Productivity and the Welfare of Nations*

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Abstract

We show how to relate the welfare of a country's infinitely-lived representative consumer to observable aggregate data. To a first order, welfare is summarized by total factor productivity and by the capital stock per capita. These variables suffice to calculate welfare changes within a country, as well as welfare differences across countries. The result holds regardless of the type of production technology and the degree of market competition. It applies to open economies as well, if total factor productivity is constructed using domestic absorption, instead of gross domestic product, as the measure of output. It also requires that total factor productivity be constructed with prices and quantities as perceived by consumers, not firms. Thus, factor shares need to be calculated using after-tax wages and rental rates and they will typically sum to less than one. These results are used to calculate welfare gaps and growth rates in a sample of developed countries with high-quality total factor productivity and capital data. Under realistic scenarios, the U.K. and Spain had the highest growth rates of welfare during the sample period 1985-2005, but the U.S. had the highest level of welfare.

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1 Introduction

Standard models in many fields of economics posit the existence of a representative household in either a static or a dynamic setting, and then seek to relate the welfare of that household to observable aggregate data. A separate large literature examines the productivity residual defined by Solow (1957), and interprets it as a measure of technical change or policy effectiveness. Yet a third literature, often termed "development accounting," studies productivity differences across countries, and interprets them as measures of technology gaps or institutional quality. To our knowledge, no one has suggested that these three literatures are intimately related. We show that they are. We start from the standard framework of a representative household that maximizes intertemporal welfare over an infinite horizon, and use it to derive methods for comparing economic well-being over time and across countries. Our results show that under a wide range of assumptions, welfare can be measured using just two variables, productivity and capital accumulation. We take our framework to the data, and measure welfare change within countries and welfare differences across countries.

In the simplest case of a closed economy with no distortionary taxes we show that to a first-order approximation the welfare change of a representative household can be fully characterized by three objects: the expected present discounted value of total factor productivity (TFP) growth as defined by Solow, the change in expectations of the same quantity, and the growth in the stock of capital per person. The result sounds similar to one that is often proven in the context of a competitive optimal growth model, which might lead one to ask what assumptions on technology and product market competition are required to obtain this result. The answer is, None. The result holds for all types of technology and market behavior, as long as consumers take prices as given and are not constrained in the amount they can buy or sell at those prices. Thus, for example, the same result holds whether the TFP growth is generated by exogenous technological change, as in the Ramsey-Cass-Koopmans model; by changes in the size of the economy combined with increasing returns to scale, as in the "semi-endogenous growth" models of Arrow (1962) and Jones (1995); or by externalities or public policy in fully-endogenous growth models, such as Romer (1986) or Barro (1990). As we discuss below, aggregate TFP can also change without any change in production technology in multi-sector models with heterogeneous distortions (for example, markups that differ across sectors): our results show that an increase in aggregate TFP due to reallocation would be as much of a welfare gain for the representative consumer as a change in exogenous technology with the same magnitude and persistence.

Our findings suggest a very different interpretation of TFP from the usual one. Usually one argues that TFP growth is interesting because it provides information on the change or diffusion of technology, or measures improvement in institutional quality, the returns to scale in the production function, or the markup of price over marginal cost. We show that whether all or none of these things is true, TFP is interesting for a very different reason. Using only the first-order conditions for optimization of the representative household, we can show that TFP is key to measuring welfare changes within a country and welfare differences across countries. We interpret TFP purely from the household side, producing what one might call "the household-centric Solow residual."[1] Here we follow the intuition of Basu and Fernald (2002), and supply a general proof of their basic insight that TFP, calculated from the point of view of the consumer, is relevant for welfare.

The intuition for our result comes from noting that TFP growth is output growth minus share-weighted input growth. The representative household receives all output, which *ceteris paribus* increases its welfare. But at the same time it supplies some inputs: labor input, which reduces leisure, and capital input, which involves deferring consumption (and perhaps losing some capital to depreciation). The household measures the cost of the inputs supplied relative to the output gained by real factor prices–the real wage and the real rental rate of capital. TFP also subtracts inputs supplied from output gained, and uses exactly the same prices to construct the input shares. The welfare result holds in a very general setting because relative prices measure the consumer's marginal rate of substitution even in many situations when they do not measure the economy's marginal rate of transformation–for example, if there are externalities, increasing returns or imperfect competition. In an extension, we show that our basic insight holds even in some situations where households are not competitive price-takers, for example, if there are multiple wages for identical workers, leading to labor market rationing.

This intuition suggests that in cases where prices faced by households differ from those facing firms, it is the former that matter for welfare. We show that this intuition is correct, and here our household-centric Solow residual differs from Solow's original measure, which uses the prices faced by firms. Proportional taxes are an important source of price wedges in actual economies. We show that the shares in the household-centric Solow residual need to be constructed using the factor prices faced by households. Since marginal income tax rates and rates of value-added taxation can be substantial, especially in rich countries, this modification is quantitatively important, as we show in empirical implementations of our results.

We then move to showing analogous results for open economies. Here we show that our previous results need to be modified substantially if we construct TFP using the standard output measure, real GDP. To the three terms discussed above we need to add the present discounted value of expected changes in the terms of trade, the present discounted value of expected changes in the rate of return on foreign assets, and the growth rate of net foreign asset holdings. Intuitively, both the terms of trade and the rate of return on foreign assets affect the consumer's ability to obtain welfare-relevant consumption and investment for a given level of factor supply. Holdings of net foreign assets are analogous to domestic physical capital in that both can be transformed into consumption at a future date.

While these results connect to and extend the existing literature, as we discuss below, they are difficult to take to the data. It is very hard to get good measures of changes in asset holdings by country for a large sample of countries.[2] Furthermore, measuring asset returns in a comparable way

[1] The term is due to Miles Kimball.

[2] The important work of Lane and Milesi-Ferretti (2001, 2007) has shed much light on this subject, but the measurement errors that are inevitable in constructing national asset stocks lead to very noisy estimates of net asset growth rates.

across countries would require us to adjust for differences in the risk of country portfolios, which is a formidable undertaking. Fortunately, we are able to show that these difficulties disappear if we switch to using real absorption rather than GDP as the measure of output.[3] In this case, exactly the same three terms that summarize welfare in the closed economy are also sufficient statistics in the open economy. Thus, we can measure welfare change empirically in ways that are invariant to the degree of openness of the economy.

These results pertain to the evolution of welfare in individual economies over time. The indexes we obtain are not comparable across countries. Thus it is natural to ask whether our methods shed any light on a pressing and long-standing question, the measurement of relative welfare across countries using a method firmly grounded in economic theory. It turns out that they do. Perhaps our most striking finding is the result that we can use data on cross-country differences in TFP and capital intensity, long the staples of discussion in the development and growth literatures, to measure differences in welfare across countries. More precisely, we show that productivity and the capital stock suffice to calculate differences in welfare across countries, with both variables computed as log level deviations from a reference country.

To understand this result, it helps to deepen the intuition offered above. Our analysis is based on a dynamic application of the envelope theorem, and it shows that the welfare of a representative agent depends to a first order on the expected time paths of the variables that the agent takes as exogenous. In a dynamic growth context, these variables are the prices for factors the household supplies (labor and capital), the prices for the goods it purchases (consumption and investment), and beginning-of-period household assets, which are predetermined state variables and equal to the capital stock in a closed economy. Apart from this last term, the household's welfare depends on the time paths of *prices*, which are exogenous to the household. Thus, the TFP that is directly relevant for household welfare is actually the *dual* Solow residual. We use the national income accounts identity to transform the dual residual into the familiar primal Solow residual.[4]

Our cross-country welfare result comes from using the link between welfare and exogenous prices implied by economic theory to ask how much an individual's welfare would differ if he faced the sequence of prices, not of his own country, but of some other country. We can perform the thought experiment of having a US consumer face the expected time paths of all goods and factor prices in, say, France, and also endow him with beginning-of-period French assets rather than US assets. The difference between the resulting level of welfare and the welfare of remaining in the US measures the gain or loss to a US consumer of being moved to France. Note that our welfare comparisons are from a definite point of view–in this example, from the view of a US consumer. In principle, the result could be different if the USA-France comparison is made by a French consumer, with different preferences over consumption and leisure. Fortunately, our empirical results are qualitatively unchanged and quantitatively little affected by the choice of the "reference country" used for these welfare comparisons.

[3] We are indebted to Mikhail Dmitriev for pointing out this result.

[4] See Barro and Sala-i-Martin (2004, section 10.2).

The same insights that apply to the time series are relevant for the cross section: TFP needs to be defined using the prices perceived by households, and if the economy is open then other terms become relevant. Thus, tax rates, terms of trade, and foreign asset holdings also matter for cross-country welfare comparisons. As before, we can reduce the measurement complications enormously by using absorption rather than GDP as the definition of output in our household-centric TFP measure.

