC_v\hat{v}_x + xC_x - C + K,$$

and since C is assumed to be convex (because marginal cost is assumed to be increasing), we have $K - C + xC_x \geq 0$. The assumptions $\Delta_x < 0$ and $C_v < 0$ therefore yield $R + K > 0$, or equivalently $px - C + K > 0$, and we can state the following result.

Result 2. *If abilities are private information of the students and if the university cost is convex, then, a social optimum is implemented by means of a tuition fee, and university revenues px must be greater than variable costs $C - K$.*

It must then be the case that part of the university fixed cost will be covered by a public subsidy, or another source of revenue. This being said, we know that the optimal policy can be implemented by a combination of tuition and sorting of abilities, and in this case, the above pricing formula has no reason to apply. In practice, universities do combine the tools of selection by money and selection by merit, they engage in the costly production of information on abilities (by means of tests and entrance examinations), and students are subsidized (e.g. Winston (1999)). These facts are more appropriately captured in an incomplete information framework. Before we start the analysis of the incomplete information case, we now contrast the "cynic" and "philanthropic views". To this end, and for the sake of completeness, we analyze the behavior of a rent-seeking (or purely for-profit) university under conditions of complete information.

2.6. The Cynic View: Rent-Seeking and For-Profit University

We now study the behavior of a deregulated rent-seeking or for-profit university. The university is deregulated in the sense that it is free to choose total enrollment x, tuition fee p, quality e, and the admission threshold z. The university is also endowed with market power; this is an approximation for a situation in which, say, a centrally governed public network of universities has a quantitatively important share of the higher education market, but the model could as well capture the behavior of a private university with a dominant position.

The cynic view assumes that the university (the faculty) seeks to maximize their rent

$$R = px - C(x, e, v(t)), \tag{23}$$

with respect to (e, x, p, z), subject to the constraints $x \leq q(t)$, $t = max(y, z)$ and $y = p + w_0 - \Delta(x, e)$, effective demand q being defined by (8b). This rent can be understood as the amount of resources that are made available to finance faculty activities other than teaching. In academic systems in which teacher's careers essentially depend on research achievement, it is likely that research will be a major faculty objective (although there is of course no guarantee), so that R might as well stand for research. But R can of course easily be interpreted as profit, and our model is then that of a for-profit university.

It can again be shown that $x = q(t)$ at the optimum. Suppose that $z < y$, then $t = y$, and an increase of p yields $dR/dp = dR/dy = x - C_v v'(t) > 0$, so that p should be increased until $q(t)$ equals x. Suppose now that $z > y$, then $t = z$, and an increase in z yields $dR/dz = -C_v v'(t) > 0$, so that z should be increased until $q(t)$ equals x. Substituting $q(t) = x$ and $p = y + \Delta - w_0$ in the expression for rent (23) yields the program,

$$max_{(e,y,z)} \{(y + \Delta(q(t), e) - w_0)q(t) - C[q(t), e, v(t)]\} \tag{24}$$

subject to $t = max(y, z)$. Finally, z (and the constraint defining t) can be eliminated from the optimization program as follows. To solve the problem, it is sufficient to choose t and y subject to $t \geq y$; if $t > y$ at the optimum, then $t = z$ and if $t = y$ then z can be arbitrarily chosen so that $y \geq z$ at the optimum. The rent-seeking university therefore wants to maximize,

$$R(e, t, y) = q(t)\left[y + \Delta(q(t), e) - w_0\right] - C(q(t), e, v(t)), \tag{25}$$

subject to the constraint $t \geq y$. Let now λ be the constraint's Lagrange multiplier. Kuhn and Tucker's necessary conditions for constrained optimality can be written:

$$(y + \Delta - w_0 + q(t)\Delta_x)q'(t) - C_x q'(t) - C_v v'(t) = -\lambda, \tag{26a}$$

$$q(t)\Delta_e = C_e, \tag{26b}$$

$$q(t) = \lambda \qquad \text{and} \qquad \lambda(t - y) = 0. \tag{26c}$$

Conditions (26) immediately yield an important qualitative result: since $q(t) = \lambda > 0$, they imply $y = t$, and the rent-seeking university will not use pre-enrollment selection to sort students; the active screening variable is the tuition fee. Using $q'/q = -f/(1-F)$ and $v'_t = -(q'/q)(v - t)$ to rearrange terms in (26), we get the following rent-maximization conditions,

$$x\Delta_x + p = C_x + C_v\left(\frac{y - v(y)}{x}\right) + \left(\frac{1 - F(y)}{f(y)}\right), \tag{27a}$$

$$x\Delta_e = C_e, \qquad \text{with} \tag{27b}$$

$$y = t^*(x), \qquad \text{and} \qquad p = t^*(x) + \Delta - w_0. \tag{27c}$$

A direct comparison of (27a)-(27c) with (18)and (19) above immediately shows that the rent-seeking university does not choose an optimal allocation (e, x, p, z). The rent-seeking university does not enroll enough students. Since it behaves as if marginal cost was higher by $(1 - F)/f$, they will typically set enrollment x below its optimal level. It follows that they will choose a tuition level which is too high with respect to the philanthropic optimum (conditional on chosen quality e). Quality will be higher or lower than its socially optimal value, depending on the sign of the cross second-order partial derivative Δ_{xe}. Equations (19) and (26b) show that the rent-seeking university will choose an optimal level of quality e if and only if it chooses an optimal level of enrollment x. These properties hold under complete information conditions, as well as if abilities are private information of the students. We can state the following result.

Result 3. *When compared with the philanthropic university, the rent-seeking university (a), chooses a sub-optimal level of enrollment, and (b), tuition is their only screening device and they do not sort students according to ability.*

A comparison of the cynic objective (24) with the philanthropic objective (17) immediately shows that the cynic solution will typically not be socially optimal. The difference between the two objectives stems from the fact that the rent-seeking university prices according to the last enrolled student's ability (or marginal ability $t^*(x)$), while the philanthropic manager takes the total value of student abilities $N\int_{t^*(x)}^{\infty} \theta f(\theta) d\theta$ (or $xv(t^*(x))$ into account. There is a close formal analogy between the complete information version of our model and the model of a monopoly choosing both quantity and quality, as studied by Spence (1975) and Sheshinski (1976).

The distortions caused by rent-seeking behavior can easily be corrected: it would be sufficient to tax the marginal value of ability and simultaneously, to pay a subsidy equal to the total value of ability. We can state the following result.

Result 4. *The rent-seeking university chooses a socially efficient policy if it is subject to a public money transfer* $T(x)$*, depending on enrollment only, and defined as,*

$$T(x) = -xt^*(x) + N \int_{t^*(x)}^{\infty} \theta f(\theta) d\theta + constant, \tag{28}$$

where $t^*(x)$ *is defined by (14).*

It is remarkable that the incentive money transfer T depends on the number of graduates x, and on distribution F only — because t^* itself only depends on F. It is also remarkable that cost observations are not needed to regulate the rent-seeking university. Enrollment x is in principle observable by the public regulator, and the ability distribution F can be estimated by an econometrician, by means of the Mincerian regression function (1), which was our point of departure. If the cynic view is correct, a public regulator of universities should collect data on students' wages and careers and perform some econometric work, to correct the distortions caused by rent-seeking behavior. Even though unbiased estimates of a regression function function like $\Delta(x, e)$, are more difficult to obtain than it would seem at first glance (e.g. Card (1999), Harmon, Oosterbek, and Walker (2003)), given the enormous amount of empirical work devoted to the topic, returns-to-education econometrics is nowadays common practice. In our simplified model, it is easy to see how two-step methods à la Heckman (e.g. Heckman (1978), Willis and Rosen (1979)) can be used to estimate F and Δ. Assume that Δ is linear and e is constant. We get the regression equation, $ln(w_i/w_0) = a + bx + \theta_i$, where i indexes individual observations. We neglect possible controls such as age and experience to simplify the discussion. Given that we observe students only, the expected value of θ_i is not zero. We get instead $E[\theta_i \mid \theta_i > y] = v(y)$, where $y = p + w_0 - bx - a$. Two-step methods (or, of course, standard maximum likelihood techniques) can be applied to estimate the regression $ln(w_i/w_0) = a + bx + v(y) + \eta_i$, where $E(\eta_i) = 0$.

