

## LIFETIME INEQUALITY AND TAX PROGRESSIVITY WITH ALTERNATIVE INCOME CONCEPTS

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The increased availability of cohort data and use of dynamic microsimulation models means that more attention is now being paid to longer term income concepts. Results are usually reported for only one income concept and a limited number of summary measures, and it is not clear whether results are influenced by the income concept used. This paper uses simulation methods to compare different approaches. For this purpose a very simple life-cycle earnings model was used to generate profiles of pre-tax incomes. Many comparisons were made using a flexible tax structure and four alternative income concepts. It was found that there was a substantial amount of agreement among the alternative concepts in making pairwise comparisons, with tentative support for the use of present values.

### I. INTRODUCTION

Studies of the redistributive impact of taxes and transfers over the lifetime are based on measures of inequality and progressivity of one of a number of alternative income concepts. For example, in studying the redistributive impact of government pension schemes, it is usual to base comparisons on the distribution of present values of gross and net income; see Creedy (1982), and Creedy, Disney, and Whitehouse (1993). A different approach has been used by some of those constructing large-scale micro-simulation models; for example, Harding (1994) based comparisons of taxes and transfers on the annual average income of individuals in alternative schemes. Other measures are available, such as the annuity that could be financed over life with the individual's wealth, or the constant value of consumption that gives the same lifetime utility as the actual stream of consumption. The latter approach requires assumptions about the preference patterns of individuals. A feature of the lifetime redistributive impact of taxes and transfers is that there is differential mortality. In computing present values, no special adjustment is made for the fact that the income and consumption streams of individuals differ in length, but the other measures allow in quite different ways for such differences.

Given the variety of alternative approaches, the important question arises of whether the conclusions reached by empirical or simulation studies are likely to be affected by the income concept used. The purpose of the present paper is to examine a set of comparisons, based on the application of a variety of tax structures to the simulated earnings of a cohort of individuals. For simplicity, it will be assumed that all individuals in the cohort enter the labour market at the same age, 20 years, and survive at least until retirement, at age 65. A process of

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differential mortality is assumed such that those with relatively higher annual average real (gross) earnings live, on average, relatively longer than those with lower average earnings. The issue may thus be stated as follows. Given an earnings stream over the periods,  $t=1, \dots, L$ , for the  $i$ -th member of a cohort,  $i=1, \dots, N$  of  $y_{it}$ , the combined effect of savings and taxes produces a stream of consumption of  $c_{it}$  (for  $t=1, \dots, T_i$  and  $T_i \geq L$ ). In producing summary measures of the streams, do comparisons depend on the income concept used? The complexity of the transformations between pre- and post-tax distributions makes the search for analytical results intractable. Hence simulations are required using a wide range of structures.

The literature on measuring the progressivity of taxes and transfers is almost all based on an annual or single-period measure of income; for a survey see Lambert (1993). The present paper also examines the performance of alternative progressivity measures applied to the different income concepts. A feature of the present context is that the application of the same tax system to all members of the cohort can lead to re-ranking of individuals when comparing net and gross income. This cannot occur in a single-period framework when all individuals face the same tax schedule. However, income variability over time combined with the separate taxation of each period's income (that is, no income averaging for tax purposes) can lead to a re-ranking of individuals.

The alternative income concepts are presented in section II. The simulation model used to generate earnings and consumption profiles is described in section III, which also describes the model of differential mortality. Inequality and progressivity comparisons are made in section IV. Brief conclusions are presented in section V.

