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# Capital income taxation, wealth distribution and borrowing constraints

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## Abstract

The theorem of zero taxation of capital income is reexamined and is shown to hinge critically on the assumptions of a long horizon and perfect markets for the inter-temporal allocation of resources. The theorem does not hold when borrowing constraints prevent individuals from insuring against idiosyncratic shocks and have a precautionary motive for savings. Structural assumptions are made such that with no taxation, aggregate savings are socially ‘excessive’ in the long-run, i.e. the rate of return is smaller than the discount rate. Sufficient conditions for a Pareto efficient taxation or subsidization of capital in the long-run depend on the correlation between individuals’ consumption and savings. A subsidy may be efficient when individuals’ incomes follow a predictable pattern of life-cycles with no negative bequest. © 2001 Elsevier Science B.V. All rights reserved.

*Keywords:* Capital income taxation; Wealth distribution; Pareto efficient tax reform; Borrowing constraints

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## 1. Introduction

The debate on capital income taxation becomes sometimes confused when the public finance view is neglected. This approach emphasizes that the tax on capital is a tax on future consumption (Feldstein, 1978). The flow of capital income is not a base like labor income which could be an opportunity for taxation. The efficiency cost of the tax is caused solely by the wedge on the prices of future consumptions. In a welfare analysis the tax impact on savings or accumulated capital is irrelevant.

Simple accounting shows that a permanent tax on capital at a constant rate creates the same distortion as an ad valorem tax on future consumption at an increasing rate which tends to infinity! It is well known that the structure of efficient taxation depends heavily on the structure of preferences. Standard studies on capital taxation generate indeterminate results because they assume a finite horizon. When the horizon is infinite, the impact of the ever widening tax wedge imposes the convergence of the efficient tax rate to zero (Chamley, 1980; Chamley, 1986)<sup>1</sup>. This result holds even when individuals have different initial endowments and seems to put in doubt the usefulness of the capital income tax for redistribution.

However, the zero tax result rests critically on the possibilities of shifting consumption between any future periods through perfect capital markets. This assumption is not satisfied in any relevant model of income distribution which should incorporate random incomes with borrowing constraints. The purpose of this paper is to reexamine the redistributive role of capital income taxation in such economies.

The issue of the relation between capital accumulation and taxation has been raised again by Aiyagari (1993), who takes the view that the level of capital in the long-run is 'too high' socially because individuals who face a credit constraint save for possible bad draws in the future lotteries of their labor income. In the steady state of the economy the rate of return on capital is smaller than the discount rate. A tax on capital income would reduce capital accumulation and be socially desirable. This intuitive argument is disproved here. The model presented in this paper is structural and generates a level of capital income that is 'too high', in the sense of Aiyagari, and for the same reason: individuals use private capital accumulation to smoothen their consumption path even when the rate of return is lower than the rate of time preference. It will be shown that the impact of capital income taxation is in general ambiguous: whether an anticipated future capital income tax is Pareto efficient depends on the random properties of the labor income.

In the present study, the operating mechanism of the capital income tax is the standard motive for insurance when markets are incomplete. (Previous studies of fiscal policies with uncertainty and incomplete markets include: Eaton and Rosen, 1980; Hellwig, 1980; Mirrlees, 1990). The tax on capital income (positive or negative) may redistribute income from the 'rich' to the poor, i.e. from individuals with a relatively low marginal utility of consumption to their contemporaries who have a relatively high marginal utility of consumption. These marginal utilities may have a positive or a negative correlation with the level of an individual's savings. When the correlation is positive (negative) the optimal tax rate is positive (negative). Both cases are found in the structural models presented here which

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<sup>1</sup>The tax rate is not zero when some incidence falls on pure profits (Correia, 1996). See also Atkeson et al., 1999. For an analysis with endogenous growth, see Chamley, 1992.

assume different random properties for individual labor incomes. Pareto efficient policies can be implemented here because individuals are *ex ante* fairly similar about their position in the long-run. The analysis shows that the sign of the optimal tax may be positive or negative, and there is a simple empirical criterion for the determination of this sign. This criterion indicates that the permanent tax on capital income should probably be positive.