3. Asymmetric Information and Entrance Examinations

Let us now come back to the study of philanthropic and cynic views of university, but under less restrictive assumptions relative to the information of the students, public regulator, and Faculty. Our fundamental assumption will now be that students do not observe their own ability θ; they are endowed with incomplete knowledge of their own talent, formed by means of noisy informative signals. Students are assumed to observe a private signal of ability; more precisely, they observe s, where, by definition,

$$s = \theta + \varepsilon, \tag{29}$$

where ability θ and ε, a zero-mean noise, are assumed to be normal and independent random variables.

In addition, an costless examination technology provides an estimation of ability which is publicly observable. The examination grade is a random variable denoted z, and is defined as follows:

$$z = \theta + \nu, \tag{30}$$

where ν is normal with a zero mean and independent from θ and ε. The grade z is known to the student and to the Faculty (*i.e.*, the university authorities). This examination can be interpreted as a national final high school exam, (such as baccalauréat in France, or Abitur in Germany), or z can be viewed as an entrance test score.

The university can set a pass mark $\bar{z}$; a student is then admitted for registration only if his or her grade z is greater than $\bar{z}$. The value of higher education is now expressed in expected terms, conditional on the two signals (s, z). Let u_1 be the expected utility of higher education,

$$u_1 = -p + \frac{\ln(w_0)}{r} + \Delta + E(\theta \mid s, z),$$

where variables, p, w, r, Δ, have the same meaning as in the above section. The utility of a non-educated worker is still

$$u_0 = -w_0 + \frac{\ln(w_0)}{r}.$$

An individual applies for higher education if and only if $u_1 \geq u_0$, that is, equivalently, if and only if,

$$E(\theta \mid s, z) \geq \bar{y} \equiv p + w_0 - \Delta(x, e). \tag{31}$$

Under our normality assumptions, a student's rational estimation of her own ability, that is, $E(\theta \mid s, z)$, is itself a normal random variable. Let σ_θ^2, σ_ε^2, σ_ν^2, be the variances of θ, ε, et ν, respectively, and let μ be the prior mean of θ. Some computations, using the normality assumption yield the classic result,

$$E(\theta \mid s, z) = \frac{s\sigma_\theta^2\sigma_\nu^2 + z\sigma_\theta^2\sigma_\varepsilon^2 + \mu\sigma_\nu^2\sigma_\varepsilon^2}{\sigma_\theta^2\sigma_\nu^2 + \sigma_\theta^2\sigma_\varepsilon^2 + \sigma_\nu^2\sigma_\varepsilon^2}. \tag{32}$$

This expression being linear with respect to s and z, we have,

$$E(\theta \mid s, z) \equiv \alpha s + \beta z + \gamma\mu \equiv y, \tag{33}$$

where the values of α, β, and γ are defined by identifying (32) et (33). The random signal y can be interpreted as the expected ability of an individual, knowing her private signal and her test score or examination grade. With this specification, a student is enrolled if she is willing to apply and if she satisfies the requirements of the entrance selection process based on z, that is, if and only if,

$$z \geq \bar{z} \quad \text{and} \quad y \geq \bar{y}. \tag{34}$$

Effective demand can then be written,

$$x = q(\bar{y}, \bar{z}) = N\,\Pr(y \geq \bar{y},\, z \geq \bar{z}),$$

that is,

$$q(\bar{y}, \bar{z}) = N \int_{\bar{z}}^{\infty} \int_{\bar{y}}^{\infty} \psi(y, z) dy dz, \tag{35}$$

where ψ is the joint normal density of (y, z).

3.1. The Philanthropic View: Optimal Examination and Tuition Fees

Expected social surplus can be written,

$$W = xE\left[u_1 \mid y \geq \bar{y}, z \geq \bar{z}\right] + (N - x)u_0 + px - C.$$

Define,

$$v(\bar{y}, \bar{z}) \equiv E\left[\theta \mid y \geq \bar{y}, z \geq \bar{z}\right]. \tag{36}$$

Function v is the expected ability of enrolled students, knowing that their grade is greater than $\bar{z}$ and that their private assessment y is greater than $\bar{y}$. After some simplifications, we get, $W = x\left[\Delta(x, e) - w_0 + v(\bar{y}, \bar{z})\right] - C[x, e, v(\bar{y}, \bar{z})] + Nu_0$, and substituting the constraint $x = q(\bar{y}, \bar{z})$ in the above expression yields the expression of social surplus, or philanthropic objective,

$$W(e, \bar{y}, \bar{z}) = q(\bar{y}, \bar{z})\left[\Delta(q(\bar{y}, \bar{z}), e) - w_0 + v(\bar{y}, \bar{z})\right] - C[q(\bar{y}, \bar{z}), e, v(\bar{y}, \bar{z})] + Nu_0. \tag{37}$$

W can be maximized with respect to $(e, \bar{y}, \bar{z})$, instead of $(e, p, \bar{z})$, given that $\bar{y} = p + w_0 - \Delta$. This maximization problem can be decomposed into two sub-problems. A first sub-problem is to maximize $xv(\bar{y}, \bar{z}) - C(x, e, v(\bar{y}, \bar{z}))$ with respect to $(\bar{y}, \bar{z})$, for given (e, x). Given that $C_v \leq 0$, this is tantamount to maximizing $v(\bar{y}, \bar{z})$ subject to $x = q(\bar{y}, \bar{z})$, with respect to $(\bar{y}, \bar{z})$, for fixed x. The necessary conditions for an optimal pair $(\bar{y}, \bar{z})$ (if it is finite!) are simply

$$\frac{v_{\bar{y}}}{v_{\bar{z}}} = \frac{q_{\bar{y}}}{q_{\bar{z}}}, \qquad \text{and} \qquad x = q(\bar{y}, \bar{z}), \tag{38}$$

where subscripts denote partial derivatives. The interpretation of condition (38) is easy if it is reminded that, $v_{\bar{y}} = \partial v/\partial \bar{y} = \partial v/\partial p$, $q_{\bar{y}} = \partial q/\partial \bar{y} = \partial q/\partial p$; it says that the marginal rate of substitution between p and $\bar{z}$ should equal its marginal rate of transformation, conditional on the fixed production target x. The second sub-problem is then to maximize W with respect to (x, e), given that the optimal $(\bar{y}, \bar{z})$ have been expressed as functions of x.

The determination of $(\bar{y}, \bar{z})$ is formally equivalent to the following problem. Assume that an examination procedure has two consecutive tests (say, Math and English), and that both tests are graded on a numerical scale. Grades in Math and English are random variables s and z respectively. A student is admitted if a weighted average y of both grades is greater than $\bar{y}$, and if — Math being considered very important— the math grade is greater than $\bar{z}$. Our "first sub-problem" above is formally equivalent to solving for the pair $(\bar{y}, \bar{z})$ which maximizes the expected ability v of admitted students, given that a certain enrollment target x should be met.

To study the existence of solutions to the system of equations (38), we state a number of technical Lemmata.

Lemma 1. *$v_y/q_y \geq v_z/q_z$ is equivalent to $h_y(\bar{y}, \bar{z}) \geq h_z(\bar{y}, \bar{z})$, where, by definition,*

$$h_y(\bar{y}, \bar{z}) \equiv E(\theta \mid y = \bar{y}, z \geq \bar{z}), \tag{39a}$$

$$h_z(\bar{y}, \bar{z}) \equiv E(\theta \mid y \geq \bar{y}, z = \bar{z}), \qquad (39b)$$

For proof, see the appendix

We now state a result which is a key for what follows.