## II. ALTERNATIVE INCOME CONCEPTS

Consider a single cohort of individuals, all members of which work for  $L$  years, obtaining nominal gross earnings in each year of  $y_{it}$ , for  $t=1, \dots, L$  and  $i=1, \dots, N$ . The assumptions that the working life is the same for all individuals and that no deaths take place until retirement begins could be relaxed with an obvious modification of the following formulae. Suppose also that the nominal interest rate,  $r_n$ , and the inflation rate,  $p$ , remain constant, so that the real rate of interest,  $r_r$ , is also constant and is given by:

$$(1) \quad r_r = (1 + r_n) / (1 + p) - 1.$$

Nominal earnings  $y_i$  are subject to a tax system in each year. Furthermore, individuals save for retirement and the interest from savings may or may not be subject to taxation. The precise nature of the tax system, and the way it is indexed over time to allow for inflation, need not be specified at this stage. If the  $i$ -th individual lives for  $d_i$  years after retirement, the length of "life," measured from entry into the labour force, is given by  $L + d_i = T_i$ . The tax scheme, combined with savings, transforms the nominal earnings stream into a stream of net consumption. It is convenient to refer to this stream of net consumption as one of "net income."

The simplest measures of the stream of gross earnings are as follows. The *present value of gross real earnings*,  $Y_i$ , is given for each individual by:

$$(2) \quad Y = \sum_{t=1}^L y_t / (1 + r_n)^{t-1}$$

where individual subscripts have been omitted for convenience. Precisely the same form is obtained if earnings in each year are first converted into real terms using the price index, and then the real rate of interest is used. It is, however, necessary to work explicitly in terms of nominal earnings if there is an interest-income tax, since nominal rather than real interest-income is subject to taxation.

The *annual average gross real earnings*,  $\bar{y}$ , of each individual are given by:

$$(3) \quad \bar{y} = \frac{1}{L} \sum_{t=1}^L y_t / (1 + p)^{t-1}$$

A common approach is to compare the dispersion of  $Y$ , the present value of gross real earnings, with that of the *present value of net real income* (or consumption),  $C$ , making no special allowance for the fact that individuals live for different periods. The latter measure is given for each individual by:

$$(4) \quad C = \sum_{t=1}^T c_t / (1 + r_n)^{t-1}$$

where, as before, individual subscripts are omitted from  $C$ ,  $T$  and  $c_t$ .

Some studies compare the inequality of the undiscounted average real gross earnings, based on  $y_t$  as in equation (3), with the equivalent measure of the flow of net income or consumption  $c_t$  ( $t = 1, T$ ). The *annual average real net income*,  $\bar{c}$ , is given by:

$$(5) \quad \bar{c} = \frac{1}{T} \sum_{t=1}^T c_t / (1 + p)^{t-1}$$

Comparisons would be made between average gross earnings and  $\bar{c}$ . In this type of comparison, neither the net nor the gross income stream is discounted, so that the shapes of the time profiles are ignored. However, the shape of the profile of  $y_t$  (for  $t = 1, \dots, L$ ) will have a role in the transformation from  $y_t$  to  $c_t$ , because it will influence the time stream of savings and their accumulation, as well as affecting the tax payments. Those individuals whose incomes are more variable over the working life will pay more tax as they will move into higher tax brackets in some years, compared with someone with the same annual average, but constant, income stream. Some studies, such as Harding (1994), which use annual average concepts do not however allow for savings and interest income taxation in examining lifetime redistribution.

### *Annuity Measures*

A different approach involves the transformation of each time-stream into a single measure based on an annuity. Thus, a *constant* value is produced which has the same present value as the actual time profile. This is achieved using the

following standard result, where  $r$  is the relevant rate of interest, with the discount factor,  $v=1/(1+r)$ . If  $a_T$  denotes the present value of an annuity in which  $T$  payments of \$1 are made, the first received at the end of the first year, then:

$$(6) \quad a_T = (1 - v^T)/r.$$

The conversion of a present value to an annuity is carried out by dividing the present value by  $a_T$ .