These results show that we can perform interesting welfare comparisons using readily-available national income accounts data. We illustrate our methods using data for several industrialized countries for which high-quality data are available: Canada, France, Italy, Japan, Spain, the United Kingdom and the United States. We show the importance of fiscal considerations in constructing measures of welfare change over time. For example, if we assume that government spending is wasteful and taxes are lump-sum, the UK has the largest welfare gain among our group of countries over our sample period, 1985-2005, while Spain lags far behind due to its low TFP growth rate. Indeed, the US, a much richer country, has faster welfare growth than Spain under these assumptions. Allowing for distortionary taxation and assuming that government expenditures are chosen optimally, Spain has the highest welfare growth among all countries, with the UK a shade behind, and the US much further back.

However these welfare growth rates are country-specific indexes, and cannot be used to compare welfare across countries. We next apply our methodology to cross country-comparisons and show how these relative welfare levels evolve over time. In our benchmark case of optimal government spending and distortionary taxation, the US is the welfare leader throughout our sample period. At the start of our sample, we find that France and the UK are closest to the US in terms of welfare, with France having a slight advantage over the UK. By the end of the sample, France and most of the other economies fall further behind the US in terms of welfare levels, with the two exceptions being Spain and the UK. Spain converges towards the US level of welfare in the first several years of the sample, and then holds steady at a constant percent gap. The UK, by contrast, converges towards the US at a relatively constant rate, and by 2005 is within a few percent of the US level of per-capita welfare.

These measures have a clear interpretation because they are derived from a well-posed optimization problem. Starting from a precise statement of the household's optimization problem also forces us to confront two issues in national income and welfare measurement. First, our derivation shows that "consumption" should be defined as any good or service that consumers value, whether or not it is included in GDP. Similarly, "capital" should include all consumption that is foregone now in order to raise consumption possibilities for the future. These items include, for example, environmental quality and intangible capital. Of course, both are hard to measure and even harder to value, since there is usually no explicit market price for either good. But our derivation is quite clear on the principle that the environment, intangibles and other non-market goods should be included in our measure of "welfare TFP." We follow conventional practice in restricting the output measure for our TFP variable to market output (and the inputs to measured physical capital and labor), but in so doing we, and almost everyone else, are mismeasuring real GDP and TFP. Second, our starting point of a representative-consumer framework implies that we automatically ignore issues of distribution that intuition says should matter for social welfare. We believe that distributional issues are very important. However, our objective of constructing a welfare measure from aggregate data alone implies that we cannot incorporate measures of distribution into our framework. Thus, we maintain the representative-consumer framework, but without in any way minimizing the importance of issues that cannot be handled within that framework.

The paper is structured as follows. The next section presents our analytical framework, and uses it to derive results on the measurement of welfare within single economies. Section 3 applies similar methods to derive results for welfare comparisons among countries. (Full derivations of the results in Sections 2 and 3 are presented in an appendix.) We present a number of extensions to our basic framework in Section 4, allowing for multiple types of goods and factors, distortionary taxes, government expenditure, an open economy, and labor market rationing. We then take the enhanced framework to the data, and discuss empirical results in Section 5. We discuss relations of our work to several distinct literature in Section 6. Finally, we conclude by summarizing our findings and suggesting fruitful avenues for future research.

2 The Productivity Residual and Welfare

Both intuition and formal empirical work link TFP growth to increases in the standard of living, at least as measured by GDP per capita.[5] The usual justification for studying the Solow productivity residual is that, under perfect competition and constant returns to scale, it measures technological change. However, should we care about the Solow residual in an economy with non-competitive output markets, non-constant returns to scale, and possibly other distortions where the Solow residual is no longer a good measure of technological progress? Here we build on the intuition of Basu and Fernald (2002) that a slightly modified form of the Solow residual is welfare relevant even in those circumstances and derive rigorously the relationship between a modified version of the productivity residual (in growth rates or log levels) and the intertemporal utility of the representative household. The fundamental result we obtain is that, to a first-order approximation, utility reflects the present discounted values of productivity residuals (plus possibly other terms).

Our results are complementary to those in Solow's classic (1957) paper. Solow established that if there was an aggregate production function then his index measured its rate of change. We now show that under a very different set of assumptions, which are disjoint from Solow's, the familiar TFP index is also the correct welfare measure. The results are parallel to one another. Solow did not need to assume anything about the consumer side of the economy to give a technical interpretation to his index, but he had to make assumptions about technology and firm behavior. We do not need to assume anything about the firm side (which includes technology, but also firm behavior and industrial organization) in order to give a welfare interpretation, but we do need to

[5] For a review of the literature linking TFP to GDP per worker, in both levels and growth rates, see Weil (2008).

assume the existence of a representative consumer. Both results assume the existence of a potential function (Hulten, 1973), and show that TFP is the rate of change of that function. Which result is more useful depends on the application, and the trade-off that one is willing to make between having a result that is very general on the consumer side but requires very precise assumptions on technology and firm behavior, and a result that is just the opposite.

2.1 Approximating around the Steady State

More precisely, assume that the representative household maximizes intertemporal utility:

$$V_t = E_t \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} U(C_{t+s}; \overline{L} - L_{t+s}) \tag{1}$$

where C_t is the per-capita consumption at time t, L_t are per-capita hours of work, $\overline{L}$ is the per-capita time endowment, and N_t is population. H is the number of households, assumed to be fixed and normalized to one from now on. Population grows at constant rate n and per capita variables at a common rate g. For a well defined steady state in which hours of work are constant while consumption and real wage share a common trend, we assume that the utility function has the King, Plosser and Rebelo (1988) form:

$$U(C_{t+s}; L - \overline{L}_s) = \frac{1}{1-\sigma} {C_{t+s}}^{1-\sigma} \nu(\overline{L} - L_{t+s}) \tag{2}$$

with $0 < \sigma < 1$ or $\sigma > 1$ and $\nu(.) > 0$.[6] Denote X_t an index for per capita variables in the sense that, along the steady state growth path, their level at time t is proportional to X_t and define $c_{t+s} = \frac{C_{t+s}}{X_{t+s}}$. We can rewrite the utility function in a normalized form as follows:

$$v_t = \frac{V_t}{N_t X_t^{(1-\sigma)}} = E_t \sum_{s=0}^{\infty} \beta^s U(c_{t+s}; \overline{L} - L_{t+s}) \tag{3}$$

where $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$ is assumed to be less than one. The budget constraint (with variables scaled by $N_t X_t$) is:

$$k_t + b_t = \frac{(1-\delta) + p_t^K}{(1+g)(1+n)} k_{t-1} + \frac{(1+r_t)}{(1+g)(1+n)} b_{t-1} + p_t^L L_t + \pi_t - p_t^C c_t \tag{4}$$

New capital goods are the numeraire, $k_t = \frac{K_t}{X_t N_t}$ denotes capital per worker normalized by X_t, $b_t = \frac{B_t}{P_t^I X_t N_t}$ are normalized real bonds. $p_t^K = \frac{P_t^K}{P_t^I}$, $p_t^L = \frac{P_t^L}{P_t^I X_t}$, $p_t^C = \frac{P_t^C}{P_t^I}$ denote, respectively, the user cost of capital, the wage per hour of effective worker, and the price of consumption goods. $(1 + r_t)$ is the real interest rate (again in terms of new capital goods) and $\pi_t = \frac{\Pi_t}{P_t^I X_t N_t}$ denotes normalized profits per capita.

[6] If $\sigma = 1$, then the utility function must be $U(C_1, .., C_G; \overline{L} - L) = \log(C) - \nu(\overline{L} - L)$. See King, Plosser and Rebelo (1988).

Log linearizing around the non stochastic steady state, intertemporal household utility can be written (to a first order approximation) as:

$$\begin{aligned} v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L L \widehat{p}^L_{t+s} + \frac{p^K k}{(1+g)(1+n)} \widehat{p}^K_{t+s} + \pi \widehat{\pi}_{t+s} - p^C_t c \widehat{p}_{t+s} \right] \\ & \quad + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} + \lambda \frac{1}{\beta} b \widehat{b}_{t-1} \end{aligned} \tag{5}$$

where v is the steady state value of utility, $\widehat{x} = \log x_t - \log x$ denote log deviation from the steady state. In obtaining this result we have used the FOCs of the household maximization problem; for time t these consist of:

$$U_{c_t} - \lambda_t p^C_t = 0 \tag{6}$$

$$U_{L_t} + \lambda_t p^L_t = 0 \tag{7}$$

$$-\lambda_t + \beta E_t \frac{(1-\delta) + p^K_{t+1}}{(1+g)(1+n)} \lambda_{t+1} = 0 \tag{8}$$

$$-\lambda_t + \beta \frac{1}{(1+g)(1+n)} E_t (1 + r_{t+1}) \lambda_{t+1} = 0 \tag{9}$$

plus the two trasversality conditions, one for bonds and one for capital. Equation (5) is an immediate consequence of the envelope theorem and expresses normalized utility as a function of the variables that the consumer takes as exogenous or predetermined, e.g. prices and the initial stock of capital and bonds. Using the log linear approximation of the budget constraint around the steady state:

$$\begin{aligned} & k\ \widehat{k}_t + b \widehat{b}_t - \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1} - p^L L \widehat{L}_t - p^L L \widehat{p}^L_t - \frac{p^K k}{(1+g)(1+n)} \widehat{p}^K_t \\ & - \pi \widehat{\pi}_t + p^C c \widehat{c}_t + p^C c \widehat{p}_t = 0 \end{aligned} \tag{10}$$

we can re-write equation (5) in terms of quantities:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s} - p^L L\ \widehat{L}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \widehat{k}_{t+s-1} \right] + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \tag{11}$$

Equation (11) says that intertemporal utility (in log deviation from the steady state) reflects the expected present discounted value of terms that represent the sum of the components of final demand (in log deviation from the steady state), weighted by their steady state contribution to demand, minus primary inputs (in log deviation from the steady state) times their respective steady state total remuneration. In addition, intertemporal utility depends upon the initial level of the capital stock.