Lemma 2. *y is a sufficient statistic for θ, and,*

$$E[\theta \mid y, z] = y \qquad (40)$$

For proof, see the appendix

Intuitively, Lemma 2 says that the random variable $y = \alpha s + \beta z + \gamma\mu$, being a statistically optimal combination of the two signals s and z, conveys all the useful (private and public) information about an individual's ability. This result has the following striking consequence.

Proposition 1. *If the grade or test result z is publicly observed and s is privately observed by the students, then, there does not exist a finite solution of (38): in this case, the optimal (philanthropic) solution involves $\bar{z} = -\infty$, i.e., admission standards are the lowest possible; optimal screening is performed by means of the tuition fee only.*

For proof, see the appendix

The meaning of Proposition 1 can be rephrased as follows. In a world in which economic agents are perfectly rational (i.e., if they are good enough statisticians), the university can safely rely on student self-selection through the pricing mechanism only. An optimal tuition p^* is therefore the only useful tool, and selection by means of an admission standard is superfluous, provided that students can assess their ability by conditioning on the publicly disclosed academic grade z.

Thus, according to the philanthropic view, social value maximization doesn't lead the university to make use of an optimal "policy mix" involving pricing and selection on the basis of test scores. We study of variants of our model leading to less radical conclusions in the following Sections. Let us now examine the pricing behavior of the philanthropic university, and the screening and pricing policies of a rent-seeking, or profit-maximizing, university.

Given Proposition 1, define

$$\hat{q}(\bar{y}) = \lim_{\bar{z} \to -\infty} q(\bar{y}, \bar{z}) \qquad (41a)$$

and

$$\hat{v}(\bar{y}) = \lim_{\bar{z} \to -\infty} v(\bar{y}, \bar{z}) = E(\theta \mid y \geq \bar{y}), \qquad (41b)$$

and rewrite the philanthropic objective (37) as

$$W = \hat{q}(\bar{y}) \left[\Delta(\hat{q}(\bar{y}), e) - w_0 + \hat{v}(\bar{y})\right] - C[\hat{q}(\bar{y}), e, \hat{v}(\bar{y})] + Nu_0 \qquad (41c),$$

to be maximized with respect to $(e, \bar{y})$. The first-order conditions for the maximization of (41c) yield, $\hat{q}\Delta_e = C_e$ with obvious notations, as a condition "determining" optimal quality e, and, after some rearrangement of terms,

$$\Delta - w_0 + \hat{q}\Delta + \hat{v} - C_x = \left(\frac{C_v - \hat{q}}{\hat{q}}\right)\left(\hat{h}_y - \hat{v}\right), \tag{42}$$

where $\hat{h}_y = E(\theta \mid y = \bar{y})$, and $\hat{v} = E(\theta \mid y \geq \bar{y})$, and we make use of the relation $\hat{v}_y/\hat{q}_y = (1/\hat{q})(\hat{h}_y - \hat{v})$. From this condition, with some reworking, we get the next result.

Proposition 2. *If z is publicly observed, the optimal tuition fee p^* of the philanthropic university is higher than marginal cost C_x. The optimal tuition is thus positive, i.e., students are not subsidized.*

Proof: To prove Proposition 2, remark first that, using the definition $p = \bar{y} + \Delta - w_0$, equation (42) can be rewritten as,

$$p^* = -\hat{q}\Delta_x + C_x + \frac{C_v}{\hat{q}}(\hat{h}_y - \hat{v}) + (\bar{y} - \hat{h}_y). \tag{42b}$$

Remark then that the first two terms on the right-hand side of (42b) are positive since by assumption, $\Delta_x \leq 0$. We show next that the last term is zero. Using the formula for conditional expectation under normality, we get

$$\hat{h}_y = E(\theta \mid \bar{y}) = \mu + \frac{Cov(y, \theta)}{Var(y)}(\bar{y} - \mu).$$

But, by definition of conditional expectation, $y - \mu = E(\theta - \mu \mid s, z)$ is a linear orthogonal projection of $\theta - \mu$ onto the space spanned by $s - \mu$ and $z - \mu$. Thus, $E[(\theta - y)(y - \mu)] = 0$, which is equivalent to $Cov(\theta, y) = E(y - \mu)^2 = Var(y)$. This immediately yields $\hat{h}_y = \bar{y}$. Finally, we get,

$$\hat{h}_y = \bar{y} \leq E[y \mid y \geq \bar{y}] = E[E(\theta \mid y) \mid y \geq \bar{y}] = E(\theta \mid y \geq \bar{y}) = \hat{v},$$

and therefore, since $C_v \leq 0$, the third term on the right-hand side of (42b) is non-negative. The right-hand side of (42b) is therefore a sum of nonnegative terms; we conclude that $p^* > C_x > 0$. *Q.E.D.*

3.2. The Cynic Approach again: the Rent-Seeking University's Policy

Let us now study the rent-seeking university in the same asymmetric information framework, where z is publicly observed. The rent-seeking university will try to maximize $R = pq(\bar{y}, \bar{z}) - C(q(\bar{y}, \bar{z}), e, v(\bar{y}, \bar{z}))$, subject to $p = \bar{y} - w_0 + \Delta(q(\bar{y}, \bar{z}), e)$. The fee p can be eliminated from the expression of rent, which becomes

$$q(\bar{y}, \bar{z})\,[\bar{y} + \Delta(q(\bar{y}, \bar{z}), e) - w_0] - C(q(\bar{y}, \bar{z}), e, v(\bar{y}, \bar{z})), \tag{43}$$

and must be maximized with respect to $(e, \bar{y}, \bar{z})$. It would be easy to prove that the rent-seeking university would not like to ration students (*i.e.,* choose to set $x < q$), for it would then always benefit from a raise of the tuition p. The rent maximization problem can be decomposed into two steps. For fixed values of (x, e), the thresholds $(\bar{y}, \bar{z})$ can first be set so as to maximize $x\bar{y} - C(x, e, v(\bar{y}, \bar{z}))$ subject to $x = q(\bar{y}, \bar{z})$. This yields the first-order conditions,

$$\frac{x}{C_v} = v_y - \frac{v_z}{q_z} q_y. \tag{44}$$

Given that $C_v < 0$, due to peer effects, and by Lemma 1, (44) implies that the rent-seeking optimum should satisfy $h_y > h_z$, but, for finite values of $\bar{z}$, this is again impossible. We can state,

Proposition 3. *If z is publicly observed, the optimal rent-seeking (or for-profit) university policy is to set $\bar{z} = -\infty$, i.e., admission standards are the lowest possible and tuition does all the screening job.*

For proof, see the appendix

If the peer effects were negligible, *i.e.,* $C_v = 0$, it would be easy to provide a proof of the latter result. It would then always be profitable to increase $\bar{y}$ by $d\bar{y} > 0$ and to reduce $\bar{z}$ by $d\bar{z} = -(q_y/q_y)d\bar{y}$, keeping $x = q(\bar{y}, \bar{z})$ (and thus the cost C) constant, for that would increase the rent R by $dR = xdp = xd\bar{y} > 0$. The matter is slightly more complicated in the presence of a peer effect, and the result is driven by the fact that $h_y < h_z$ holds for every finite value of $\bar{z}$.

With the help of definitions (41a)-(41b), rewrite the rent as $R = (\bar{y} + \Delta(\hat{q}(\bar{y}), e) - w_0)\hat{q}(\bar{y}) - C(\hat{q}, e, \hat{v}(\bar{y}))$, to be maximized with respect to $(e, \bar{y})$. A comparison of (41c) and the above expression of rent obviously shows that the rent-seeking policy is not optimal in the philanthropic sense. But the rent-seekers become dedicated philanthropists if they are subjected to a certain public incentive transfer.