The flow of gross income  $y_t$  ( $t=1, L$ ) would need to be converted, if there were no taxes, into an annuity which is capable of lasting the whole of the lifetime, that is,  $T$  periods. Hence the two relevant annuities,  $A_y$  and  $A_c$  for the gross and net profiles respectively, are defined as:

$$(7) \quad A_y = Yr_r \left\{ 1 - \left( \frac{1}{1+r_r} \right)^T \right\}^{-1}$$

$$(8) \quad A_c = Cr_r \left\{ 1 - \left( \frac{1}{1+r_r} \right)^T \right\}^{-1}.$$

It might be argued, however, that an annuity as calculated above may not give the individual the same lifetime utility as the actual stream of net income, depending on the inter-temporal elasticity of substitution and the rate of time preference. A constant amount giving the same lifetime utility as the actual stream was used by Nordhaus (1973) and is called the *utility equivalent annuity*. This concept is similar to an equally distributed "equivalent" measure used in the measurement of inequality. With a concave utility function, income fluctuations are to a certain extent "wasteful" and a constant annual amount which is less than the arithmetic mean income would provide the same total utility for the individual. The calculation of utility equivalent annuities obviously requires strong assumptions about each individual's preferences. Nordhaus (1973) assumed that all individuals have the same tastes, but in section III a specification for the joint distribution of taste parameters is introduced.

The utility equivalent annuity can be derived as follows. Define  $\beta = -(1 - 1/\eta)$ , where  $\eta$  is the inter-temporal elasticity of substitution, and let  $\xi$  denote one plus the individual's time preference rate. Write an individual's lifetime utility,  $U$ , as:

$$(9) \quad U = \sum_{t=1}^T \xi^{-(t-1)} c_t^{-\beta}.$$

If  $\delta$  is one plus the real rate of interest and  $C$  is, as above, the present value of net real income, then utility is maximized subject to the lifetime budget constraint:

$$(10) \quad \sum_{t=1}^T \delta^{-(t-1)} c_t = C.$$

This formulation assumes that there is no interest income tax and that capital markets are "perfect," so that corner solutions can be ignored. The solution to

this problem is given by:

$$(11) \quad c_t = \left( \frac{\delta}{\xi} \right)^{\eta(t-1)} (qC),$$

$$(12) \quad q = \left\{ \sum_{t=1}^T (\xi \delta^\beta)^{-\eta(t-1)} \right\}^{-1}.$$

It can be shown that the utility equivalent annuity,  $A_c^U$  is given by:

$$(13) \quad A_c^U = qC \left\{ \left( \frac{1-\xi^{-1}}{1-\alpha} \right) \left( \frac{1-\alpha^T}{1-\xi^{-T}} \right) \right\}^{-1/\beta},$$

$$(14) \quad \alpha = (\delta/\xi)^{\eta/\delta}.$$

The annuity in the absence of taxes and transfers,  $A_y^U$ , is given by replacing  $C$  in (13) by  $Y$ .

### III. A SIMULATION MODEL

#### *Generating Earnings Profiles*

This section presents a simulation model which can be used to generate the profiles of gross and net income for individuals in a single cohort. The components of the model include the earnings profiles, the process of differential mortality and retirement income, along with income taxation. The model uses few parameters and has a very simple structure, but is adequate for present purposes given the focus on the performance of alternative income concepts. The tax system is highly stylized but very flexible and is described by a small number of parameters.

Individuals' earnings are modelled as consisting of a systematic component which follows the growth pattern of the geometric mean of earnings in each year and a random component which introduces a measure of earnings mobility. Relative earnings are defined as the ratio of earnings  $y_{it}$  to geometric mean earnings  $m_t$  in the year  $t$ . Let  $u_{it}$  be a random variable which is distributed independently of income and previous proportional changes, then if  $z_{it} = \log (y_{it}/m_t)$  the generating process is:

$$(15) \quad z_{it} - z_{i,t-1} = u_{it}.$$

If  $u_{it}$  has a constant variance of  $\sigma_u^2$  and if  $\sigma_t^2$  denotes the variance of  $z_{it}$  then (15) implies that the variance of the logarithms of income in each year grows linearly over time. The geometric mean of earnings over time is assumed to follow a typical parabolic pattern. Let  $\mu_t$  denote the logarithm of the geometric mean income in year  $t$ , then:

