

# Master PPD - Public Economics: Tax & Transfer Policies

Final Exam, March 10, 2015 - 17h30-19h30

*The exam is 2 hours long and can be done either in French or English. **No** document whatsoever is allowed.*

## **Exercise 1 : Global warming and carbon taxes (7 points)**

### **Part A : Pigouvian Corrective Taxation (5 points)**

Consider an economy with a continuum of agents  $i$  in  $[0, 1]$ . There are two goods : a non-energy good ( $c$ ) and an energy good ( $x$ ). Each agent has the same utility function :

$$U_i = U_i(c_i, x_i, X) = c_i^{1-\alpha} \cdot x_i^\alpha \cdot X^{-\lambda}$$

where :

- $c_i$  is the individual consumption level of the non-energy good,
- $x_i$  is the individual consumption level of the energy good,
- $X$  is the aggregate consumption level of the energy good,
- $0 < \alpha < 1$  and  $0 < \lambda < 1$

In this framework, earnings of individuals are equal to  $y_i$  and the prices of energy and non-energy goods are set to 1.

- 1) Why do  $x$  and  $X$  affect differently the utility of individuals? (1 point)
- 2) Compute the optimal levels of  $c_i$  and  $x_i$  in a laissez-faire economy. (1 point)
- 3) In a planned economy, a social planner wants now to maximize social welfare of individuals. Define the social welfare function and the aggregate budget

constraint of the social planner.(0.5 point)

*Notes :  $C$  and  $X$  are respectively the aggregate levels of consumption of the non-energy good and of the energy good.  $Y$  is the national income, i.e aggregate earnings of all individuals.*

4) Compute the socially optimal values of  $C$  and  $X$  in the planned economy. (0.5 point)

5) Show and explain briefly why the consumption of energy good is lower in the planned economy than in the laissez-faire economy. (0.5 point)

In order to implement the social optimum, the social planner introduces a corrective tax  $\tau$  on energy consumption. For simplicity, we assume that the revenues from the tax are not redistributed to individuals.

6) Define the new budget constraint of individuals and compute the optimal levels of  $c_i$  and  $x_i$ .(1 point)

7) Compute the tax rate  $\tau^*$  that allows to obtain the socially optimal value of consumption of the energy good. (0.5 point).

### **Part B : Modified golden rule (2 points)**

In the theoretical framework, the economic costs of global warming depend on  $\lambda$  but also on the modified golden rule :  $r^* = \delta + \gamma \cdot g$ .

8) What are the parameters  $\delta$ ,  $\gamma$  and  $g$ ? (1 point)

9) What is the implication of a higher economic growth rate? (0,5 point)

10) What is the implication of a higher  $\gamma$ ? (0,5 point)

### **Exercise 2 : Optimal capital taxation (8 points)**

Consider a discrete-time closed economy economy with a continuum  $[0; 1]$  of infinite-horizon dynasties.

For simplicity, assume a two-points distribution of wealth : dynasties can either own a large capital stock  $k_t^A$  or a zero capital stock  $k_t^B = 0$ . The proportion of high-capital dynasties is exogenous and equal to  $\lambda$  (and the proportion of zero-capital dynasties equal to  $1 - \lambda$ ).

Zero-wealth dynasties have only labor income, which they consume entirely (zero savings). High-wealth dynasties are the only dynasties to own wealth and to save. Assume they maximize a standard dynastic utility function :

$$U_t = \sum_{t \geq 0} \frac{U(c_t)}{(1 + \theta)^t}$$

All dynasties supply exactly one unit of (homogeneous) labor each period. Output per labor unit is given by a standard production function  $f(k_t)$  ( $f'(k) > 0, f''(k) < 0$ ), where  $k_t$  is the average capital stock per capita of the economy at period  $t$ .  $r_t$  and  $w_t$  are respectively the interest rate and wage rate. Markets for labor and capital are assumed to be fully competitive.

1) Compute the marginal products of capital and labor in function of  $r_t$  and  $w_t$ . (0.5 point)

2) Explain why interest rate  $r^*$  should be equal to  $\theta$  at the steady-state? (No computation is required.) (1 point)

The government wants to introduce a linear redistributive capital taxation into this model. That is, capital income  $r_t \cdot k_t$  of the capitalists is taxed at tax rate  $\tau$ , and the tax revenues are used to finance a wage subsidy  $s_t$  to all workers.

3) What does the Golden rule of capital accumulation imply at the steady-state? Explain the effect of the introduction of the capital taxation on the wealth of capitalist dynasties. (1 point)

4) What is the long run income of the workers  $y_t$  equal to? (0.5 point)

5) what is the capital tax rate  $\tau$  maximizing workers' income  $y_t^*$ ? Explain the result. (1 point)

6) How does the elasticity of capital supply play on this result ? (1 point)

7) Why is the optimal capital tax rate equal to 0 in the Atkinson-Stiglitz model ? (Explicit the hypothesis made by the authors) (1.5 points)

8) According to you, what are the rationales for capital taxation ? (1.5 points)

## Exercise 2 : Optimal linear taxation (5 points)

We consider an economy made up of individuals who have identical preferences defined over consumption  $c$  and labor  $l$ , but different wage rates  $w_i$ . Assume that each agent  $i$  has utility function :

$$u(c, l) = c - \frac{l^{1+\mu}}{\mu + 1}$$

where  $\mu > 0$  is a given fixed parameter.

An individual with wage rate  $w$  supplying labor  $l$ , earns  $z = w \cdot l$  (pre-tax earnings) and consumes  $c = z - T(z)$  where  $T(\cdot)$  is the (possibly nonlinear) income tax.

1) How has the top marginal income tax rate evolved in the U.S. since 1900 ? (1 point)

2) Consider a linear income tax system  $T(z) = -R + \tau \cdot z$  where  $R > 0$  is the lump-sum income transfer and  $\tau$  is a flat tax rate. Compute the optimal level of labor supply that agent  $i$  makes (1 point).

3) Show that the tax rate maximizing total tax revenue is equal to  $\tau^* = \frac{1}{1 + 1/\mu}$  (taking  $R$  as given). (1 point)

4) What is the parameter  $\frac{1}{\mu}$  ? Interpret the formula. (1 point)

5) In Piketty-Saez-Stantcheva, « Optimal Taxation of Top Labor Incomes : A Tale of Three Elasticities » (AEJ 2013), the authors compute an « augmented » optimal tax formula such as :

$$\tau = \frac{1 + t \cdot a \cdot e_2 + a \cdot e_3}{1 + a \cdot e}$$

What is the meaning of  $e_2$  and  $e_3$  ? (1 point)