Proposition 4. *The rent-seeking university chooses a socially optimal policy if it is subjected to the following transfer T, defined as,*

$$T(e, x, p) = -x\bar{y} + x\hat{v}(\bar{y}) + T_0, \tag{45}$$

where T_0 is any constant.

The proof of this result is obvious since $R + T \equiv W +$ Constant. The result is more interesting because equation (45) shows that the transfer T depends on $\bar{y}$, x and knowledge of $\hat{v}$. Since $\bar{y} = p + w_0 - \Delta(x, e)$, the transfer depends also on e. So it seems that the public authority or public regulator can only compute the transfer if they observe x, p, and e. But it is in fact the knowledge of the Mincerian regression function $ln(w) = ln(w_0) + \Delta(x, e) + \theta$, involving an estimation of distribution parameters μ and σ_θ^2 which is required, with the addition of the covariance $Cov(y, \theta)$ (equal to $Var(y)$ here), which are needed to compute the average ability v. This is because

$$\hat{v}(\bar{y}) = \frac{\int_{\bar{y}}^{\infty} \int_{-\infty}^{\infty} \theta \hat{\psi}(\theta, y) d\theta dy}{\int_{\bar{y}}^{\infty} \int_{-\infty}^{\infty} \hat{\psi}(\theta, y) d\theta dy},$$

where $\hat{\psi}$ is the joint density of θ and y. Quality e, intervening only through its effect on Δ, can be captured, in principle, as a fixed university effect. So the problem is to estimate the distribution of $y = E(\theta \mid s, z)$. This latter distribution can in principle be estimated from regression work, where wages are regressed on higher education achievement, secondary education test scores, plus family and social background control variables for a cross section of individuals. Again, a regulator which is also an (expert) econometrician can, in principle, compute and implement the incentive transfer T, but the task is not really easy.

In practice, we know that the rent-seeking faculty will tend to underestimate the social value of educating students, because they take $\bar{y}$, the marginal student's value into account instead of $\hat{v}(\bar{y})$, the average student's value, and that $\bar{y} \leq \hat{v}(\bar{y})$ (as shown by the proof of Proposition 2 above). It seems that the transfer function can be approximated by the sum of two terms, a per capita subsidy, which aims at correcting the gap between marginal and average values, minus a lump sum tax, formally $T \approx s_0 x + T_0$, where $s_0 > 0$ and $T_0 < 0$.

The conclusions reached in this section are somewhat unpleasant, because the social optimum doesn't rely on exams or test scores, but only on money. But these results are probably less robust than it seems, for they depend on the property that the university's information set is strictly included in that of the student. A balance between the two screening tools appears to be an optimum if we assume a form of *bilateral asymmetric information*, in which the university knows something that the student doesn't take into account about his (her) own ability. We now turn to the study of such a setting, in a variant of our model.

4. The Case of Bilateral Asymmetric Information

It seems reasonable to assume that the university is endowed with information about student abilities that students themselves do not have, while continuing to assume that students observe a noisy private signal of ability s. If we interpret z as a private signal of the university about the student's ability, we can easily construct a variant of our model in which informational asymmetries are bilateral. Assume then to fix ideas that z is the result of an admission test which is compulsory, costless, and observed by academic authorities only, and that the admission pass-mark is $\bar{z}$. Using the same notation as above, unless specified otherwise, an individual will apply for registration if and only if $u_1 \geq u_0$, where,

$$u_1 = Pr(z \geq \bar{z} \mid s) \left[-p + \frac{ln(w_0)}{r} + \Delta + E(\theta \mid s) \right] + Pr(z < \bar{z} \mid s) \left[w_0 + \frac{ln(w_0)}{r} \right],$$

and

$$u_0 = w_0 + \frac{ln(w_0)}{r}.$$

It follows that $u_1 \geq u_0$ if and only if $E(\theta \mid s) \geq w_0 + p - \Delta \equiv \bar{y}$. Redefine now

$$y = E(\theta \mid s). \tag{46}$$

A student applies for registration if and only if $y \geq \bar{y}$. Due to normality assumptions, we have,

$$y = \alpha_0 s + (1 - \alpha_0)\mu, \tag{47a}$$

where

$$\alpha_0 = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2}. \tag{47b}$$

The number of enrolled (admitted) students is still $q(\bar{y}, \bar{z}) = NPr(y \geq \bar{y}, z \geq \bar{z})$.

4.1. Philanthropic Optimum under Bilateral Asymmetric Information

The philanthropic objective of the university is still the same as above, that is, $W = x(\Delta(x, e) - w_0 + v(\bar{y}, \bar{z})) - C[x, e, v(\bar{y}, \bar{z})] + Nu_0$, where $v = E[\theta \mid y \geq \bar{y}, z \geq \bar{z}]$, and of course $x = q(\bar{y}, \bar{z})$.

It follows that the optimal $(\bar{y}, \bar{z})$ for given (x, e) are still a solution of system (38), i.e., $v_y/q_y = v_z/q_z$, and $x = q$, and by Lemma 1, which still applies, (38) is equivalent to $h_y = h_z$, where the h functions are still defined by (39a)-(39b).

There are differences with the above version of the model, starting from this point. Intuitively, z and y now play a symmetric informational part, and y is no longer a sufficient statistic for (y, z). We can state,

Lemma 3.

$$E[\theta \mid y, z] = a_0 y + b_0 z, \tag{48a}$$

where,

$$a_0 = \frac{\sigma_\nu^2}{\sigma_\nu^2 + (1 - \alpha_0)\sigma_\theta^2} \tag{48b}$$

and $a_0 + b_0 = 1$, α_0 being defined by (47).

For proof, see the appendix.

In this new setting, the system of equations (38) has a (finite) solution. Let $\phi(x) = (2\pi)^{-1/2} e^{-x^2/2}$ denote the normal density and $\Phi(x) = \int_{-\infty}^{x} \phi(u) du$ denote the normal c.d.f. We can state the following.

Proposition 5. *If z is a private information of the university and s is a private information of the student, the optimal policy of the philanthropic university involves a mix of non-trivial admission standards $\bar{z}^*$ and a tuition p^*. Formally, equations (38) possess a solution $(\bar{y}^*, \bar{z}^*)$ for every given $x > 0$. More precisely, this solution is fully characterized as follows:*

$$\bar{y}^* = (1 - \alpha_0)\mu + \alpha_0(\bar{z}^* - \sigma_0 \xi^*), \tag{49a}$$

where ξ^* solves the equation,

$$\xi = \left(\Phi(\xi) - \frac{\sigma_\nu^2}{\sigma_\epsilon^2 + \sigma_\nu^2} \right) \frac{\phi(\xi)}{(1 - \Phi(\xi))\Phi(\xi)}, \tag{49b}$$

$\bar{z}^*$ solves,

$$x = q[(1 - \alpha_0)\mu + \alpha_0(\bar{z} - \sigma_0 \xi^*), \bar{z}], \tag{49c}$$

and finally, $\sigma_0 = \sqrt{\sigma_\epsilon^2 + \sigma_\nu^2}$.

For Proof, see the Appendix.

Proposition 5 provides us with a much more reasonable description of the world than Proposition 1. To clarify its meaning, assume for instance that $\sigma_\epsilon^2 = \sigma_\nu^2$ (*i.e.*, signals z and s are "equally noisy"). Then, by (49b), we find that $\xi^* = 0$ is the only solution (because $\Phi(0) = 1/2$), and by (49a), we get $\bar{y}^* = (1-\alpha_0)\mu + \alpha_0 \bar{z}^*$: the first order condition in (38) has provided us with a linear relationship between $\bar{y}$ and $\bar{z}$. The "production level" x pins down the appropriate value of $\bar{z}$, as indicated by (49c).