$$(16) \quad \mu_t = \mu_0 + \theta t - \gamma t^2.$$

It is possible to allow for productivity growth by assuming that every worker benefits equally, so that the rate of growth can be added to the parameter  $\theta$ . To generate lifetime earnings profiles for a set of individuals, rewrite (15) as:

$$(17) \quad y_{it} = y_{i,t-1} \exp \{ (\mu_t - \mu_{t-1}) + u_{it} \}$$

which can be used to generate the  $y_{it}$ s given a set of random variates from an

TABLE 1  
PARAMETER ESTIMATES OF AGE-EARNINGS  
PROFILES

| Parameter    | Estimate |
|--------------|----------|
| $\sigma_v^2$ | 0.1817   |
| $\sigma_u^2$ | 0.00575  |
| $\mu_0$      | 9.612    |
| $\theta$     | 0.0385   |
| $\gamma$     | 0.00086  |

*Source:* Creedy (1992, pp. 51-53).

$N(0, \sigma_v^2)$  distribution. To generate  $y_{it}$ , earnings in the first year of working life (set at age twenty), suppose that  $v_i$  is randomly selected from the standard normal distribution,  $N(0, 1)$ . Then  $y_{it} = \exp(\mu_0 + v_i \sigma_u)$ .

The five parameters of the model were estimated using information about the age and earnings of a sample of males using the Australian *Income Distribution Survey* 1985/86. The full results are reported in Creedy (1992), but the relevant point estimates are shown in Table 1.

### *Differential Mortality*

Age at death is assumed to vary systematically with annual average real earnings relative to the (geometric) mean annual average earnings of the cohort, such that those with relatively high lifetime earnings tend to live longer. Only those who survive to retirement are considered so there are no deaths before  $L$ , which is set at sixty-five for each individual. If, as above,  $d_i$  is the number of years person  $i$  survives after retirement,  $d$  is the average of the  $d_i$ s,  $x_i$  is person  $i$ 's annual average gross real earnings as defined in equation (3),  $x_g$  is the geometric mean value of the  $x_i$ s, and  $e_i$  is a random variable distributed as  $N(0, \sigma_e^2)$ , the relationship between earnings and the number of years of retirement is:

$$(18) \quad d_i = d + \tau \log(x_i/x_g) + e_i.$$

The value of  $d$  was set at 14 years to give an expectation of life of 79 years. The values of  $x_i$  and  $x_g$  are obtained directly from the lifetime earnings simulations. After experimentation, a value of 8 for  $\tau$  and 50 for  $\sigma_e^2$  were found to give a good fit to the survival curve for Australian males. This specification of differential mortality was first used by Creedy (1982) and has recently been found to provide a good fit to available data; see Creedy, Disney, and Whitehouse (1993).

### *Retirement Income*

A large variety of superannuation arrangements is available, but since this paper is not directly concerned with these complexities, the very simple assumption is made that accumulated savings are converted into an annuity at retirement. Savings are simply assumed to be a constant proportion of disposable income in

each period. What is not saved each year of the working life is spent, and savings are accumulated at a fixed nominal rate of interest. No allowance is made for bequests. The assumption that the retiree converts wealth at retirement into an annuity means that he takes a constant real amount each year from the stock of wealth. This does not constitute a constant yearly level of consumption in the presence of an interest income tax, but the assumption avoids the computational difficulties associated with calculating the real amounts that would allow a constant level of consumption.