Credit constraints are essential in any discussion of income distribution. In economies with perfect credit markets, income variations could be smoothened, and the problem of income distribution would be greatly alleviated (Loury, 1981). In the present paper, individuals face idiosyncratic variations of period endowment (e.g. labor income), and cannot borrow. Other types of risk such as a consumption risk could be considered (Atkeson and Lucas, 1992). The income uncertainty is chosen here both for its empirical relevance, and because it generates simple models with liquidity constraints and different tax properties. I will not derive the constraints on borrowing and insurance from first principles. My purpose is to focus on the most basic mechanisms which imply that the asymptotic tax rate on capital income may not be equal to zero. The analysis makes a number of assumptions about model structure and methodology which need be stated and discussed now.

(i) Individuals are infinitely lived (or belong to dynasties), have a standard additive utility function and receive in each period a random exogenous income. (Extensions to cases with endogenous labor income will be discussed briefly.)

(ii) The rate of return on capital is fixed and chosen such that with perfect capital markets there would be no saving in the long-run. This assumption is made for two reasons. First, a fixed rate of return is equivalent to that of fixed producer prices and is standard in an analysis of optimal consumer's taxation. We know that the Ramsey rules depend essentially on the structure of preferences. In order to focus on the impact of imperfect markets on the efficient tax rules, producer prices are fixed. The results of the paper do not depend on this expository assumption, as discussed in the concluding section. Second, the model generates no saving in the long-run with perfect markets: the individual motivation for saving is the precaution against low future labor income. The model provides the basis for an analysis of the efficient taxation of precautionary savings.

(iii) Tax reforms are analyzed in the following way. An initial position of an economy is assumed (with or without taxation). A tax change which may be permanent or transitory is announced at time 0 to be implemented at time  $T$ , where  $T$  is sufficiently large. There are two reasons to proceed in this way. First, when  $T$  is large, the initial positions of individuals have little impact on their state in period  $T$ . Hence, the tax reforms (implemented at time  $T$ ), have a qualitative effect which is uniform on all individuals: they are Pareto improving. Second, the focus on the long-run alleviates the well-known problems of time-consistency which arise with sudden changes of tax rules. Note that the welfare criterion will not be limited to the long-run, but will take into account the entire horizon of individuals.

(iv) The taxation or the subsidization of capital income is balanced by a uniform lump-sum transfer in the same period (unless stated explicitly otherwise). This methodological device is important in order to focus on the specific properties of the capital income tax. Models which combine this tax with the public debt or other taxes (such as the wage tax) in order to meet the government budget, generate results which may be driven by the properties that are specific to these other fiscal instruments. This set of fiscal instruments enables us to focus on the trade-off between redistribution with distortions through the capital income tax and the efficient (but inequitable) transfer through lump-sum taxation.

The paper is articulated as follows. Section 2 presents the model and the main result. Whether capital income should be taxed or subsidized in the long-run depends on the correlation between savings and marginal utility function in the long-run. The result can be formulated in terms of Pareto efficiency because all individuals have asymptotically identical positions, *ex ante*.

A first application is given in Section 3 in the context of the standard model with independent random shocks to labor income (Foley and Hellwig, 1975; Shechtman, 1976; Shechtman and Escudero, 1977; Clarida, 1987). In this class of models, capital income should be taxed in the long-run.

Section 4 presents an opposite result in the context of a model with random length of lives and where altruistic agents cannot borrow from their descendants: a subsidy to capital income is Pareto efficient after some future date which does not have to be randomized (in contrast to Section 3).

In conclusion, the structure of the shocks to individuals' labor incomes with a borrowing constraint generate a stationary distribution of wealth and consumption. Whether capital income should be taxed or subsidized depends on the correlation between individual savings and consumption and not on the overall level of savings or the difference between the discount rate and the rate of return.