The distance between $\bar{y}$ and $\alpha_0 \bar{z}$ depends on the ratio $\sigma_\epsilon^2/\sigma_\nu^2$. To see this, define,

$$\lambda = \frac{\sigma_\nu^2}{\sigma_\epsilon^2 + \sigma_\nu^2}.$$

Then, a simple application of the Implicit Function Theorem shows that

$$\frac{\partial \xi^*}{\partial \lambda} < 0,$$

i.e., ξ^* decreases when $\sigma_\nu^2/\sigma_\epsilon^2$ increases. Using (49a), it is also easy to see that the distance $\bar{y}^* - \alpha_0 \bar{z}^*$ increases when λ increases, that is, formally,

$$\frac{\partial(\bar{y}^* - \alpha_0 \bar{z}^*)}{\partial \lambda} = -\alpha_0 \sigma_0 \frac{\partial \xi^*}{\partial \lambda} > 0.$$

This result is intuitive, if test scores z become more noisy than private signals s, then, less weight should be placed on selection by means of test scores, *i.e.,* $\bar{y}^* - \alpha_0 \bar{z}^*$ should increase. This can be achieved if admission standards $\bar{z}^*$ are lowered and (or) tuition fees are raised conditional on (x, e). This is because optimal tuition (conditional on (x, e)) is of the form,

$$p^* = \bar{y}^* + \Delta(x, e) - w_0. \tag{49d}$$

Now, maximization of the philanthropic objective W with respect to $(e, \bar{y}, \bar{z})$ yields — after some rearrangement of terms — the following first-order necessary conditions,

$$q\Delta_e = C_e, \qquad h_y = h_z, \tag{50a}$$

$$q\Delta_x + \Delta - w_0 + v - C_x = \left(1 - \frac{C_v}{q}\right)(v - h_y). \tag{50b}$$

But it is no longer possible to show that (49d), (50a) and (50b) jointly imply $p^* > 0$. It happens that p^* could be negative, a personal subsidy instead of a fee. More precisely, we get the following result.

Proposition 6. *Assume that peer effects are negligible, i.e., $C_v = 0$. Then, in the bilateral asymmetric information version of the model, the optimal tuition is smaller than marginal cost, or even negative if the test score z is accurate enough as a measure of ability. Formally, for sufficiently small σ_ν, $p^* < 0$.*

For Proof, see the Appendix

Proposition 6 captures in part the idea that some higher education institutions would at the same time be highly selective, and subsidize talented students to lure them into their classrooms. Winston (1999) shows that top universities and colleges in the US do indeed at the same time seem to be those who offer the highest subsidy to students, in view of unit cost information. Intuitively, if the student's private signals s are very poor as indicators of talent, but if the university admission test technology is very precise, it could be optimal to select only the very best and to subsidize them heavily, to be sure that no good element is deterred by the price. Remark that the result does not depend on a redistribution motive on the part of the philanthropic university (because their objective W is quasi-linear with respect to period 0 income, and utilitarian in nature). Proposition 6 only shows that negative fees can improve selection when the university can condition admission on sufficiently accurate information about student's talents. In the next section, we develop a version of the model in which students also face a financial constraint: the presence of liquidity constrained students is then also a motive for tuition fee subsidization. But before we turn to the study of this question, let us compare the philanthropic and rent-seeking universities under the same bilateral asymmetric information assumptions.

4.2. The Cynic View under Bilateral Asymmetric Information

The rent R is still $R = (\bar{y} + \Delta(x,e) - w_0)x - C[x, e, v(\bar{y}, \bar{z})]$, with $x = q(\bar{y}, \bar{z})$. Decomposing the problem again, it is easy to see that for fixed (x, e), the rent-seeking faculty should choose $(\bar{y}, \bar{z})$ so as to maximize $x\bar{y} - C[x, e, v(\bar{y}, \bar{z})]$, subject to the constraint $x = q(\bar{y}, \bar{z})$. A necessary condition is therefore obtained from the first-order conditions for this latter sub-problem. We must have (44) again, that is, $x/C_v = v_y - (v_z/q_z)q_y$. This implies $v_y/q_y > v_z/q_z$ (recall that $q_y < 0$ and $C_v < 0$), and, by Lemma 1, $h_y > h_z$. The screening policy of the rent-seeking university will therefore not be socially optimal, because optimality requires equality of the latter two terms.

To see what happens in this case, use condition $h_y > h_z$, and with the help of the statement and proof of Proposition 5 above, it can be shown that the solution will be as that given by (49a) above, except that ξ^* is replaced with a value $\xi^r < \xi^*$. It follows that $\bar{y} - \alpha_0 \bar{z}$ will be greater under rent-seeking than at the (philanthropic) optimum. From this, and the above remark, we conclude,

Result 5. *The screening policy of the rent-seeking university is not socially optimal. The rent-seeking university will set higher fees and (or) lower selection standards $\bar{z}$ than the philanthropic university, for any given value of (x, e).*

5. Borrowing Constraints and Asymmetric Information

We now address the question of financial constraints in human capital accumulation, *i.e.*, the question of poor talented students, who cannot convince a banker that they deserve credit. Would the presence of this market imperfection, due to incomplete information, change the analysis presented above? We will see that it does, although not fundamentally, in an extension of our model taking the distribution of student's "initial financial endowments" and borrowing constraints into account. Some students, due to the borrowing constraint, would not be able to pay the tuition fee, in spite of having received very good signals relative to their future ability and earnings.

To perform the analysis, we come back to the informational assumptions of Section 3 (*i.e.* where z is publicly observed). For simplicity, we assume that the student's "initial financial endowment" (or "asset") is a normal random variable a with mean $\bar{a}$ and variance σ_a^2. Assume that a is independent of θ, ϵ and ν (in fact, assume a independent of every other random source in the model). Assume that if $p > a$, a student must borrow, and that the lender attaches a "score" to each student, where the score is defined as $(\eta + \Delta)$, where Δ is defined by (1) and (6) above, and η is an independent normal random noise with mean $\mu = E(\theta)$ and variance σ_η. This random noise reflects the errors of appreciation made by the banker. Now, we assume that the lender will finance the student's education project if and only if

$$p \leq a + \kappa(\Delta + \eta), \tag{51}$$

where κ is a coefficient satisfying $0 \leq \kappa \leq 1$. Defining a new random variable $t = a + \kappa\eta$, the liquidity constraint (51) can be expressed as,

$$t \equiv a + \kappa\eta \geq p - \kappa\Delta \equiv \bar{t}, \tag{52}$$

or simply $t \geq \bar{t}$. Assume that, as in the asymmetric information model above, t, z and s are observed by the students and that the university observes z only. Given independence, the effective demand for education is now

$$\tilde{q}(\bar{y}, \bar{z}, \bar{t}) = N \Pr(y \geq \bar{y}, z \geq \bar{z}) \Pr(t \geq \bar{t}) = q(\bar{y}, \bar{z}) \Pr(t \geq \bar{t}). \tag{53}$$

With the addition of the borrowing constraint, because of independence, the average ability of students does not change, that is,

$$v(\bar{y}, \bar{z}) = E(\theta \mid y \geq \bar{y}, z \geq \bar{z}, t \geq \bar{t}) = E(\theta \mid y \geq \bar{y}, z \geq \bar{z}). \tag{54}$$

It follows that the philanthropic objective W can still be expressed as $W = x(\Delta(x, e) - w_0 + v) + Nu_0 - C(x, e, v)$, where $x = \tilde{q}(\bar{y}, \bar{z}, \bar{t})$. Another difference with the analysis of Section 4 above is that $\bar{t}$ depends on $\bar{y}$. To see this, recall that $p = \bar{y} + \Delta - w_0$, so that

$$\bar{t} \equiv \bar{y} + (1 - \kappa)\Delta - w_0, \tag{55}$$

and one should keep in mind that $\partial\bar{t}/\partial\bar{y} = 1$.