### *Income Taxation*

A very flexible functional form to describe the income tax structure is given by the following expression. Tax,  $T(y)$ , is subject to a “standard rate,”  $t_1$ , between the thresholds  $a_1$  and  $a_2$ , and thereafter, marginal rates increase non-linearly towards a maximum of  $t_2$ , such that:

$$\begin{aligned}
 (19) \quad T(y) &= 0 & y < a_1 \\
 &= t_1(y - a_1) & a_1 \leq y \leq a_2 \\
 &= t_1(a_2 - a_1) + (t_2 - hy^{-k})(y - a_2) & y > a_2
 \end{aligned}$$

with

$$(20) \quad h = (t_2 - t_1)a_2^k.$$

The parameter  $k$  is less than one in order to ensure concavity of the tax function. The relation in (20) ensures that the marginal rate at  $a_2$ , beyond which marginal tax rates increase, is equal to  $t_1$ . As  $k$  increases, marginal rates increase more rapidly towards the maximum of  $t_2$ .

Some of the comparisons reported below involve the use of transfer payment, in the form of a minimum income guarantee (MIG). Hence, those whose net income after income taxation is below a specified level receive a transfer payment in order to bring their disposable income up to the specified level.

### *Heterogeneous Preferences*

The utility equivalent annuity requires assumptions to be made about each individual's values of  $\xi$  and  $\eta$ . Where this concept has previously been used, the assumption has been made that all individuals have the same preferences. However, it is of interest to allow for some heterogeneity by assuming that  $\xi$  and  $\eta$  are jointly distributed. It is of course unlikely that direct information about the precise extent of population heterogeneity will ever become available, though the assumption of identical tastes is obviously unrealistic. Any assumption is obviously highly arbitrary, but the following specification is offered as a convenient and flexible method of generating a simulated distribution. Suppose that  $\xi$  and  $\eta$  are jointly lognormally distributed as  $\Lambda(\xi, \eta | \mu_\xi, \sigma_\xi^2, \mu_\eta, \sigma_\eta^2, \rho)$ , where  $\mu$  and  $\sigma^2$  are the mean of logarithms and variance of logarithms of the relevant variables and  $\rho$  is the correlation between the logarithms of  $\xi$  and  $\eta$ . The following procedure can then be used to generate a set of values. Given a random observation  $u_i$  from a  $N(0, 1)$  distribution, person  $i$ 's value of  $\xi$  can be obtained from the

marginal distribution using  $\xi_i = \exp(\mu_\xi + u_i\sigma_\xi)$ . The value of  $\eta$  is obtained from the corresponding conditional distribution using

$$(21) \quad \eta_i = \exp[\mu_\eta + \rho(\sigma_\eta/\sigma_\xi)\{\log \xi_i - \mu_\xi\} + v_i\sigma_\eta(1 - \rho^2)^{0.5}].$$

Where  $v_i$  is another random observation from an  $N(0, 1)$  distribution. This completes the description of the simulation model.

#### IV. SIMULATION RESULTS

This section produces simulation results for alternative income concepts and summary measures of inequality and progressivity. The various summary measures are described in the following subsection. The procedure adopted is to examine the measures for alternative tax structures. A wide range of comparisons are made to see if the alternative measures and income concepts give different results. The first stage of the analysis is to generate the earnings profiles. Using the earnings parameters from Table 1, with additional productivity growth of 0.015, the earnings of 1,000 individuals were generated. The nominal rate of interest used was 0.10 and inflation was 0.07. The distribution of taste parameters was specified as follows. The arithmetic mean values of  $\xi$  and  $\eta$  are set at 0.05 and 1.8 respectively, while their variances (of logarithms) are 0.002 and 0.01. These variances are small in absolute terms but they translate into a substantial amount of taste heterogeneity. The correlation coefficient between the logarithms of  $\xi$  and  $\eta$  is assumed to be  $-0.75$ . The tax thresholds were indexed at the rate of 0.07 per year.

The assumption was made that each individual saves a fixed proportion, 0.15, of net income in each year of the working life. This assumption regarding saving behaviour is obviously simplistic, but unfortunately very little information about savings patterns with age and income is available in Australia. Experiments were carried out using a "humped savings" profile and also allowing the savings rate to increase with income, but these did not affect the results. The computer programs used to produce the simulations and comparisons, allowing for alternative structures and saving assumptions, are available from the author.