## 2. The model

There is a continuum of individuals with a total mass normalized to 1. All individuals have the same utility function

$$U = E\left[\sum_{t \geq 0} \beta^t u(c_t)\right], \quad \text{with } u' > 0, u'' < 0, \quad \text{and } \lim_{c \rightarrow 0} u'(c) = \infty \quad (1)$$

Each individual has an exogenous random income  $w_t$  in period  $t$ . It is a function  $w_t = g(x_t)$ , where  $x_t$  is a random variable which is specific to the individual and takes values in the space of the real numbers. For each individual, the cumulative distribution of the determinant of his income  $x_t$ , is a function  $F(x_{t-i})$  of  $x_{t-i}$ . The function  $F$  is the same for all agents, but the random drawings of  $x_t$  are independent across agents for each period.

Individuals can transfer wealth between periods through a positive amount of

savings in capital. Here the technology to transfer resources into the future is assumed to linear, in order to simplify the analysis. The linear assumption does not restrict the generality of the results. A unit of good saved in any period yields an amount of good  $R$  in the next period. As discussed in Section 1, we want to consider a case where the accumulation of capital is ‘excessive’ from a social point of view in the sense that in the steady state the rate of return is strictly smaller than the social discount rate.

**Assumption 1.**  $\beta R < 1$

The distribution of savings in the initial period is given. Its specification is not important since we will focus on long-term properties. Denoting by  $k_t$  an agent’s level of savings carried into period  $t$  and  $w_t$  the labor income in period  $t$ , the accumulation equation between periods is defined by

$$k_{t+1} = Rk_t + w_t - c_t$$

Agents are credit constrained and  $k_t \geq 0$  for any  $t$ . An agent determines his consumption level in period  $t$  after receiving his income  $w_t$ . (One could also assume that consumption takes place before the realization of the random income.) Agents thus face a standard problem of stochastic dynamic programming (Stokey and Lucas, 1989). We will obviously suppose that this problem has a solution. An individual’s level of consumption in any period is determined by a policy function of  $k_t$  and  $x_t$ . By an abuse of notation,

$$c_t = c(k_t, x_t)$$

The consumption function defines a saving function  $s(k_t, x_t)$  such that

$$k_{t+1} = s(k_t, x_t) = Rk_t + w_t - c(k_t, x_t)$$

Denote by  $\Phi_t(k, x; k_0, x_0)$  the conditional cumulative distribution of  $(k, x)$  in period  $t$ , for an individual with initial conditions  $(k_0, x_0)$ , and define the following ergodic property.

**Definition.** (Property  $\mathcal{S}$ ) The model has the property  $\mathcal{S}$  when for any  $k \geq 0$ , if  $t \rightarrow \infty$ ,  $\Psi_t(k, x; k_0, x_0) \rightarrow \Phi(k, x)$  uniformly in  $(k_0, x_0)$ .

The structural models which will be considered in the following sections will be shown to satisfy this assumption.

### 2.1. Capital income taxation

A fiscal policy is implemented as follows: a tax on capital income is announced in period 0 for period  $t$  ( $t > 0$ ) and that period only. The tax is proportional at the

rate  $\theta$  to the savings accumulated at the beginning of period  $t$ . Since the government has no real expenditures and the tax is used only for redistributive purposes its proceeds are distributed by a uniform lump-sum payment which is paid to each agent. The value of  $\theta > 0$  is assumed to be small, and the value of  $t$  will be large (in a sense which will be made more precise). Because  $\theta$  is small, incentive effects can be neglected in a first-order calculus and the welfare effect evaluated in period 0 for an individual in the state  $(k_0, x_0)$ , actualized in period  $t$ , is equal to

$$M_t(k_0, x_0) = \theta E_{(k_0, x_0)} [\bar{k}_t u'(g(k_t, x_t)) - \bar{k}_t u'(g(k_t, x_t))]$$