The optimal screening policy $(\bar{y}, \bar{z})$ maximizes $xv - C(x, e, v)$, for fixed (e, x), subject to $x = \tilde{q}(\bar{y}, \bar{z}, \bar{t})$. Introducing a Lagrange multiplier τ for the latter constraint leads to the following system of first order optimality conditions,

$$\begin{aligned}(x - C_v)v_y &= -\,\tau(\tilde{q}_y + \tilde{q}_t)\\ (x - C_v)v_z &= -\,\tau\tilde{q}_z.\end{aligned}$$

Eliminating τ from the system and using the fact that $\tilde{q}_y = q_y \Pr(t \geq \bar{t})$ yields,

$$\left(\frac{\tilde{q}_y}{\tilde{q}_y + \tilde{q}_t}\right)\frac{v_y}{q_y} = \frac{v_z}{q_z},$$

Using then the result of Lemma 1, that is $v_z/q_z = (1/q)(h_z - v)$, etc., yields the equivalent expression,

$$\left(\frac{\tilde{q}_y}{\tilde{q}_y + \tilde{q}_t}\right)h_y + \left(\frac{\tilde{q}_t}{\tilde{q}_y + \tilde{q}_t}\right)v = h_z \tag{56}$$

Since by Lemma 2 we have $h_y = \bar{y}$ when z is publicly observed, we get the still equivalent,

$$\frac{\tilde{q}_t}{\tilde{q}_y}(v - h_z) = h_z - \bar{y},$$

In this case, by Lemma 2 again, we also have

$$h_z = E(\theta \mid y \geq \bar{y}, z = \bar{z}) = E(y \mid y \geq \bar{y}, z = \bar{z}).$$

Let $g(t; \bar{a}, \sigma_t)$ be the Gaussian density of t, and $G(t; \bar{a}, \sigma_t)$ its c.d.f (recall that $t \sim \mathcal{N}(\bar{a} + \kappa\mu, \sigma_t^2))$. Using $\tilde{q}_y = (1 - G(\bar{t}; \bar{a}, \sigma_t))q_y$, and $\tilde{q}_t = -g(\bar{t}; \bar{a}, \sigma_t)q$, we can rewrite again (56) as follows,

$$\frac{g(\bar{t}; \bar{a}, \sigma_t)}{(1 - G(\bar{t}; \bar{a}, \sigma_t))}\big(v(\bar{y}, \bar{z}) - h_z(\bar{y}, \bar{z})\big) = \frac{-q_y(\bar{y}, \bar{z})}{q(\bar{y}, \bar{z})}\big(h_z(\bar{y}, \bar{z}) - \bar{y}\big). \tag{57}$$

There is no obvious impossibility to solve (57) for a finite $(\bar{y}, \bar{z})$, because, even if z is publicly observed, the borrowing constraint is binding for some students. In the proof of Proposition 1, the optimality condition $h_y = h_z$ boils down to $\bar{y} = E(y \mid y \geq \bar{y}, z = \bar{z})$ which holds only asymptotically, *i.e.*, if $\bar{z} = -\infty$ (or if $\bar{y} = +\infty$).

Equation (57) is a generalization of (38) above. To see this, assume that the weight of liquidity constraints vanishes, *i.e.*, formally, assume that $\bar{a} \to +\infty$ and $\sigma_t \to 0$. Then, in (57), the ratio $g/(1 - G) \to 0$ and it follows that the equation boils down to $h_z = \bar{y}$ which is impossible, except asymptotically, if $\bar{z} = -\infty$.

Equation (57) is quite complex and hard to study, but it cannot be trivially solved with $\bar{z} = -\infty$.

Proposition 7. *If $x > 0$, the solution $(\bar{y}, \bar{z})$ of the system comprising equation (57), and $x = \tilde{q}(\bar{y}, \bar{z}, \bar{t})$, where $\bar{t}$ is given as a function of $\bar{y}$ by (55), if it exists, is finite ($\bar{z} = -\infty$ is not an asymptotic solution of the system).*

Proof:
Assume that $\bar{z} \to -\infty$, then, we get $h_z \to \bar{y}$; $v \to E(y \mid y \geq \bar{y})$; $-q_y/q$ tends towards a finite, positive limit $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \psi(\theta, \bar{y}, z) d\theta dz / \Pr(y \geq \bar{y})$; and finally $g/(1-G) > 0$ doesn't change, because it doesn't depend on $\bar{z}$. It follows that when $\bar{z} \to -\infty$, the left-hand side of (57) tends towards a positive limit, while the right-hand side tends towards 0. This is impossible for a finite value of $\bar{y}$. If in addition to $\bar{z}$, we let $\bar{y} \to +\infty$, then $g/(1-G) \to 0$, and (57) boils down to $0 = 0$, but this also implies $\tilde{q} = 0$, which contradicts the assumption $x > 0$. *Q.E.D.*
The addition of borrowing constraints justifies the recourse to selection on the basis of test scores and a form of direct student subsidization through rebates on tuition fees.

We have found two cases in which below cost pricing of higher education can be justified: it is either because universities have knowledge about student abilities that students themselves do not have (the case of bilateral asymmetric information), or because some good students are liquidity constrained, even if students and the university are equally informed (in the sense that they exploit the information conveyed by test scores rationally). In the latter case, it will also be optimal to raise the admission standards, and hence, simultaneously decrease tuition (*i.e.*, decrease $\bar{y}$), to reduce the number of poor-but-talented students inefficiently deterred by price. (This is true since $\bar{z}$ is finite, and q is decreasing with respect to both $\bar{y}$ and $\bar{z}$.)

6. Conclusion

We have shown that tuition fees and selection on the basis of test scores are both used as screening instruments in an optimal university policy, if some students are liquidity constrained, or if asymmetric information is bilateral (in the sense that both the university and the students possess useful, non-redundant private information about students' abilities). Our theory describes how the two screening instruments should be combined. We also showed that the optimal tuition fee can entail an element of direct subsidy, a rebate, which is an increasing function of the university's information accuracy. Optimal fees can therefore be smaller than marginal cost. Price covers marginal cost only if the student's information set includes the university information set (and if students are fully rational), in which case entrance examinations are also useless. This means that a social optimum will often be characterized by the need for outside resources in the form of public money or donations, which are thus fully justified, to balance the university budget. Rent-seeking or for-profit teaching institutions will typically set prices too high and enroll too few students. Finally, it does not seem that tuition fees alone constitute a solution to the university budget problem. There is probably some scope for tuition increases in European countries, where public universities and free access are dominant, and symmetrically, tuition charges cannot be the universal remedy in the United States.

7. Appendix

Proof Lemma 1.

Note first that,

$$v(\bar{y},\bar{z}) = \frac{\int_{\bar{y}}^{\infty}\int_{\bar{z}}^{\infty}\int_{-\infty}^{\infty}\theta\psi(y,z,\theta)dydzd\theta}{\int_{\bar{y}}^{\infty}\int_{\bar{z}}^{\infty}\int_{-\infty}^{\infty}\psi(y,z,\theta)dydzd\theta},$$

where ψ is the joint normal density of (y,z,θ). The denominator of v is just q/N, so it will be convenient to define a mapping g as follows,

$$v(\bar{y},\bar{z}) = \frac{g(\bar{y},\bar{z})}{q(\bar{y},\bar{z})}.$$

Now we get,

$$v_y \equiv \frac{\partial v}{\partial \bar{y}} = \frac{g_y}{q} - \frac{g q_y}{q^2},$$

which is equivalent to,

$$\frac{v_y}{q_y} = \frac{1}{q}\left(\frac{g_y}{q_y} - v\right).$$

Likewise, $v_z/q_z = (1/q)\left((g_z/q_z) - v\right)$. Now remark that,

$$\frac{g_y}{q_y} = \frac{-N\int_{\bar{z}}^{\infty}\int_{-\infty}^{\infty}\theta\psi(\bar{y},z,\theta)dzd\theta}{-N\int_{\bar{z}}^{\infty}\int_{-\infty}^{\infty}\psi(\bar{y},z,\theta)dzd\theta} = E[\theta \mid y=\bar{y}, z\geq\bar{z}] = h_y.$$

Likewise, $g_z/q_z = E[\theta \mid y\geq\bar{y}, z=\bar{z}] = h_z$. We can therefore conclude that $v_y/q_y > v_z/q_z$ if and only if $h_y > h_z$.
Q.E.D.