##### *Progressivity Measures*

The progressivity measures used are described briefly as follows; for more details see Lambert (1993). For notational convenience, suppose that as above  $y_i$  is a measure of individual  $i$ 's pre-tax income, and this is transformed into post-tax income  $c_i$ . Furthermore, individuals are ranked in ascending order so that  $y_1 < y_2 < \dots < y_N$ . The Atkinson and Gini inequality measures are used. The former is expressed as  $1 - \bar{y}/y_e$ , where  $y_e$  is the equally distributed equivalent income. That is, social welfare, specified as  $\sum_i y_i^{1-\varepsilon}/(1-\varepsilon)$ , is the same under the actual distribution as when everyone receives  $y_e$ , and  $\varepsilon$  is a measure of the relative inequality aversion of the welfare function. The Gini measure of inequality of



pre-tax income,  $G_y$ , is usefully expressed in terms of the following covariance:

$$(22) \quad G_y = \left( \frac{2}{\bar{y}} \right) \text{Cov} (y, F(y))$$

where  $F(y)$  is the distribution function, with  $F(y_i) = i/N$ . On the use of covariance expressions in this context, see Jenkins (1988). Pre-tax income  $y$  is transformed to  $c$  using the tax system  $c = y - t(y)$ , and the corresponding inequality measure of post-tax income,  $G_c$ , may be obtained in the same way, after re-ranking individuals according to  $c$ . The concentration index,  $C_t$ , of tax paid, is defined by:

$$(23) \quad C_t = \left( \frac{2}{\bar{t}} \right) \text{Cov} (t(y), F(y)).$$

Kakwani's index of progressivity,  $K$ , which focuses on the disproportionality of tax payments is the difference between the tax concentration index and the Gini measure of pretax income:

$$(24) \quad K = C_t - G_y.$$

Jenkins (1988) has shown that the Suits measure of progressivity,  $S$ , can be expressed as:

$$(25) \quad S = \left( \frac{2}{\bar{t}} \right) \text{Cov} (t(y), F_1(y)) - G_y$$

where  $F_1(y)$  is the first moment distribution of  $y$ , with  $F_1(y_i) = \sum_{j=1}^i y_j / \sum_{i=1}^N y_j$ .

It is also of interest to relate the changes in inequality to a measure of social welfare, in view of the fact that specific welfare rationales are available for the Atkinson and Gini measures of inequality. The rationales differ significantly, but both give rise to an "abbreviated" social welfare function, when the latter is defined in terms of only arithmetic mean income and inequality. The abbreviated social welfare function based on the inequality measure,  $I$ , and arithmetic mean,  $\mu$ , is, for both Atkinson and Gini inequality measures, expressed as:

$$(26) \quad W = \mu(1 - I).$$

Social welfare for a proportional tax which raises the same net revenue as the actual tax,  $W_p$ , is given by:

$$(27) \quad W_p = (1 - g)\bar{y}(1 - I_y).$$

The social welfare from the progressive tax,  $W_c$ , is, since  $\bar{c} = (1 - g)\bar{y}$ , given by:

$$(28) \quad W_c = (1 - g)\bar{y}(1 - I_c).$$

The welfare "premium,"  $\Pi$ , from progressive tax is thus  $W_c - W_p$  and is, from Lambert (1993) given by:

$$(29) \quad \Pi = (1 - g)\bar{y}(I_y - I_c).$$

Other forms of social welfare functions are available although they are not used below. For example, Kakwani (1980) has provided a rationale for  $W = \mu / (1 + G)$

which gives rise to a welfare premium from progression equal to  $(1-g)\mu_y(G_y - G_c)/(1+G_y)(1+G_c)$ .