2.2 Connecting with the Productivity Residual

We are now close to relating utility to a modified version of the productivity residual. Let us start by obtaining a first order approximation for the level of utility in terms of the log level productivity residual. In doing so, we will use the fact that, to a first order approximation, the level of normalized value added in deviation from the steady state is given by:

$$\widehat{y}_t = \log y_t - \log y = s_c \widehat{c}_t + s_i \widehat{i}_t \tag{12}$$

where $s_c \equiv \frac{P^C CN}{P^Y Y}$ and $s_i \equiv \frac{P^I I}{P^Y Y}$ are respectively the steady-state shares of consumption goods and investment goods out of value added. Using (12), equation (11) can be re-written as:

$$\frac{v_t - v}{\lambda p^Y y} = E_t \sum_{s=0}^{\infty} \beta^s \left[\widehat{y}_{t+s} - \ s_L \ \widehat{L}_{t+s} - s_K \ \widehat{k}_{t+s-1} \right] + \frac{1}{\beta} \frac{k}{p^Y y} \widehat{k}_{t-1} \tag{13}$$

where $s_L = \frac{p^L L}{p^Y y}$ and $s_K = \frac{p^K k}{p^Y y(1+g)(1+n)}$ denote the distributional shares of labor and capital respectively. If $\sigma < 1$, so that $v_t > 0$, $(v_t - v)$ can be approximated by $v(\log v_t - \log v) = v(\log \frac{V_t}{N_t} - \log \left(\frac{V_t}{N_t}\right)_{SS})$, where the subscript SS denotes the value of time varying variables along the steady state growth path. Then the equation above can be rewritten as:

$$\frac{v}{\lambda p^Y y} \left(\log \frac{V_t}{N_t} - \log \left(\frac{V_t}{N_t} \right)_{SS} \right) = E_t \sum_{s=0}^{\infty} \beta^s (\log PR_{t+s} - (\log PR_{t+s})_{SS}) + \frac{1}{\beta} \frac{k}{p^Y y} \left(\log \frac{K_{t-1}}{N_{t-1}} - \left(\log \frac{K_{t-1}}{N_{t-1}} \right)_{SS} \right) \tag{14}$$

where:

$$\log PR_{t+s} = \log \frac{Y_t}{N_t} - s_L \log L_t - s_K \log \frac{K_{t-1}}{N_{t-1}} \tag{15}$$

$\log \frac{Y_t}{N_t}$ is the sum of log consumption and log investment per capita, each weighted by its steady state expenditure shares. Productivity, $\log PR_t$, is defined $\log \frac{Y_t}{N_t}$ minus the log level of factor inputs per capita, $\log L_t$ and $\log \frac{K_{t-1}}{N_{t-1}}$ multiplied by their respective steady state distributional shares, s_L and s_K.[7]

In order to interpret equation (14), notice that $\frac{v}{\lambda p^Y y}$ measures the percentage increase in income necessary to generate a one percentage point increase in lifetime utility (see the Appendix for details). Therefore the right hand side of equation (14), $\frac{v}{\lambda p^Y y} \left(\log \frac{V_t}{N_t} - \log \left(\frac{V_t}{N_t} \right)_{SS} \right)$, represents the income equivalent corresponding to the log deviation of per-capita utility from the steady state. This quantity is an increasing function of the sequence of log deviation from the steady state of productivity residuals, appropriately discounted and the log deviation from the steady state of the initial level of the per-capita capital stock, $\frac{K_{t-1}}{N_{t-1}}$.

[7] See details in the appendix

When $\sigma > 1$, so that $v_t < 0$, the interpretation remains essentially the same but the left hand side of equation (14) now equals $\frac{v}{\lambda p^Y y}\left(\log\frac{-V_t}{N_t} - \log\left(\frac{-V_t}{N_t}\right)_{SS}\right)$. This quantity is greater than zero for positive deviations of per capita utility from its value along the steady state growth path.

In order to illustrate the relationship between the change in per-capita welfare and the Solow residual, we return to (14) and take its difference through time. In this case, we will rely on the following (Divisia) definition of growth in value added:

$$\Delta\log Y_t = s_c\Delta\log(C_tN_t) + s_i\Delta\log I_t \tag{16}$$

where the growth rate of each demand component is aggregated using constant steady state shares[8]. Using this definition of value added in growth terms and taking first differences of equation (14), we show in the Appendix that the money-value of the growth rate of per-capita welfare, as a proportion of GDP, can be written as follows[9]:

$$\begin{aligned}\frac{v}{\lambda p^Y y}\Delta\log\frac{V_t}{N_t} \quad &= \quad E_t\sum_{s=0}^{\infty}\beta^s\Delta\log PR_{t+s} \\ &\quad +\sum_{s=0}^{\infty}\beta^s\left[E_t\log PR_{t+s} - E_{t-1}\log PR_{t+s}\right] \\ &\quad +\frac{1}{\beta}\frac{k}{p^y y}\Delta\log\frac{K_{t-1}}{N_{t-1}}\end{aligned} \tag{17}$$

where $\Delta\log PR_{t+s}$ denotes the "modified" Solow productivity residual:

$$\Delta\log PR_{t+s} = \Delta\log\frac{Y_{t+s}}{N_{t+s}} - s_L\Delta\log L_{t+s} - s_K\Delta\log\ \frac{K_{t+s-1}}{N_{t+s-1}} \tag{18}$$

and $\Delta\log Y_t = s_c\Delta\log N_tC_t + s_i\Delta\log I_t$. We use the word "modified," for two reasons. First, we do not assume that the distributional shares of capital and labor add to one, as they would if there were zero economic profits and no distortionary taxes. Zero profits are guaranteed in the benchmark case with perfect competition and constant returns to scale, but can also arise with imperfect competition and increasing returns to scale, as long as there is free entry, as in the standard Chamberlinian model of imperfect competition. Second, the distributional shares are calculated at their steady state values and, hence, are not time varying (as it is conventionally assumed). Rotemberg and Woodford (1991) argue that in a consistent first-order log-linearization of the production function the shares of capital and labor should be taken to be constant, and Solow's (1957) use of time-varying shares amounts to keeping some second-order terms while ignoring others.

The term $E_t\log PR_{t+s} - E_{t-1}\log PR_{t+s}$ represents the revision in expectations of the log level of the productivity residual, based on the new information received between t-1 and t. In addition, $\Delta\log\frac{K_{t-1}}{N_{t-1}}$ captures the change in the initial endowment of the capital stock per capita.

[8] Here we are slightly departing from convention, as value added is usually calculated with time-varying shares.
[9] Also in this case, when $\sigma > 1$, so that $v_t < 0$, the left hand side of equation (17) equals $\frac{v}{\lambda p^Y y}\Delta\log\frac{-V_t}{N_t}$

Note that the revision terms in the second summation will reduce to a linear combination of the innovations in the stochastic shocks affecting the economy at time t. Moreover, if we assume that the modified log level productivity residual follows a univariate autoregressive process, then only the innovation of such process matters. In addition, all the terms in the first summation are simply a function of current and past values of productivity.

Summarizing, by focusing only on the representative consumer optimization problem, we have shown that the Solow residual is a welfare relevant object. Since we have made no assumption about the production function or about product market structure, this result is very general. It holds not only in competitive economies with constant return to scale technologies and no externalities, but also in distorted economies with externalities. Even though in those economies the Solow residual does not capture technology, it matters for welfare, together with the initial endowment of capital.