where the expectation depends on the initial condition  $(k_0, x_0)$ , and  $k_t$  is the level of individual savings. (Since the population is normalized to one and all individuals are identical,  $k_t$  represents the savings of any particular agent or of the whole economy.) The first term originates in the lump-sum transfer while the second represents the effect of the tax on capital. The previous expression is equivalent to

$$M_t = \theta E_{(k_0, x_0)} [(\bar{k}_t - k_t) u'(c(\bar{k}_t, x_t))]$$

If Property  $\mathcal{S}$  holds, the distribution of  $u'(c(k, x))$  converges to a stationary distribution, uniformly with respect to  $(k_0, x_0)$ . Therefore,  $M_t$  converges also uniformly to some limit. If this limit is different from zero, there exists  $T$  such that a tax on capital income for period  $t$ , with  $t > T$ , has an income effect with the same sign for all agents, i.e. the sign of  $\lim_{t \rightarrow \infty} (M_t(k_0, x_0))$ . Hence the next result.

**Theorem 1.** *If Property  $\mathcal{S}$  holds and if an individual's marginal utility of consumption is negatively (positively), correlated with his savings in the long-run, there exists  $T$  such that for any  $t > T$ , a capital levy (subsidy), in time  $t$  announced in period 0, is Pareto efficient.*

The introduction in period 0 of a permanent tax to be implemented after period  $T$  can be decomposed as an infinite series of capital levies for all periods  $t > T$ . Theorem 1 implies the next result.

**Corollary 1.** *If Property  $\mathcal{S}$  holds and if the asymptotic distributions of the individuals' marginal utilities are negatively (positively), correlated with their accumulated assets, there exists  $T$  such that a permanent tax (subsidy), on capital income for  $t > T$ , announced in period 0, is Pareto efficient.*

## 2.2. Discussion

The two previous results indicate only the direction for an efficient tax reform, and are not statements on the magnitude of the tax or the subsidy. They focus on

tax changes from an initial regime with no taxation. By continuity, they apply to an initial regime with a permanent tax rate in a neighborhood of 0. These results support the efficiency of a permanent tax rate (positive or negative), on capital income in two different ways. First, a current permanent tax rate may be the consequence of a past commitment, which was Pareto efficient at the time of its announcement. Second, the announcement of a reduction of the current tax rate, for the future, may reduce the present welfare of all agents. Theorem 1 provides a general criterion for a positive or negative tax rate on capital income. A plausible structural model is presented for each of the two cases in the next sections.

### 3. Independent shocks to labor income

In this section the shocks to labor income are independent. Since there are no market instruments to ensure the random income, individuals get partial self-insurance through saving. The motive for saving is the standard precautionary motive. The next assumption is introduced to simplify the analysis.

#### Assumption 2.

- (i) An individual's random income in period  $t$ ,  $w_t$  is serially independent, and  $w_t \in [0,1]$ . The probability that  $w_t=0$  is equal to  $p>0$ .
- (ii) The utility function  $u$  (in (1)), is such that there exist  $\bar{c}$  and a finite  $\delta>0$ , with

$$-\frac{cu''(c)}{u'(c)} < \sigma \quad \text{for all } c > \bar{c}$$

- (iii)  $R>1$ .

Since  $u'(1) < \beta R p \lim_{c \rightarrow 0} u'(c) = \infty$ , the level of savings at the end of the period for an individual with a positive labor income is greater than some strictly positive lower bound  $\alpha$ . The total capital is therefore greater than  $(1-p)\alpha > 0$ . Note also that an individual faces a strictly positive probability of no income in the next period, and always leaves a strictly positive level of saving. Therefore, the distribution of savings in any period has no mass at 0.