Proof of Lemma 2.

Given our normality assumptions, $E[\theta-\mu \mid y,z]$ is a linear orthogonal projection (theoretical regression) of $\theta-\mu$ onto the sub-space spanned by $y-\mu$ and $z-\mu$. Let us denote $\sigma_{ab} = Cov(a,b)$ and $\sigma_a^2 = Var(a)$ for any random variables a, b. Applying a classic result, it then follows, with matrix notation,

$$\hat{\theta} \equiv E[\theta \mid y,z] - \mu = (\sigma_{\theta y},\sigma_{\theta z})\mathbf{\Lambda}^{-1}\begin{pmatrix} y-\mu \\ z-\mu \end{pmatrix},$$

where

$$\mathbf{\Lambda} = \begin{pmatrix} \sigma_y^2 & \sigma_{yz} \\ \sigma_{yz} & \sigma_z^2 \end{pmatrix}.$$

We then easily obtain,

$$\mathbf{\Lambda}^{-1} = \frac{1}{det\mathbf{\Lambda}}\begin{pmatrix} \sigma_z^2 & -\sigma_{yz} \\ -\sigma_{yz} & \sigma_y^2 \end{pmatrix},$$

where $det\mathbf{\Lambda} = \sigma_y^2\sigma_z^2 - \sigma_{yz}^2$. Thus, the coefficients of $(y-\mu)$ and $(z-\mu)$ in the expression of $\hat{\theta}$ are, respectively,

$$a = \frac{\sigma_z^2\sigma_{\theta y} - \sigma_{\theta z}\sigma_{yz}}{\sigma_y^2\sigma_z^2 - \sigma_{yz}^2}, \qquad \text{and} \qquad b = \frac{\sigma_y^2\sigma_{\theta z} - \sigma_{\theta y}\sigma_{yz}}{\sigma_y^2\sigma_z^2 - \sigma_{yz}^2}.$$

Now, since $y = E[\theta \mid s, z]$ is an orthogonal projection of θ onto the space spanned by s and z, we must have $Cov(\theta - y, y) = 0$, which implies $\sigma_{\theta y} = \sigma_y^2$. Since $\theta - y \perp y$, in particular, we also have $\theta - y \perp z$, so that $Cov(\theta - y, z) = 0$, or, equivalently, $\sigma_{\theta z} = \sigma_{yz}$. With the help of these remarks, it is easy to check that $a = 1$ and $b = 0$. Thus, finally $E[\theta \mid y, z] = \mu + a(y - \mu) = y$.
Q.E.D.

Proof of Proposition 1.

From Lemma 1, we must solve $h_y = h_z$ to solve (38). From Lemma 2, we derive,

$$h_y = E[\theta \mid y = \bar{y}, z \geq \bar{z}] = E[E(\theta \mid y, z) \mid y = \bar{y}, z \geq \bar{z}] = \bar{y},$$

and, using (33), we get,

$$\begin{aligned}
h_z &= E\big[E(\theta \mid y, z) \mid y \geq \bar{y}, z = \bar{z}\big] \\
&= \alpha E[s \mid y \geq \bar{y}, z = \bar{z}] + \beta\bar{z} + \gamma\mu \\
&= \alpha E\left[s \,\middle|\, s \geq \frac{\bar{y} - \beta\bar{z} - \gamma\mu}{\alpha}\right] + \beta\bar{z} + \gamma\mu \\
&= \alpha\sigma_s E\left[\frac{s-\mu}{\sigma_s} \,\middle|\, \frac{s-\mu}{\sigma_s} \geq \frac{\bar{y} - \beta\bar{z} - (1-\beta)\mu}{\alpha\sigma_s}\right] + \beta\bar{z} + (1-\beta)\mu \\
&= \alpha\sigma_s\big[E(\zeta \mid \zeta \geq \zeta_0) - \zeta_0\big] + \bar{y},
\end{aligned}$$

where $\zeta \sim \mathcal{N}(0,1)$ and,

$$\zeta_0 = \frac{\bar{y} - \beta\bar{z} - (1-\beta)\mu}{\alpha\sigma_s}.$$

It follows that $h_y = h_z$ is equivalent to,

$$E(\zeta \mid \zeta \geq \zeta_0) = \zeta_0,$$

which is impossible, except if $\zeta_0 = +\infty$.

If $\bar{y} = +\infty$, then $x = 0$. But $\lim_{\bar{z}\to-\infty} q(\bar{y}, \bar{z})$ is well defined and positive. It is now routine work to show that if there existed an optimum in which $\bar{z}$ is finite, then, decreasing $\bar{z}$ slightly would increase W, so that the optimal philanthropic solution involves $\bar{z} = -\infty$.
Q.E.D.

Proof of Proposition 3.

Since $C_v < 0$ and $q_y < 0$, by (44), we know that $v_y/q_y \geq v_z/q_z$ at a maximum of rent R. By Lemma 1, this is equivalent to $h_y \geq h_z$, but the proof of Proposition 1 above shows that this is equivalent to $\zeta_0 \geq E(\zeta \mid \zeta \geq \zeta_0)$, where $\zeta_0 = (1/\alpha\sigma_s)(\bar{y} - \beta\bar{z} - (1-\beta)\mu)$, which is impossible, except if $\zeta_0 = +\infty$. If $\bar{y} = +\infty$, then $x = 0$. The limits of q and v as $\bar{z} \to -\infty$ are well defined. It is again routine work to show that if a rent maximum involved a finite value of $\bar{z}$, then R could be increased slightly by decreasing $\bar{z}$ slightly. It follows that $\bar{z} = -\infty$ is an optimal solution for the rent-seeking university.
Q.E.D.

Proof of Lemma 3.

As in the proof of Lemma 2, the classic result on conditional expectations under normality assumptions yields,

$$\begin{aligned} E[\theta \mid y, z] =& \mu + (\sigma_{\theta y}, \sigma_{\theta z}) \begin{pmatrix} \sigma_y^2 & \sigma_{yz} \\ \sigma_{yz} & \sigma_z^2 \end{pmatrix}^{-1} \begin{pmatrix} y-\mu \\ z-\mu \end{pmatrix} \\ =& \mu + \frac{(\sigma_{\theta y}, \sigma_{\theta z})}{(\sigma_y^2\sigma_z^2 - \sigma_{yz}^2)} \begin{pmatrix} \sigma_z^2 & -\sigma_{yz} \\ -\sigma_{yz} & \sigma_y^2 \end{pmatrix} \begin{pmatrix} y-\mu \\ z-\mu \end{pmatrix}. \end{aligned}$$

The coefficients of $(y-\mu)$ and $(z-\mu)$ in the expression of $\hat{\theta}$ are, respectively,

$$a_0 = \frac{\sigma_z^2\sigma_{\theta y} - \sigma_{\theta z}\sigma_{yz}}{\sigma_y^2\sigma_z^2 - \sigma_{yz}^2}, \qquad \text{and} \qquad b_0 = \frac{\sigma_y^2\sigma_{\theta z} - \sigma_{\theta y}\sigma_{yz}}{\sigma_y^2\sigma_z^2 - \sigma_{yz}^2}.$$

Simple algebra then yields the stated result, using $\alpha_0 = \sigma_\theta^2/\sigma_s^2$, because

$$\sigma_y^2 = \alpha_0^2\sigma_s^2 = \alpha_0\sigma_\theta^2; \qquad \sigma_z^2 = \sigma_\theta^2 + \sigma_\nu^2;$$

$$\sigma_{\theta y}^2 = Cov(\theta, \alpha_0 s) = \alpha_0\sigma_{\theta s} = \alpha_0\sigma_\theta^2; \qquad \sigma_{yz}^2 = Cov(\alpha_0 s, z) = \alpha_0 Cov(\theta, \theta) = \alpha_0\sigma_\theta^2;$$

and finally $\sigma_{\theta z} = \sigma_\theta^2$.
Q.E.D.