To aid the quantitative interpretation of our the results, we can express them in terms of "equivalent consumption" per capita, denoted by C_t^*. C_t^* is defined as the level of consumption per capita at time t that, if it grows at the steady state rate g from t onward, with leisure set at its steady state level, delivers the same intertemporal utility per capita as the actual stream of consumption and leisure. More precisely, C_t^* satisfies:

$$\begin{aligned} \frac{V_t}{N_t} &= \sum_{s=0}^{\infty} \frac{(1+n)^s}{(1+\rho)^s} \left(C_t^*(1+g)^s\right)^{1-\sigma} \nu(\overline{L}-L)) \\ &= \frac{1}{(1-\sigma)\,(1-\beta)} C_t^{*1-\sigma} \nu(\overline{L}-L) \end{aligned} \tag{19}$$

Taking log differences of equation (19) and using equation (17), one obtains:

$$\Delta \log C_t^* = \frac{(1-\beta)}{s_c} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s} + \frac{1}{\beta} \frac{k}{p^y y} \Delta \log \frac{K_{t-1}}{N_{t-1}} \right] \tag{20}$$

It is easily seen that this final expression holds for σ smaller or greater than one. This means that if we multiply the right hand side of (17) by $\frac{(1-\beta)}{s_c}$, we can interpret it as the growth rate of per-capita equivalent consumption between t and $t-1$.

3 Implications for Cross Country Analysis

We can use the framework developed here in order to make cross country comparisons in welfare and to show how they are affected by differences in productivity and capital accumulation. Welfare comparisons across countries have been investigated recently by Jones and Klenow (2010). Instead of focusing on single period (or steady state) utility, as they do, we consider lifetime utility and consider out of steady state dynamics and how they are related to capital accumulation and productivity. This allows us to properly take into account the dynamic welfare effects associated with the fact that lower consumption or leisure today may raise capital accumulation and support greater consumption in the future. We do not, however, allow for cross country differences in life expectancy or in inequality as in the Jones and Klenow paper. In our approach the cross-country differences in welfare are summarized solely in terms of productivity and capital endowments and not in terms of consumption and leisure.

A comparison of welfare across countries requires either assuming that their respective representative agents possess the same utility function, or making the comparison from the perspective of the representative agent in a reference country. We will in any case assume that the discount factor, β, is the same in each country. These assumptions suggest that it is more reasonable to focus on a subset of countries that are relatively homogenous and this is exactly what we will do by using core OECD countries in our empirical illustration.

Let us go back to a modified version of equation (5) and assume we have taken the log-linear expansion around the US steady state:

$$\begin{aligned} v_t^i &= v^{us} + E_t \sum_{s=0}^{\infty} \beta^s \lambda^{us} [p^{L,us} L^{us} (\log p_{t+s}^{L,i} - \log p^{L,us}) + \frac{p^{K,us} k^{us}}{(1+g)(1+n)} (\log p_{t+s}^{K,i} - \log p^{K,us}) \\ &+ \pi^{us} (\log \pi_{t+s}^i - \log \pi^{us}) - p^{C,us} c^{us} (\log p_{t+s}^i - \log p^{us})] \\ &+ \lambda^{us} \frac{1}{\beta} k^{us} (\log k_{t-1}^i - \log k^{us}) + \lambda^{us} \frac{1}{\beta} b^{us} (\log b_{t-1}^i - \log b^{us}) \end{aligned} \quad (21)$$

If preferences are common across countries, v_t^i, can be interpreted as the normalized lifetime utility of an individual in country i. Alternatively, v_t^i, can be thought of representing normalized lifetime welfare of a US individual when facing the sequence of prices, initial endowments, profits and X_t of country i.

In both cases, using the log-linearized version of the budget constraint in equation (10) expanded around the US steady state, we can obtain (proceeding in a way parallel to the one used in the previous section):[10]

$$\begin{aligned} \frac{v^{us}}{\lambda^{us} p^{Y,us} y^{us}} (\log v_t^i - \log v^{us}) &= E_t \sum_{s=0}^{\infty} \beta^s [s_c^{us} (\log c_{t+s}^i - \log c^{us}) + s_i^{us} (\log i_{t+s}^i - \log i^{us}) \\ &- s_L^{us} (\log L_{t+s}^i - \ln L^{us}) - s_K^{us} (\log k_{t+s-1}^i - \log k^{us})] \\ &+ \frac{1}{\beta} \frac{k^{us}}{p^{Y,us} y^{us}} (\log k_{t-1}^i - \log k^{us}) \end{aligned} \quad (22)$$

Define the productivity term obtained using US shares as $\overline{PR}_{t+s}^i = \left(s_c^{us} \log C_{t+s}^i + s_i^{us} \log \frac{I_{t+s}^i}{N_{t+s}^i} \right) -$

[10] When v_t^i represents the normalized utility of and individual from country i, it is obvious that (22) follows from (10). When v_t^i is normalized utility of the US individual when faced with the price sequence and the endowments of country i, the result still holds. It does not follow that the US individual will choose the quantities of country i unless the utility function is identical across countries. However,after expanding the budget constraint around the US steady state, the algebraic sum of the terms in prices for the US individual facing country i prices and endowments equals the algebraic sum of the terms in the quantities for the individual from country i. Hence (22) holds in this case too.

$s_L^{us}\log L_{t+s}^i - s_K^{us}\log\frac{K_{t+s-1}^i}{N_{t+s-1}^i}$. Using this modified measure of productivity, where input shares are common across countries, we show in the appendix that equation (22) can be re-written in per-capita terms as:

$$\frac{v^{us}}{\lambda^{us}p^{Y,us}y^{us}}\left[\log\frac{V_t^i}{N_t^i}-\log\left(\frac{V_t^{us}}{N_t^{us}}\right)_{SS}\right] = E_t\sum_{s=0}^{\infty}\beta^s\left[\log\overline{PR}_{t+s}^i-\log(PR_{t+s}^{us})_{SS}\right] + \frac{1}{\beta}\frac{k^{us}}{p^{Y,us}y^{us}}\left(\log\frac{K_{t-1}^i}{N_{t-1}^i}-\log\left(\frac{K_{t-1}^{us}}{N_{t-1}^{us}}\right)_{SS}\right) \tag{23}$$

If we expand the US intertemporal utility around the US steady state and subtract the resulting equation from (23), we can write:

$$\frac{v^{us}}{\lambda^{us}p^{Y,us}y^{us}}\left[\log\frac{V_t^i}{N_t^i}-\log\frac{V_t^{us}}{N_t^{us}}\right] = E_t\sum_{s=0}^{\infty}\beta^s\left[\log\overline{PR}_{t+s}^i-\log PR_{t+s}^{us}\right]+\frac{1}{\beta}\frac{k^{us}}{p^{Y,us}y^{us}}\left(\log\frac{K_{t-1}^i}{N_{t-1}^i}-\log\frac{K_{t-1}^{us}}{N_{t-1}^{us}}\right) \tag{24}$$

Welfare differences across countries, therefore, are summarized by two components. The first component depends on the differences in the capital endowments. The second component is related to the well-known log difference between TFP levels, which accounts empirically for most of the difference is per capita income across countries (Hall and Jones (1999)). In the development accounting literature it is interpreted as a measure of technological or institutional differences between countries. This interpretation, however, is valid under restrictive assumptions on market behavior and technology (i.e. perfect competition, constant returns to scale, etc.). We provide a different interpretation of the relevance of cross country differences in TFP, by showing that a "modified" version of the log-difference between TFP levels is essential for welfare comparison across countries, together with differences in their per capita capital endowment. This last term is similar to the capital intensity term used in the development accounting literature. Our result holds for any kind of technology and market structure, as long as a representative consumer exists, takes prices as given and is not constrained in the amount he can buy and sell at those prices. Notice however, that our measure of TFP is modified with respect to the traditional growth accounting measure in three ways. First, measuring welfare differences requires comparing not only current log differences in TFP but the present discounted value of present and future ones. Second, the distributional and expenditure shares used to compute the log differences in TFP need to be calculated at their steady-state value in the reference country. Third, as we will argue in section 4.4, domestic absorption is used, instead of GDP, in calculating the productivity residual.