### *Simulation Results*

In view of the fact that analytical results cannot be obtained, it is important to consider a wide variety of tax structures using the flexible functional form introduced in the previous section. Values of the Gini and Atkinson measures of inequality of net income, the Kakwani and Suits progressivity measures, and the Gini and Atkinson based welfare premia from progression, were calculated for a simulated cohort of individuals for each tax structure and income concept. The degree of inequality aversion used for the Atkinson measure was 0.5; for further discussion of appropriate orders of magnitude, see Amiel and Creedy (1994). The tax function contains five parameters, and "high" and "low" values of each parameter were specified: these were as follows. The low values of  $a_1$ ,  $a_2$ ,  $t_1$ ,  $t_2$  and  $k$  were 5,000, 10,000, 0.25, 0.6 and 0.5 respectively, while the high values were 10,000, 20,000, 0.3, 0.8 and 0.8 respectively. All combinations of these values gave a total of 32 tax structures, covering a wide range of types of structure. With  $N$  tax structures there are  $N(N-1)/2$  pairwise comparisons, so that 496 pairwise comparisons are made for each measure.

It should be recognized that each of the 32 tax structures produces a different total tax revenue over the lifetime of the cohort. No attempt was made to ensure revenue neutrality. The imposition of revenue neutrality would involve the simultaneous adjustment of some other aspect of the tax system, such as transfer payments, and would require an iterative search process.

The approach used can be described as follows. For each of the 32 tax structures, the six inequality and progressivity measures were available for each of the four income concepts examined. For each type of comparison (inequality, progressivity and welfare premium) a "base measure" was chosen. For example, in examining inequality, the ranking of each of the 496 pairs of tax structures was obtained using the Gini measure of the present value of lifetime income. This ranking was compared, for each pair of tax structures, with that found for the Atkinson measure of the present value; any disagreement was recorded. The ranking of each pair was also compared with that obtained by the use of the Gini and Atkinson measures of each of the other three income concepts. In each case, any disagreement in the ranking, compared with that given by the base measure and income concept, was recorded. Although information is available about precisely which pair of tax structures are involved in cases where the rankings differ, only the aggregate number of "conflicts" are reported below. The exercise was repeated for the progressivity and welfare premium measures.

A summary of results is shown in Table 2. Each entry in the table indicates the number of exceptions, compared with the "base" measure, out of 496 pairwise comparisons; each row thus involves 3,472 comparisons. The table is divided into three blocks, according to the "base" measure used; these are the Gini inequality measure, the Kakwani progressivity measure and the welfare premium using the Gini measure, in each case for the present value income concept. Consider first the inequality measures, where for each pair of tax structures, the Gini measure

TABLE 2  
TAX STRUCTURE COMPARISONS—SUMMARY OF EXCEPTIONS

| Base Measure         |        | Present Values |   | Annual Average |    | Annuity |   | Utility Equivalent Annuity |   |
|----------------------|--------|----------------|---|----------------|----|---------|---|----------------------------|---|
|                      |        | G              | A | G              | A  | G       | A | G                          | A |
| Gini                 | DM (i) | —              | 0 | 10             | 10 | 1       | 1 | 0                          | 0 |
|                      | (ii)   | —              | 0 | 11             | 9  | 0       | 0 | 0                          | 0 |
|                      | CM     | —              | 0 | 12             | 12 | 0       | 0 | 0                          | 0 |
|                      |        | K              | S | K              | S  | K       | S | K                          | S |
| Kakwani              | DM (i) | —              | 6 | 14             | 23 | 1       | 6 | 0                          | 6 |
|                      | (ii)   | —              | 7 | 9              | 20 | 0       | 6 | 0                          | 7 |
|                      | CM     | —              | 7 | 26             | 28 | 0       | 7 | 2                          | 7 |
|                      |        | Premium        |   | Premium        |    | Premium |   | Premium                    |   |
|                      |        | G              | A | G              | A  | G       | A | G                          | A |
| Gini-Welfare Premium | DM (i) | —              | 1 | 6              | 14 | 0       | 1 | 0                          | 1 |
|                      | (ii)   | —              | 1 | 4              | 13 | 0       | 1 | 0                          | 1 |
|                      | CM     | —              | 1 | 7              | 4  | 0       | 1 | 0                          | 1 |