Under Assumption 2 and because labor income  $w_t$  is bounded, one can show as an exercise (Shechtman and Escudero, 1977), that there exists  $k$  such that if  $k_t > k$ , then for any  $w_t$ ,  $k_{t+i} \leq k_t$ . Each item in Assumption 2 has an intuitive interpretation: (i) if the distribution of  $w_t$  were unbounded there could be a string of very high labor incomes which could lead to a high level of capital accumulation; (ii) the upper bound on the elasticity of the marginal utility rules out an unbounded risk aversion which could also lead to an unbounded level of capital accumulation. There exists therefore a value  $k$  such that the distribution of

individual savings is contained in an interval  $[0, k]$  for every period, including the initial one.

We introduce the distribution of  $k_t$  conditional on  $k_0$  in period  $t$ :  $\Psi_t(k; k_0)$ . It tends to a limit  $\Psi(k)$  when  $t$  tends to infinity (see Clarida, 1987, or for a more general exposition, Stokey and Lucas, 1989, Theorem 12.12). The uniform convergence is shown in the next result, which is proven in Appendix A.

**Lemma 1.** *Under Assumption 2, Property  $\mathcal{S}$  holds: for any  $a \in [0, \hat{k}]$ ,  $\Psi_t(a; k_0)$  converges to  $\Psi(a)$  uniformly with respect to  $k_0 \in [0, k]$  when  $t \rightarrow \infty$ .*

The consumption function  $g(Rk + w)$  is increasing in its argument (see, for example, Stokey and Lucas, 1989, Chap. 9, or Laitner, 1992), and the marginal utility  $u'(g(\cdot))$  is therefore decreasing in  $k$ . Since  $w$  is independent on  $k$ ,  $u'$  and  $k$  have a negative correlation in the ergodic distributions. Using Theorem 1, we have the next result.

**Proposition 1.** *Under Assumption 2, there exists  $T$  such for any  $t > T$ , there is a tax on capital income for period  $t$  only, or for all periods  $t > T$  which is Pareto efficient.*

#### 4. Random life-cycles

In this section the time span between generations, within a family, is random. Such a randomness generates an ergodic distribution of states for any period  $t$  which is the same for all families and thus enables Pareto efficient tax reforms.

##### 4.1. The model

There is a continuum of families, and each family is made of consecutive generations. In any period, there is one individual per family, who lives 2 or 3 periods. This individual learns in his first period before his consumption decision, whether he will live 2 or 3 periods. The length of a life time is determined randomly with probability  $\pi$  for a life of 3 periods. Following the last period of an individual's life, another person is born in the same family. The lengths of life of different individuals are independent. All individuals of the same family have the (altruistic) utility function in their first period (taken to be zero):

$$U = E\left[\sum_{t \geq 0} \beta^k u(c_t)\right], \quad \text{with } \beta < 1$$

where  $c_t$  measures the consumption of the living member of the family in period  $t$ , and  $u$  is strictly concave and increasing. A family's income is determined as follows: only the young (individuals in their first period), have an exogenous labor income equal to 1. As in the previous sections, one can transfer resources from one

period to the next with a total return  $R$  per unit of savings, and  $\beta R < 1$ . Individuals cannot borrow against the incomes of their descendants.

In order to determine the optimal programs of consumption, consider two fictitious types of individuals, type 1 who lives 2 periods and type 2 who lives three periods. Each individual  $i$  ( $i = 1, 2$ ), has an income of 1 in the first period only, and the utility function

$$U_i = \sum_{j=0}^{j=i} \beta^j u(c_{ij})$$

where  $c_{ij}$  is the consumption of an individual of type  $i$  in his period  $j$ . We now choose specific values  $c_{ij}$  which satisfy the following first-order conditions and budget constraints:

$$\begin{cases} u'(c_{10}) = \beta R u'(c_{11}), & c_{11} = R(1 - c_{10}), \\ u'(c_{20}) = \beta R u'(c_{21}) = (\beta R)^2 u'(c_{22}), & c_{22} = R(R(1 - c_{20}) - c_{21}) \end{cases} \quad (2)$$

These are the optimal levels of consumption for ‘life-cyclers’ with a life of 2 or 3 periods, respectively.