As in the within-country case, we can conduct the comparison by using the concept of equivalent consumption. In this context, for any country i, $C_t^{*,i}$, is defined for a constant level of leisure, fixed at the US steady state level.

$$\frac{V_t^i}{N_t^i} = \frac{1}{(1-\sigma)(1-\beta)} \left(C_t^{*,i}\right)^{1-\sigma} \nu(\overline{L} - L^{us}) \tag{25}$$

It then follows that:

$$\log C_t^{*,i} - \log C_t^{*,us} = \frac{(1-\beta)}{s_c^{us}} \left[E_t \sum_{s=0}^{\infty} \beta^s \left(\log \overline{PR}_{t+s}^i - \log PR_{t+s}^{us} \right) + \frac{1}{\beta} \frac{k^{us}}{p^{Y,us} y^{us}} \left(\log \frac{K_{t-1}^i}{N_{t-1}^i} - \log \frac{K_{t-1}^{us}}{N_{t-1}^{us}} \right) \right] \tag{26}$$

Note that along the steady state growth path, assuming that g is common across countries, (26) can be rewritten after simple algebra as:

$$\begin{aligned}
& \log(C_t^{*,i})_{SS} - \log(C_t^{*,us})_{SS} = \\
= \; & \frac{1}{s_c^{us}} \left[\left(s_c^{us} \log(C_t^i)_{SS} + s_i^{us} \log \left(\frac{I_t^i}{N_t^i} \right)_{SS} \right) - \left(s_c^{us} \log(C_t^{us})_{SS} + s_i^{us} \log \left(\frac{I_t^{us}}{N_t^{us}} \right)_{SS} \right) \right] \\
& - \frac{(1-\delta^{us}) - (1+g^{us})(1+n^{us})}{(1+g^{us})(1+n^{us})} \frac{k^{us}}{p^{Y,us} y^{us}} \left(\log \left(\frac{K_{t-1}^i}{N_{t-1}^i} \right)_{SS} - \log \left(\frac{K_{t-1}^{us}}{N_{t-1}^{us}} \right)_{SS} \right) \\
& - \frac{s_L^{us}}{s_c^{us}} (\log L^i - \log L^{us})
\end{aligned} \tag{27}$$

The second and third line of equation (27) contain the difference in the log value of demand components (aggregated with US expenditure weights), adjusted by the difference in capital depreciation (on the third line). The fact that the latter term represents differences in depreciation is most easily seen when g and n are set to zero. This result is in the spirit of Weitzman (1976, 2003) who emphasizes the role of Net National Product (NNP) in welfare comparisons.[11] However, the term on the fourth line of (27) suggests that differences in per capita NNP (or a log linear approximation to it) are not a sufficient statistic for welfare even in the steady state and have to be adjusted for differences in leisure.

4 Extensions

We now show that our method of using TFP to measure welfare can be extended to cover multiple types of capital, labor and consumption goods, taxes, and government expenditure. These extensions also modify in obvious ways the formulas for within and across countries welfare comparisons. The first extension modifies our baseline results in only a trivial way, but the others all require more substantial changes to the formulas above. These results show that the basic idea of using TFP to measure welfare holds in a variety of economic environments, but also demonstrate the advantage of deriving the welfare measure from an explicit dynamic model of the household. The model shows exactly what modifications to the basic framework are required in each case.

[11] We thank Chad Jones for drawing our attention to this point.

4.1 Multiple Types of Capital, Labor and Consumption Goods

The extension to the case of multiple types of labor, capital and consumption goods is immediate. For simplicity, we could assume that each individual is endowed with the ability to provide different types of labor services, $L_{h,t}$ and that the utility function can be written as:

$$U(C_{1,t+s},..,C_{Z,t+s},L_{1,t+s},...,L_{H_L,t+s})=\frac{1}{1-\sigma}C(C_{1,t+s},..,C_{Z,t+s})^{1-\sigma}\nu\left[\overline{L}-L(L_{1,t+s},...,L_{H_L,t+s})\right] \tag{28}$$

where $L(.)$ and $C(.)$ are homogenous functions of degree one, H_L is the number of types of labor and Z is the number of consumption goods. Denote the payment to a unit of $L_{h,t}$, $P_t^{L_h}$.[12] Similarly consumers can accumulate different types of capitals $K_{h,t}$ and rent them out at $P_t^{K_h}$. Take capital good 1 as the numeraire. Equation (11) now becomes:

$$\begin{aligned} v_t-v &= E_t\sum_{s=0}^{\infty}\beta^s\lambda\left[\sum_{i=1}^{Z}p_i^C c_i\widehat{c}_{i,t+s}+\sum_{h=1}^{H_K}p_h^I i_h\widehat{i}_{h,t+s}-\sum_{h=1}^{H_K}p_h^L L_h\ \widehat{L}_{h,t+s}-\sum_{h=1}^{H_K}\frac{p_h^K k_h}{(1+g)(1+n)}\ \widehat{k}_{h,t+s-1}\right] \\ &\quad +\sum_{h=1}^{H_K}\lambda\frac{1}{\beta}k_h\widehat{k}_{h,t-1} \end{aligned} \tag{29}$$

Redefine normalized real GDP in deviation from SS as:

$$\widehat{y}_t=\sum_{i=1}^{Z}s_{c_i}\widehat{c}_{i,t}+\sum_{h=1}^{H_K}s_{i_h}\widehat{i}_{h,t} \tag{30}$$

Using the two equations above, we get:

$$\begin{aligned} \frac{v_t-v}{\lambda p^Y y} &= E_t\sum_{s=0}^{\infty}\beta^s\left[\widehat{y}_{t+s}-\sum_{h=1}^{H_K}s_{L_h}\ \widehat{L}_{h,t+s}-\sum_{h=1}^{H_K}s_{K_h}\ \widehat{k}_{h,t+s-1}\right] \\ &\quad +\sum_{h=1}^{H_K}\frac{1}{\beta}\frac{k_h}{p^Y y}\widehat{k}_{h,t-1} \end{aligned} \tag{31}$$

where $s_{L_h}=\frac{p_h^L L_h}{p^Y y}$ and $s_{K_h}=\frac{p_h^K k_h}{(1+g)(1+n)p^Y y}$. Proceeding exactly as in the previous section, equations (16), (19), (23) and (25) continue to characterize the relationship between utility and the productivity residual, with the only difference that the latter is defined now as:

$$\log PR_t=\log\frac{Y_t}{N_t}-\sum_{h=1}^{H_K}s_{L_h}\log L_{h,t}-\sum_{h=1}^{H_L}s_{K_h}\log\frac{K_{h,t-1}}{N_{h,t-1}} \tag{32}$$

[12] We assume that the nature of the utility function is such that positive quantities of all types of labors are supplied.

4.2 Taxes

The derivations in Section 2.2 can be modified to accommodate an environment with either distortionary and/or lump-sum taxes. Since the prices in the budget constraint (11) are those faced by the consumer, if there are taxes, prices should all be interpreted as after-tax prices. At the same time, the variable that we have been calling "profits," π, can be viewed as comprising any transfer of income that the consumer takes as exogenous. Thus, it can be interpreted to include lump-sum taxes or rebates. Finally one should think of b_t as including both government and private bonds (assumed to be perfect substitutes, purely for ease of notation).

More precisely, in order to modify (11) to allow for taxes, let τ_t^K be the tax rate on capital income, τ_t^R be the tax rate on revenues from bonds, τ_t^L be the tax rate on labor income, τ_t^C be the *ad valorem* tax on consumption goods, and τ_t^I be the corresponding tax on investment goods[13]. We assume that the revenue so raised is distributed back to individuals using lump-sum transfers. (We consider government expenditures in the next sub-section.) Equation (4) now takes the following form:

$$\begin{aligned} v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L \left(1-\tau^L\right) L \widehat{p}_{t+s}^L + \frac{p^K \left(1-\tau^K\right) k \, \widehat{p}_{t+s}^K}{(1+g)(1+n)} + \frac{r \left(1-\tau^R\right) b \widehat{r}_{t+s}}{(1+g)(1+n)} + \pi \widehat{\pi}_{t+s} - p^C \left(1+\tau^C\right) c \widehat{p}_{t+s} \right] \\ & \quad - E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L \tau^L L \widehat{\tau}_{t+s}^L + \frac{p^K \tau^K k}{(1+g)(1+n)} \widehat{\tau}_{t+s}^K + \frac{r \tau^R b}{(1+g)(1+n)} \widehat{\tau}_{t+s}^R - p^C \tau^C c \widehat{\tau}_{t+s}^C \right] \\ & \quad + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} + \lambda \frac{1}{\beta} b \widehat{b}_{t-1} \end{aligned} \tag{33}$$

As in the benchmark case, this equation is an immediate consequence of the envelope theorem and expresses normalized utility as a function of those variables that the household takes as exogenous. However, differently than in the benchmark case, the exogenous variables in the household's maximization are not only the prices and the initial stocks of capital and bonds, but also the tax rates on labor and capital income, consumption and investment. Using the new log-linearized individual budget constraint:

$$\begin{aligned} 0 &= k \, \widehat{k}_t - \frac{(1-\delta) + p^K \left(1-\tau^K\right)}{(1+g)(1+n)} k \widehat{k}_{t-1} - \frac{p^K \left(1-\tau^K\right) k}{(1+g)(1+n)} \widehat{p}_t^K + \frac{p^K \tau^K k}{(1+g)(1+n)} \widehat{\tau}_t^K \\ & \quad - p^L \left(1-\tau^L\right) L \widehat{L}_t - p^L \left(1-\tau^L\right) L \widehat{p}_t^L + p^L \tau^L L \widehat{\tau}_t^L \\ & \quad + p^C \left(1+\tau^C\right) c \widehat{c}_t + p^C \left(1+\tau^C\right) c \widehat{p}_t^C + p^C \tau^C c \widehat{\tau}_t^C - \pi \widehat{\pi}_t \\ & \quad + b \widehat{b}_t - \frac{\left(1 + r\left(1-\tau^R\right)\right) b}{(1+g)(1+n)} \widehat{b}_{t-1} - \frac{r \left(1-\tau^R\right) b}{(1+g)(1+n)} \widehat{r}_t - \frac{r \tau^R b}{(1+g)(1+n)} \widehat{\tau}_t^R \end{aligned} \tag{34}$$

[13] For simplicity, we are assuming no capital gains taxes and no expensing for depreciation. These could obviously be added at the cost of extra notation.