*Notes:* G: Gini. A: Atkinson. K: Kakwani. S: Suits. DM: Differential Mortality. Case (i)—No transfer; Case (ii) MIG of \$9,000. CM: Common mortality with no transfer payment. Inequality Aversion coefficient: 0.5. Each entry shows the number of exceptions, compared with the base measure, out of 496 comparisons. Hence each row involves 3,472 comparisons.

of inequality of the present value of net income is taken as the “base case.” When comparing one tax structure with another, the extent to which the same ranking is given by the other income concepts was examined. Similarly, the extent to which the same ranking is given by the Atkinson inequality measure was also examined. The first row of Table 2 refers to the case where there is differential mortality and no transfer payments, and it can be seen that the annual average income measure produces a difference in the ranking of tax structures in ten pairwise comparisons for both Gini and Atkinson inequality measures, compared with the Gini measure of the present value of income. These exceptions were found to involve the same tax structure comparisons for the two inequality measures.

The Gini measure of annual average lifetime net income moves in the opposite direction from the Gini measure of the present value of lifetime income in 10 out of 496 comparisons. The case where a transfer payment is in operation is shown in the second row of each block of Table 2. Here a minimum income guarantee of \$9,000 (indexed in line with inflation) was applied in each year of the working life. The number of exceptions produced by the annual average measures is very similar to the first row, while no exceptions out of 496 pairwise comparisons are found for each of the other concepts and inequality measures. The third row shows the results for the assumption of common mortality (each individual lives 14 years after retirement) with no transfer payments. Again only the annual average measure displays any exceptions.

The middle section of Table 2 reports the number of exceptions when the Kakwani progressivity measure using the present value income concept is used as the base measure. The number of exceptions between income concepts, and

between the Kakwani and Suits measures, is somewhat higher than for the inequality measures. However, the majority of these exceptions are obtained for the annual average income concept. The final set of results shows exceptions for rankings of tax structures according to the welfare premium, taking the Gini-based premium with the present value concept as the base measure.

Each row of Table 2 involves 3,472 comparisons between tax structures and income concepts, so that in producing the table, a total of 31,248 comparisons were made. In aggregate, fewer than 1 percent gave exceptions, and the large majority of these were for the annual average income concept. Investigations showed that the use of homogenous preferences made no difference to the number of exceptions when using the utility equivalent annuity concept. Comparisons using a multi-step tax function produce very similar results (and the computer programs mentioned earlier allow for the choice of tax function, along with alternative progressivity measures).

## V. CONCLUSIONS

The increased availability of cohort data and use of dynamic microsimulation models has meant that more attention is being paid to longer term income concepts. Despite the variety of concepts available and range of inequality and progressivity measures, results are usually reported for only one income concept and a limited number of summary measures. Since different authors use different concepts, it is not clear whether results are in any way influenced by the income concept used. The purpose of this paper has been to provide, using simulation methods, comparisons among different approaches. For this purpose a very simple life-cycle earnings model was used to generate profiles of pre-tax incomes.

It is not possible to obtain analytical results and simulation results, by their nature, cannot be conclusive. For this reason, a very large number of comparisons were made using a higher flexible tax structure, and it was found that there was a substantial amount of agreement among the alternative concepts in making pairwise comparisons. The largest number of exceptions was found for the annual average income concept, perhaps suggesting some caution in the use of this concept. For the other concepts the substantial agreement is a convenient result because it would not be reasonable always to require researchers to produce results for such a range of income concepts, given the computational problems and, in the case of utility equivalent annuities, the arbitrary nature of extra assumptions required. There is thus some tentative support for the use of present values, given the substantial agreement with the more complex annuity concepts.

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