Define by  $v_{ij}$  the marginal utility of consumption  $v_{ij} = u'(c_{ij})$ , where  $c_{ij}$  are the same as in the previous equations, and by  $k_{ij}$  the savings of an individual of type  $i$  in his period  $j$ . Denoting by  $\lambda$  the product  $\beta R$ ,  $\lambda = \beta R < 1$ , we make the following assumption.

**Assumption 3.**

$$v_{11} > \lambda((1 - \pi)v_{10} + \pi v_{20})$$

Since  $v_{10} = \lambda v_{11} > v_{11}$ , there exists  $\pi^*$  such that if  $\pi < \pi^*$ , the previous assumption is satisfied. It will be discussed further at the end of the section. We have obviously  $v_{22} > v_{11}$ : an individual of type 2 (with three periods), consumes less in his last period than an individual of type 1 in his second period. Therefore,

$$v_{22} > \lambda((1 - \pi)v_{10} + \pi v_{20}) \quad (3)$$

Assumption 3 and the inequality (3) show that with the programs of consumption defined by Eq. (2) an individual of type 1 or 2 who represents a family and is in his last period would like to receive resources from the next generation. This transfer is impossible because he cannot borrow against the income of his descendants. Given the borrowing constraint, a bequest of 0 is optimal and the programs of consumption  $\{c_{10}, c_{11}\}$  and  $\{c_{20}, c_{21}, c_{22}\}$ , which are determined by optimizing life-cyclers, are indeed optimal for the families.

*4.2. The ergodic distribution*

From the previous discussion, in any period each family is in one of five possible states. Each state defines completely the level of savings and of

consumption of the family. Denote by  $y_t$  the vector where the first two components are the probabilities to be young and old with a life span of 2, and the last three components are the probabilities to be in one of the three periods with a life span of 3.

The vector  $y_{t+i}$  is determined as a function of  $y_t$  by the equation

$$y_{t+i} = y_t A, \quad \text{with} \quad A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 - \pi & 0 & \pi & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 - \pi & 0 & \pi & 0 & 0 \end{pmatrix}$$

Define the vector

$$e = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Because  $y_t$  is a vector of probabilities,  $y_0 \cdot e = 1$ . The ergodic distribution of the states in a family is given by the following property, which is proven in Appendix A.

**Lemma 2.** For any initial value  $y_0$  such that  $y_0 \cdot e = 1$ ,

$$\lim_{t \rightarrow \infty} y_t = \omega = \left( \frac{1 - \pi}{2 + \pi}, \frac{1 - \pi}{2 + \pi}, \frac{\pi}{2 + \pi}, \frac{\pi}{2 + \pi}, \frac{\pi}{2 + \pi} \right)$$

Since there are five possible states for the families in the initial period, the convergence of  $y_t$  to  $w$  is uniform with respect to the initial state of an agent and Property  $\mathcal{S}$  holds.

#### 4.3. The subsidization of capital

Normalizing the total population to one, the level of capital is equal to

$$k = \frac{1}{1 + \pi} ((1 - \pi)k_{11} + \pi(k_{21} + k_{22}))$$

The correlation between savings and marginal utilities is equal to

$$C = \frac{1}{1 + \pi}((1 - \pi)(-kv_{10} + (k_{11} - k)v_{11}) \\ + \pi(-kv_{20} + (k_{21} - k)v_{21} + (k_{22} - k)v_{22}))$$

Take the first  $\pi$  to be vanishingly small. Since  $k \approx k_{11}$ , we have  $C \approx kv_{10} < 0$ . Therefore there exists  $\pi^*$  such that if  $\pi < \pi^*$  both Assumption 3 and  $C < 0$  are satisfied. We can apply Theorem 1.