Proof of Proposition 5.

We must solve $h_y = h_z$ in the bilateral asymmetric information version of the model.

$$\begin{aligned} h_y =& E\big[E(\theta \mid z, y) \;\big|\; y = \bar{y}, z \geq \bar{z}\big] \\ =& E[a_0 y + b_0 z \mid y = \bar{y}, z \geq \bar{z}] \\ =& a_0\bar{y} + b_0 E[z \mid y = \bar{y}, z \geq \bar{z}]. \end{aligned}$$

Define $\delta = \nu - \epsilon$, and denote $\sigma_0 = \sigma_\delta = \sqrt{\sigma_\nu^2 + \sigma_\epsilon^2}$. Using $z = s + \delta$, we get,

$$\begin{aligned} E[z \mid y = \bar{y}, z \geq \bar{z}] =& E\left[s + \delta \;\middle|\; s = \frac{\bar{y} - (1-\alpha_0)\mu}{\alpha_0},\ s + \delta \geq \bar{z} \right] \\ =& \frac{\bar{y} - (1-\alpha_0)\mu}{\alpha_0} + E\left(\delta \;\middle|\; \delta \geq \frac{\alpha_0 \bar{z} - \bar{y} + (1-\alpha_0)\mu}{\alpha_0} \right) \\ =& \sigma_0 \big(-\xi_0 + E(\xi \mid \xi \geq \xi_0) \big) + \bar{z} \\ =& \sigma_0 \left[-\xi_0 + \frac{\phi(\xi_0)}{1 - \Phi(\xi_0)} \right] + \bar{z}. \end{aligned}$$

where $\xi \sim \mathcal{N}(0,1)$, and

$$\xi_0 = \frac{\alpha_0 \bar{z} + (1-\alpha_0)\mu - \bar{y}}{\alpha_0 \sigma_0}.$$

Likewise,

$$\begin{aligned} h_z =& E\big[E(\theta \mid y, z) \mid y \geq \bar{y}, z = \bar{z} \big] \\ =& E[a_0 y + b_0 z \mid y \geq \bar{y}, z = \bar{z}] \\ =& a_0 E[y \mid y \geq \bar{y}, z = \bar{z}] + b_0 \bar{z}, \end{aligned}$$

and,

$$\begin{aligned} E[y \mid y \geq \bar{y}, z = \bar{z}] =& (1-\alpha_0)\mu + \alpha_0 E\left(s \;\middle|\; s \geq \frac{\bar{y} - (1-\alpha_0)\mu}{\alpha_0},\ s + \delta = \bar{z} \right) \\ =& \alpha_0 \bar{z} + (1-\alpha_0)\mu - \alpha_0 E\left(\delta \;\middle|\; \delta \leq \frac{\alpha_0 \bar{z} - \bar{y} + (1-\alpha_0)\mu}{\alpha_0} \right) \\ =& \alpha_0 \sigma_0 \big(\xi_0 - E(\xi \mid \xi \leq \xi_0) \big) + \bar{y} \\ =& \alpha_0 \sigma_0 \left[\xi_0 + \frac{\phi(\xi_0)}{\Phi(\xi_0)} \right] + \bar{y}. \end{aligned}$$

Therefore, $h_y = h_z$ is equivalent to,

$$b_0 \left(-\xi_0 + \frac{\phi(\xi_0)}{1 - \Phi(\xi_0)} \right) = a_0 \alpha_0 \left(\xi_0 + \frac{\phi(\xi_0)}{\Phi(\xi_0)} \right).$$

Since $a_0 \alpha_0 / b_0 = \sigma_\nu^2 / \sigma_\epsilon^2$, the above equation can be rewritten,

$$\frac{\phi(\xi_0)}{1 - \Phi(\xi_0)} - \frac{\sigma_\nu^2}{\sigma_\epsilon^2} \left(\frac{\phi(\xi_0)}{\Phi(\xi_0)} \right) = \left(\frac{\sigma_\nu^2}{\sigma_\epsilon^2} + 1 \right) \xi_0,$$

which is equivalent to (49b), that is,

$$0 = (\Phi(\xi) - \lambda) \frac{\phi(\xi)}{(1 - \Phi(\xi))\Phi(\xi)} - \xi \equiv f(\xi; \lambda),$$

where $\lambda = \sigma_\nu^2/(\sigma_\nu^2 + \sigma_\epsilon^2)$.

It remains to show that $f(x;\lambda) = 0$ has a solution for every $\lambda \in (0,1)$. To perform this task, we study f's limiting behavior.

$$\lim_{x \to +\infty} f(x;\lambda) = \lim_{x \to +\infty} \left[\frac{\phi(\Phi - \lambda) - x\Phi(1-\Phi)}{\Phi(1-\Phi)} \right].$$

Now, this ratio goes to 0/0 since $x(1 - \Phi(x)) \to 0$. Using l'Hôpital's rule, and $\phi'(x) = -x\phi(x)$, we get

$$\begin{aligned}
\lim_{x \to +\infty} f(x;\lambda) &= \lim_{x \to +\infty} \left[\frac{-x\phi(1-\lambda-\Phi) + \phi^2 - \Phi(1-\Phi)}{\phi(1-2\Phi)} \right] \\
&= \lim_{x \to +\infty} \left[\frac{x(1-\lambda-\Phi) - \phi}{(2\Phi - 1)} \right] + \lim_{x \to +\infty} \left[\frac{(1-\Phi)}{(2\Phi-1)} \right] \lim_{x \to +\infty} \left(\frac{\Phi}{\phi} \right) \\
&= \frac{(+\infty)(-\lambda) - 0}{1} + \frac{0}{1}.0 = -\infty.
\end{aligned}$$

This is because, using l'Hôpital's rule again,

$$\lim_{x \to +\infty} \left(\frac{\Phi}{\phi} \right) = \lim_{x \to +\infty} \left(\frac{\phi}{-x\phi} \right) = 0.$$

The same type of reasoning would show that,

$$\lim_{x \to -\infty} f(x;\lambda) = +\infty.$$

Therefore, f being continuous, by the Intermediate Value Theorem, there exists a point $\xi_0(\lambda)$ such that $f(\xi_0(\lambda), \lambda) = 0$ for every λ in $(0,1)$. In addition, it is not difficult to check that $\xi_0(\lambda) \to +\infty$ if $\lambda \to 0$ and $\xi_0(\lambda) \to -\infty$ if $\lambda \to 1$. Equations (49a) and (49c) are immediate consequences of the definition of ξ_0^*, of the condition $x = q$, and of the fact that q is strictly decreasing with respect its arguments $\bar{y}$ and $\bar{z}$.
Q.E.D.

Proof of Proposition 6.

Using the results obtained in the proof of Proposition 5, if $\sigma_\nu \to 0$, then $b_0 \to 0$, $a_0 \to 1$, $h_z \to \bar{z}$ and $h_y \to E[z \mid z \geq \bar{z}, y = \bar{y}]$. At the optimum, since $h_z = h_y$, it must then be true that

$$E[z \mid z \geq \bar{z}^*, y = \bar{y}^*] \to \bar{z}^*.$$

But this is possible only if $\bar{z}^* \to +\infty$ and (or) $\bar{y}^* \to -\infty$. From (49d), (50b) and $C_v = 0$, we get,

$$p^* = -q\Delta_x + C_x + (y - h_y).$$

If σ_ν is sufficiently close to 0, we get $p^* < 0$ because $(\bar{y}^* - h_y) \sim (\bar{y}^* - \bar{z}^*) \to -\infty$.
Q.E.D.

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