Normalized utility can be re-expressed in terms of the endogenous variables[14]:

$$\begin{aligned} v_t - v &= \lambda E_t \sum_{s=0}^{\infty} \beta^s [\left(1+\tau^C\right) p^C c\widehat{c}_{t+s} + \left(1+\tau^I\right) i\widehat{i}_{t+s} - \left(1-\tau^L\right)\ p^L L\widehat{L}_{t+s} \\ & \quad - \frac{\left(1-\tau^K\right) p^K k}{(1+g)\,(1+n)} \widehat{k}_{t+s-1}] + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} \end{aligned} \tag{35}$$

At this point, it is interesting to notice that changes in the tax rates do not appear in the right-hand side of the equation above. The intuition for this result is that the endogenous quantities already reflect variations in the tax rates. In other words, changes in the tax burden are already captured by the changes in consumption, investment and labor supply that they determined.

To make contact with the data, note that the national accounts at market prices define nominal expenditure using prices as perceived from the demand side. Thus, equation (12) can be written exactly as before and still be consistent with standard national accounts data, where s_c and s_i in $\widehat{y}_t = s_c\widehat{c}_t + s_i\widehat{i}_t$ are inclusive of indirect taxes (subsidies) on consumption and investment. On the other hand, the national accounts measure factor payments as perceived by firms, before income taxes. Hence we can write:

$$\frac{v_t - v}{\lambda p^Y y} = E_t \sum_{s=0}^{\infty} \beta^s [\widehat{y}_t - \left(1-\tau^L\right) s_L \widehat{L}_{t+s} - \ \left(1-\tau^K\right) s_K \widehat{k}_{t+s-1}] + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} \tag{36}$$

where s_L and s_K are the gross income shares of labor and capital respectively. Thus, the data-consistent definition of the welfare residual with taxes needs to be based on a new definition of $\log PR_t$, where the shares of labor and capital returns are net of taxes. More specifically, equation (15) can be re-written as:

$$\log PR_{t+s} = \log \frac{Y_{t+s}}{N_{t+s}} - \left(1-\tau^L\right) s_L \log L_{t+s} - \left(1-\tau^K\right) s_K \log \frac{K_{t+s-1}}{N_{t+s-1}} \tag{37}$$

In conclusion, after properly redefining $\widehat{y}_{t+s}$ and $\log PR_t$, the results discussed in the second and the third section of the paper can be generalized in a model with distortionary time-varying taxes on consumption and investment goods and on the household income coming from labor, capital or financial assets.

4.3 Government Expenditure

With some minor modification, our framework can be extended to allow for the provision of public goods and services. We illustrate this under the assumption that government activity is financed with lump-sum taxes. Using the results from the previous subsection, it is straightforward to extend the argument to the case of distortionary taxes.

Assume that government spending takes the form of public consumption valued by consumers. We rewrite the instantaneous utility function as

[14] See the Appendix for details on the derivation.

$$U(C_{t+s}, C_{G,t+s}, L_{t+s}) = \frac{1}{1-\sigma} C(C_{t+s}; C_{G,t+s})^{1-\sigma} \nu(\overline{L} - L_{t+s}) \tag{38}$$

where C_G denotes per-capita public consumption and $C(.)$ is homogenous of degree one in its arguments. Equation (11) now becomes:

$$\begin{aligned} v_t - v &= \lambda E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{U_{c_G} c_G \widehat{c}_{G,t+s}}{\lambda} + p^C c \widehat{c}_{t+s} + i \widehat{i}_{t+s} - \; p^L L \widehat{L}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \widehat{k}_{t+s-1} \right] \\ & \quad + \lambda \frac{1}{\beta} k \widehat{k}_{t-1} \end{aligned} \tag{39}$$

where $c_{G,t} = \frac{C_{G,t}}{X_t}$. The definition of GDP in deviation from steady state is now:

$$\widehat{y}_t = s_c \widehat{c}_t + s_i \widehat{i}_t + s_{c_G} \widehat{c}_{G,t} \tag{40}$$

where $s_{c_G} = \frac{P^G C_G}{P^Y Y}$ and P^G is the public consumption deflator. Let $s^*_{c_G} = \frac{U_{c_G} c_G}{\lambda}$. Then we can write:

$$\frac{v_t - v}{\lambda p^Y y} = E_t \sum_{s=0}^{\infty} \beta^s \left[\widehat{y}_{t+s} - s_L \widehat{L}_{t+s} - s_K \widehat{k}_{t+s-1} + \left(s^*_{c_G} - s_{c_G} \right) \widehat{c}_{G,t+s} \right] + \frac{1}{\beta} \frac{k}{p^Y y} \widehat{k}_{t-1} \tag{41}$$

Hence in the presence of public consumption the Solow residual needs to be adjusted up or down depending on whether public consumption is under- or over-provided (i.e., $s^*_{c_G} > s_{c_G}$ or $s^*_{c_G} < s_{c_G}$ respectively). If the government sets public consumption exactly at the utility-maximizing level for the household, $s^*_{c_G} = s_{c_G}$ and no correction is necessary. In turn, in the standard neoclassical case in which public consumption is pure waste $s^*_{c_G} = 0$, the welfare residual is computed on the basis of private final demand – i.e., GDP minus government purchases.

What if government purchases also yield productive services to private agents? This could be the case if, for example, the government provides education or health services, or public infrastructure, which may be directly valued by consumers and may also raise private-sector productivity. In such case, the above expression remains valid, but it is important to note that the net contribution of public expenditure to welfare would not be fully by captured by $\left(s^*_{c_G} - s_{c_G} \right) \widehat{c}_{G,t+s}$. To this term we would need to add a measure of the productivity of public services, which in the expression is implicitly included in the productivity residual $\widehat{y}_{t+s} - s_L \widehat{L}_{t+s} - s_K \widehat{k}_{t+s}$.

4.4 Open Economy

In a closed economy without government b_t represents the net stock of domestic bonds. In deriving our basic equation (13), however, we have not made use of the fact that net bond holdings equal zero in equilibrium when b_t denotes private bonds. Therefore (13) applies also to an open economy. In the latter case, we interpret b_t as net foreign assets, and replace GDP in (13) with domestic absorption as a measure of output. However, we can still write:

$$\frac{v_t - v}{\lambda p^a a} = E_t \sum_{s=0}^{\infty} \beta^s \left[\widehat{a}_t - s_L \ \widehat{L}_{t+s} - s_K \ \widehat{k}_{t+s} \right] + \frac{1}{\beta} \frac{k}{\lambda p^a a} \widehat{k}_{t-1} \tag{42}$$

where a denotes domestic absorption and is defined as:

$$\widehat{a}_t = s_c \widehat{c}_t + s_i \widehat{i}_t \tag{43}$$

and s_L, s_K, s_c and s_i are also shares out of domestic absorption. Suppose one wants to use a standard measure of output, real GDP, defined as consumption, plus investment, plus net exports. Then utility can be written as a function of a more conventionally defined productivity residual and of additional components that capture terms of trade and capital gains effect. Moreover the initial conditions should also include the initial value of net foreign assets. We can show this by starting from the definition of a country's current account:

$$CA_t = B_t - B_{t-1} = i_t B_{t-1} + P_t^{EX} EX_t - P_t^{IM} IM_t \tag{44}$$

where B_t is now the value of the net foreign assets, EX_t and IM_t are total exports and total imports and P_t^{EX} and P_t^{IM} are their respective prices. In normalized form (44) becomes:

$$b_t = \frac{(1 + r_t)}{(1 + g)(1 + n)} b_{t-1} + p_t^{EX} ex_t - p_t^{IM} im_t \tag{45}$$

Log-linearizing we obtain:

$$b\widehat{b}_t = \frac{(1 + r)\, b}{(1 + g)(1 + n)} \widehat{b}_{t-1} + \frac{br}{(1 + g)(1 + n)} \widehat{r}_t + p^{EX} ex \widehat{p}_t^{EX} - p^{IM} im \widehat{p}_t^{IM} + ex\widehat{ex}_t - p^{IM} im\widehat{im}_t \tag{46}$$

where $ex_t = \frac{EX_t}{P_t^I X_t N_t}$; $im_t = \frac{IM_t}{P_t^I X_t N_t}$; $p_t^{EX} = \frac{P_t^{EX}}{P_t^I}$ and $p_t^{IM} = \frac{P_t^{IM}}{P_t^I}$. Equation (11) can now be rewritten as:

$$\begin{aligned} v_t - v \;\; = \;\; & E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c\widehat{c}_{t+s} + i\widehat{i}_{t+s} + b\widehat{b}_{t+s} - \frac{(1 + r)\, b\widehat{b}_{t+s-1}}{(1 + g)(1 + n)} - p^L L \ \widehat{L}_{t+s} - \frac{p^K k\widehat{k}_{t+s-1}}{(1 + g)(1 + n)} \right] \\ & + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} + \lambda \frac{1}{\beta} b\widehat{b}_{t-1} \end{aligned} \tag{47}$$