**Proposition 2.** *There exist  $T^*$  and such that if  $\pi < \pi^*$ , a subsidization of capital in period  $T > T^*$  is Pareto efficient.*

The condition  $\pi < \pi^*$  is not very restrictive: *some* randomization of life-cycles is sufficient to generate the *same* ergodic distribution for all individuals living in the first period. The model is a small perturbation of a model with overlapping generations.

As in the previous section, the policy can subsidize capital for one period, or introduce a permanent subsidization of capital. The property applies only for sufficiently distant periods and large  $T$ . We have seen that this is not a restriction. The result does not make any claim on the rate of subsidization. However, it justifies the existence of some existing rate of subsidization as the consequence of a past commitment to subsidization.

#### 4.4. Other fiscal instruments

##### 4.4.1. Capital and labor income taxation

Suppose now that instead of using lump-sum transfers, a government relies on a tax on labor income. This tax falls on the young, who have the lowest marginal utility of consumption. The same argument can be repeated, and the result of Proposition 2 holds.

The result is also valid in the important extension of an endogenous labor supply. For small tax rates, which are relevant here, the distortion of the labor tax is second-order with respect to the amount of revenues and can be neglected.

##### 4.4.2. The public debt

In the present model, families set aside resources for the lean years because they cannot borrow against future income. This model is not much different from the model of Samuelson (1958), who made the extreme assumption of  $R=0$ . As shown in Samuelson, the public debt can be socially valuable in this setting: it is more efficient to carry claims on future consumption in the form of paper rather than idling physical goods (who yield a return smaller than the discount rate).

These two previous fiscal instruments have an important role in the redistribu-

tion of resources and in the determination of the rate of return in relation to the social discount rate<sup>2</sup>. For this very reason, we have ignored them in this study of capital income taxation in order to focus on the essential mechanism of that tax.

## 5. Conclusion

No unambiguous property emerges once the assumption of infinite horizon and perfect capital markets is removed. However, the models presented here highlight how departures from the zero taxation generate Pareto welfare increases.

The assumption of a fixed return on capital has been useful here for the analysis of the main mechanisms. The results which involve marginal substitutions in the allocation of resources, do not depend on the expository assumption of a linear technology of production. The results can easily be extended with a standard production function in capital and labor.

Credit constraints prevent the consumption smoothing of individuals who are subject to labor income fluctuations. This restriction affects critically the social efficiency of the capital income taxation. The general lesson from this study is that a tax or a subsidy is efficient if an individual's savings are positively or negatively correlated with his consumption. When individuals have an optimizing behavior, the sign of this correlation depends on the stochastic properties of his income fluctuations. If they are independent over time, one can expect savings to be positively related to consumption as shown in the structural model of Section 3, in which a tax on capital income can be Pareto efficient. Section 4 provided a structural example of a negative correlation between savings and consumption which calls for which a subsidization of capital income can be Pareto efficient. In that example the negative correlation is generated by the 'very old' who are poor and consume little.

To summarize, capital market imperfections can generate a difference between the social discount rate and the rate of return on capital in the long-run. The argument presented here shows that this difference is irrelevant for the evaluation of the efficiency of the capital income taxation in the long-run. Such an evaluation depends on the structural assumptions of the model and on the stochastic properties of individual incomes. When the rate of return is smaller than the discount rate, some structures call for a permanent tax, others for a permanent subsidy.

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<sup>2</sup>Pestieau (1974) shows that with taxes on capital and labor and an optimal public debt, the rate of return and the social discount rate must be equal in the long run.