Define the normalized real GDP in deviation from the steady state as:

$$\widehat{y}_t = s_c \widehat{c}_t + s_i \widehat{i}_t + s_x \widehat{ex}_t - s_m \widehat{im}_t \tag{48}$$

where s_c, s_i, s_x and s_m are respectively the shares of consumption, investment, exports and imports out of total value added. Using the equations (46) and (48) into (47), we get:

$$\begin{aligned}\frac{v_t - v}{\lambda p^Y y} &= E_t \sum_{s=0}^{\infty} \beta^s \left[\widehat{y}_{t+s} - s_L \widehat{L}_{t+s} - s_K \; \widehat{k}_{t+s-1} + (\frac{br/p^Y y}{(1+g)(1+n)} \widehat{r}_{t+s} + s_x \widehat{p}^{EX}_{t+s} - s_m \widehat{p}^{IM}_{t+s}) \right] \\ &\quad + \frac{1}{\beta} \frac{k}{p^Y y} \widehat{k}_{t-1} + \frac{1}{\beta} \frac{b}{p^Y y} \widehat{b}_{t-1} \qquad (49)\end{aligned}$$

where s_L and s_K are also shares out of total value added. Hence, in an open economy the standard Solow residual needs to be adjusted for the returns on net foreign assets, $\frac{br/p^Y y}{(1+g)(1+n)} \widehat{r}_t$, and for terms capturing the terms of trade, $s_x \widehat{p}^{EX}_t - s_m \widehat{p}^{IM}_t$. An improvement in the terms of trade has effects analogous to an increase in TFP - both give the consumer higher consumption for the same input of capital and labor (and therefore higher welfare). See Kohli (2004) for a static version of this result.

The terms in $\widehat{r}_{t+s}$ also capture present and expected future capital gains and losses on net foreign assets due either to exchange rate movements or to changes in the foreign currency prices of the assets. Finally, the initial conditions include not only the (domestic) capital stock, but also the net stock of foreign assets.

Conceptually, it makes very good sense that all these extra terms come into play when taking the GDP route to the measurement of welfare in the open economy. Measuring them empirically poses major challenges, however. One needs reliable measures of changes in foreign asset holdings for a large sample of countries. Asset returns would have to be measured in risk-adjusted terms to make them comparable across countries. In addition, forecasts of future asset returns and the terms of trade would be required as well. In contrast, all these problems disappear if the measurement of welfare is based on real absorption rather than GDP as the measure of output, in which case the same terms that summarize welfare in the closed economy suffice to measure it in the open economy. The implication is that we can measure welfare empirically in ways that are invariant to the degree of openness of the economy.

4.5 Multiple Wages and Labor Market Rationing

So far we have assumed that the household is a price-taker in goods and factor markets, and that it faces no constraints other than the intertemporal budget constraint. We have exploited the insight that under these conditions relative prices measure the representative consumer's marginal rate of substitution between goods, even when relative prices do not measure the economy's marginal rate of transformation. We now ask whether our conclusions need to be modified in environments where the household does not behave as a price taker. We present two examples, and then draw some tentative conclusions about the robustness of our previous results.

Our examples focus on the labor market. It seems reasonable to assume that consumers are price-takers in capital markets; most individuals take rates of return on assets as exogenously given.

The assumption is still tenable when it comes to the purchase of goods, although some transaction prices may be subject to bargaining. The price-taking assumption seems most questionable when it comes to the labor market, and indeed several literatures (on labor search, union wage setting, and efficiency wages, to name three) begin by assuming that households are not price takers in the labor market. Thus, we study two examples. One is in the spirit of the dual labor markets literature, where wages are above their market-clearing level in some sectors but not in other. We do not model why wages are higher in the primary sector, but this can be due to the presence of unions or government mandates in formal but not in informal employment, or efficiency wage considerations in some sectors but not in others. Wages in the secondary market are set competitively. The second example is in the spirit of labor market search, and has households face a whole distribution of wages. In both cases households would prefer to supply all their labor to the sector or firm that pays the highest wage, but are unable to do so. In this sense, both examples feature a type of labor market rationing. (In both cases the different wages are paid to identical workers, and are not due to differences in human capital characteristics.)

First, consider the case in which the household can supply labor in two labor markets. The primary market pays a high wage $\overline{P_t^L}$ and the secondary market pays a lower wage $\underline{P_t^L} < \overline{P_t^L}$. Although the worker prefers to work only in the primary sector, the desirable jobs are rationed; he cannot supply more than $\widetilde{L}$ hours in the high-wage job in each period. The representative household faces the following budget constraint:

$$P_t^I K_t + B_t = (1-\delta) P_t^I K_{t-1} + (1+i_t) B_{t-1} + N_t \overline{P_t^L} \widetilde{L} + N_t \underline{P_t^L} (L_t - \widetilde{L}) + P_t^K K_{t-1} + \Pi_t - P_t^C N_t \quad (50)$$

Assuming that the labor rationing constraint is binding, our previous derivations remain valid and equations (16), (19), (23) and (25) continue to hold, as long as we redefine the modified productivity residual as:

$$\log PR_t = \log \frac{Y_t}{N_t} - \underline{s_L} \log L_{h,t} - s_K \log \frac{K_{h,t-1}}{N_{h,t-1}} \quad (51)$$

where the distributional share of labor $\underline{s_L} \equiv \frac{\underline{P^L} L}{P^Y Y}$ is computed using the *low* wage, paid in the dual labor market, rather than the average wage. The intuition for this result comes from the fact that the marginal wage for the household is $\underline{P_t^L}$ while $N_t(\overline{P_t^L} - \underline{P_t^L})\widetilde{L}$ can be considered as a lump-sum transfer and can be treated exactly like the profit term in the budget constraint. (Thus, we can also allow for arbitrary variations over time in the primary wage $\overline{P_t^L}$ or the rationed number of hours $\widetilde{L}$ without changing our derivations.)

This example shows that in some cases our methods need to be modified if the household is no longer a price-taker. However, in this instance the modification is not too difficult—one can simply decrease the labor share by the ratio of the average wage to the competitive wage. Furthermore, this example shows that imperfect competition in factor markets can introduce an additional gap between the welfare residual and the standard Solow residual that is like a tax wedge, making our modifications to standard TFP even more important if one wants to use TFP data to capture welfare. As is the case with taxes, welfare rises with increases in output holding inputs constant, even if there is no change in actual technology.

Note that we would get a qualitatively similar result if, instead of labor market rationing, we assumed that the household has monopoly power over the supply of labor, as in many New Keynesian DSGE models. As in the example above, we would need to construct the true labor share by using the household's marginal disutility of work, which would be less than the real wage. In this environment, we would obtain the welfare-relevant labor share by dividing the observed labor share by an assumed value for the average markup of the wage over the household's marginal rate of substitution between consumption and leisure.

The second example shows that there are situations where our previous results in sections 2 and 3 are exactly right and need no modification, even with multiple wages and labor market rationing. Consider a household that comprises a continuum of individuals with mass N_t. Suppose that each individual can either not work, or work and supply a fixed number of hours $\widehat{L}$. In this environment, the household can make all its members better off by introducing lotteries that convexify their choice sets. Suppose that the household can choose the probability q_t for an individual to work. The representative household maximizes intertemporal utility:

$$V_t = \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} \left[q_t U(C_t^0; \overline{L} - \widehat{L}) + (1-q_t) U(C_t^1; \overline{L}) \right] \tag{52}$$

where $q_t U(C_t^0; \overline{L} - \widehat{L}) + (1-q_t) U(C_t^1; \overline{L})$ denotes expected utility prior to the lottery draw. C_t^0 and C_t^1 denote respectively per-capita consumption of the employed and unemployed individuals, while average per-capita consumption, C_t is given by:

$$C_t = q_t C_t^0 + (1-q_t) C_t^1 \tag{53}$$

Assume that the individuals that work face an uncertain wage P_t^L, which is observed only after labor supply decisions have been made. More specifically, individual wages in period t are iid draws from a distribution with mean $E_t P_t^L$. Notice that, by the law of large numbers, the household does not face any uncertainty regarding its total wage income. Thus, the budget constraint for the household becomes:

$$P_t^I K_t + B_t = (1-\delta) P_t^I K_{t-1} + (1+i_t) B_{t-1} + q_t N_t E_t P_t^L \widehat{L} + P_t^K K_{t-1} + \Pi_t - P_t^C \left(q_t C_t^0 + (1-q_t) C_t^1 \right) N_t \tag{54}$$

Following Rogerson and Wright (1988) and King and Rebelo (1999),[15] we can rewrite the per-period utility function as:

[15] In obtaining this result we use the fact that the marginal utility of consumption of the individuals in the household needs to be equalized at the optimum. This implies: $c_t^0 = c_t^1 \left(\frac{\nu(\overline{L} - \widehat{L})}{\nu(\overline{L})} \right)^{\frac{1}{\sigma}}$.

$$U(C_t; L_t) = \frac{1}{1-\sigma} {C_t}^{1-\sigma} \nu^*(L_t) \tag{55}$$

where $L_t = q_t \widehat{L}$ denotes the average number of hours worked and:

$$\nu^*(L_t) = \left(\frac{L_t}{\widehat{L}} \nu(\overline{L} - \widehat{L})^{\frac{1}{\sigma}} + (1 - \frac{L_t}{\widehat{L}}) \nu(\overline{L})^{\frac{1}{\sigma}} \right)^{\sigma} \tag{56}$$

In summary, the maximization problem faced by the household is exactly the same as the one described in section 2, even if identical workers are paid different wages [16]. All the results we have derived previously also apply in this new setting. The second example leads to a different result from the first for two reasons. First, it is an environment with job search rather than job queuing. Second, the number of hours supplied by each worker is fixed. Under these two assumptions, the labor supply decision is made *ex ante* and not *ex post*.