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## Appendix A

### Proof of Lemma 1

One first shows that  $\Psi_t(a; k_0)$  satisfies a monotone property of first-order stochastic dominance:

$$\text{if } k_0 \geq k'_0, \quad \text{then } \Psi_t(k; k_0) \leq \Psi_t(k; k'_0) \text{ for all } t \geq 1 \text{ and any } a \in [0, k]$$

The property holds trivially for  $t=0$ , and is shown by induction. Suppose that it holds for  $t>0$ . By the independence of labor income in period  $t$  and  $k_t$ ,

$$\Psi_{t+1}(a; k_0) = \int_{s(Rk_t + w) \leq a} dG(w) d\Psi_t(k_t; k_0)$$

where  $G$  is the cumulative distribution of labor income, and  $s$  is the saving function. Since this function is increasing (Stokey and Lucas, 1989),

$$\Psi_{t+1}(a; k_0) = \int_{w \leq s^{-1}(a) - Rk_t} dG(w) d\Psi_t(k_t; k_0)$$

or

$$\Psi_{t+1}(a; k_0) = \int G(s^{-1}(a) - Rk_t) d\Psi_t(k_t; k_0)$$

Since  $G(s^{-1}(a) - Rkt)$  is decreasing in  $k_t$ , by application of a standard property of the first-order stochastic dominance  $\Psi_{t+i}(k; k_0) \leq \Psi_{t+i}(k; k'_0)$ .

Suppose now that Lemma 1 does not hold. There exists  $a \in [0, k]$  and a sequence  $\{k(n)\}$  such that  $|\Psi_{T(n)}(a, k(n)) - \Psi(a)| > \eta$  for some value  $\eta > 0$  and  $T(n) \rightarrow \infty$ . Taking a subsequence, we can assume that  $k(n)$  converges to  $k_0^* > 0$ . Suppose first that  $k_0^* > 0$ . We can suppose that  $k(n) \geq y_1 = k_0^* - \gamma_1$  for all  $n$ , and some  $\gamma_1 > 0$  which can be vanishingly small. By the monotone property, for all  $n$ ,  $\Psi_{T(n)}(a; k(n)) \leq \Psi_{T(n)}(a; y_1)$ , and

$$\text{LimSup } \Psi_{T(n)}(a, k(n)) \leq \text{Lim } \Psi_{T(n)}(a, y_1) = \Psi(a)$$

Using a similar argument, there exists  $\gamma_2 > 0$  such that  $k(n) \leq \gamma_2 = k_0 + \gamma_2$  for all  $n$ . In the same way,

$$\text{LimInf} \Psi_{T(n)}(a, k(n)) \geq \Psi(a)$$

which leads to a contradiction of the non-convergence hypothesis.

If  $k_0^* = 0$ , the previous argument applies with  $\gamma_1 = 0$  and  $\gamma_2 > 0$ .

### *Proof of Lemma 2*

The characteristic polynomial of  $A$  is

$$P(x) = -x^2(x-1)(x^2+x+\pi)$$

The vector  $w$  in the lemma is the row eigenvector of  $A$  associated to the eigenvalue 1. The eigenspace of the eigenvalue 1 is of dimension 1.

The other eigenvalues of  $A$  have all a modulus strictly less than 1. Any row vector  $y_0$  such that  $y_0 \cdot e = 1$  can be written as

$$y_0 = \alpha \omega + g \tag{A.1}$$

where  $\alpha$  is a scalar, and  $g$  is in the subspace of the eigenvectors of  $A$  which are not homothetic to  $\omega$ .

For any row eigenvector  $f$  of  $A$ , associated to an eigenvalue  $\mu$ , since  $Ae = e$ ,

$$fAe = \mu fe = fe$$

Therefore, either  $\mu = 1$ , or  $f \cdot e = 0$ . Hence the vector  $g$  in Eq. (A.1) is orthogonal to  $e$ .

Multiplying both sides of Eq. (A.1) on the right by  $e$ , one finds that  $\alpha = 1$  and

$$y_0 = \omega + g$$

Multiplying  $y_0$  by  $A^t$ ,

$$y_t = \omega + gA^t$$

where  $g$  is spanned by vectors associated to eigenvalues of modulus less than 1. Therefore,

$$gA^t \rightarrow 0, \quad \text{and} \quad y_t \rightarrow \omega$$

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