From these two examples, it is clear that dropping the assumption that all consumers face the same price for each good or service can—but need not—change the precise nature of the proxies we develop for welfare. Even in the case where the measure changed, however, our conclusion that welfare can be summarized by a forward-looking TFP measure and capital intensity remained robust. While the exact nature of the proxy will necessarily be model-dependent, we believe that our basic insight applies under fairly general conditions.

5 Sources of Welfare Changes and Differences

In this section we discuss how our index of welfare changes over time for each country in our data set. Expressed in terms of equivalent consumption, our calculations are based on the following equation, based on (20), that encompasses all the cases we will consider:

$$\Delta \log (C_t)^* = \frac{(1-\beta)}{(s_C + s^*_{C_G})} \left[E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} + \sum_{s=0}^{\infty} \beta^s \Delta E_t \log PR_{t+s} + \frac{1}{\beta} \frac{k}{p^y y} \Delta \log \frac{K_{t-1}}{N_{t-1}} \right] \tag{57}$$

where productivity change is defined as:

$$\Delta \log PR_t = s_C \Delta \log C_t + s_I \Delta \log \frac{I_t}{N_t} + s^*_{C_G} \Delta \log C_{G,t} - \left(1 - \tau^L\right) s_L \Delta \log L_t - \left(1 - \tau^K\right) s_K \Delta \log \frac{K_{t-1}}{N_{t-1}} \tag{58}$$

[16] King and Rebelo (1999) show that in this framework the representative agent has an infinite Frisch labor supply elasticity. This result follows from the assumption that all agents in the household have the same disutility of labor. Mulligan (2001) shows that even when all labor is supplied on the extensive margin, one can obtain any desired Frisch elasticity of labor supply for the representative agent by allowing individual agents to have different disutilities of labor. In a more elaborate example, we could use Mulligan's result to show that the only restrictions on the preferences of the representative agent are those that we assume in Section 2.

where the shares are $s_c \equiv \frac{(1+\tau^C)P^C CN}{P^C CN+P^I I+P^G C_G N}$; $s_i \equiv \frac{(1+\tau^I)P^I I}{P^C CN+P^I I+P^G C_G N}$ and, as explained earlier, the value of $s^*_{C_G}$ depends on the assumptions made about government consumption. If the latter is chosen so as to optimize the representative household's welfare, then $s^*_{C_G} = s_{c_G} \equiv \frac{P^G C_G N}{P^C CN+P^I I+P^G C_G N}$. Alternatively, if government consumption is pure waste, $s^*_{C_G} = 0$. Recall that C_t and $C_{G,t}$ denote private and public consumption per capita, while $\frac{I_t}{N_t}$ denotes investment per capita. Equation (57), based on domestic absorption, is appropriate also in the open economy case, and allows us to consider a variety of cases with respect to taxation and government spending: 1) wasteful government spending with lump sum taxes (in which case distortionary taxes are set to zero in the productivity equation); 2) optimal government spending with lump sum taxes; 3) wasteful government spending with distortionary taxes; 4) optimal government spending with distortionary taxes.

For the cross country welfare comparisons, our empirical calculations are based on the following equation, which generalizes (26):

$$\ln C_t^{*,i} - \ln C_t^{*,us} = \frac{(1-\beta)}{(s_C^{us} + s_{C_G}^{*us})} \left[E_t \sum_{s=0}^{\infty} \beta^s \left(\log \overline{PR}_{t+s}^i - \log PR_{t+s}^{us} \right) + \frac{1}{\beta} \frac{k^{us}}{p^{Y,us} y^{us}} \left(\log \frac{K_{t-1}^i}{N_{t-1}^i} - \log \frac{K_{t-1}^{us}}{N_{t-1}^{us}} \right) \right] \tag{59}$$

where the productivity of the reference country (US in our benchmark case) is:

$$\log PR_t^{us} = s_C^{us} \log C_t^{us} + s_I^{us} \log \frac{I_t^{us}}{N_t^{us}} + s_{C_G}^{*us} \log C_{G,t}^{us} - \left(1 - \tau^{L,us}\right) s_L^{us} \log L_t^{us} - \left(1 - \tau^{K,us}\right) s_K^{us} \log \frac{K_{t-1}^{us}}{N_{t-1}^{us}} \tag{60}$$

while the productivity for country i is calculated as:

$$\log \overline{PR}_t^i = s_C^{us} \log C_t^i + s_I^{us} \log \frac{I_t^i}{N_t^{is}} + s_{C_G}^{*us} \log C_{G,t}^i - \left(1 - \tau^{L,us}\right) s_L^{us} \log L_t^i - \left(1 - \tau^{K,us}\right) s_K^{us} \log \frac{K_{t-1}^i}{N_{t-1}^i} \tag{61}$$

Shares and tax rates used in the calculation of $\log \overline{PR}_t^i$and $\log PR_{t+s}^{us}$ are now those of the reference country (the US in our basic set of results) and shares are defined as in the within case.

5.1 Data and Measurement

In order to discuss how our index of welfare changes over time within a country and how it compares across countries, we use yearly data on consumption, investment, capital and labor services for the years 1985-2005 and for seven countries: the US, the UK, Japan, Canada, France, Italy and Spain. We are unable to include Germany in the sample, since data for unified Germany are available only since 1995 in EU-KLEMS. We use two different data sets to compare welfare within a country and across countries.

To analyze welfare changes over time within a country, we combine data coming from the OECD

Statistical Database and the EU-KLEMS dataset[17]. Our index of value added is constructed from the OECD dataset as the weighted growth of household final consumption, gross capital formation and government consumption (where appropriate) at constant national prices, using as weights their respective shares of value added. According to our theory, these shares should be kept constant at their steady state level, but in practice we use shares averaged across the twenty years in our sample.

In constructing the growth rate of the "modified" productivity residual, we subtract from the log-changes in value added, the log changes in the capital and labor stocks used to produce it, weighted by their average respective shares out of total compensation. Data on aggregate production inputs are provided by EU-KLEMS. Log-changes in capital are constructed using the estimated capital stock constructed by applying the perpetual inventory method on investment data. In our benchmark specification, log-changes in the labor stock are approximated by log-changes in the amount of hours worked by persons engaged. Alternatively, we use a labor service index which is computed as a translog function of types of workers engaged (classified by skill, gender, age and sex), where weights are given by the average share of each type of worker in the value of total labor compensation. We assume that economic profits are zero in the steady state so that we can recover the gross (tax unadjusted) share of capital as one minus the labor share.

In order to compare welfare across countries, we combine data coming from the Penn World Tables and the EU-KLEMS dataset. More specifically, our basic measure of value added is constructed from the Penn World Tables as the weighted average of PPP converted log-private consumption, log-gross investment and log-government consumption, using as weights their respective shares of value added in the reference country; as in the within case, we use shares that are averaged across the twenty years in our sample.

To construct the modified log-productivity residual in each country, we subtract from this measure of value added the amount of capital and the amount of labor used to produce it, weighted by their respective share of compensation in the reference country, also in this case kept constant at their average value. The stock of capital in the economy is constructed using the perpetual inventory method on the PPP converted investment time series from the Penn World Tables. The stock of labor is computed in two different ways using the EU-KLEMS dataset. In our benchmark specification, the amount of labor services used in the economy is approximated by the hours worked. In the alternative specification, it is computed by aggregating over different types of persons engaged using a translog function, where weights are given by the shares of compensation to each type of labor out of total compensation to labor and are kept constant at their average value in the reference country.

Finally, to correct our welfare calculations for the presence of distortionary taxation, we use data on average tax rates on capital and labor provided by Boscá et al. (2005). The tax rates are computed by combining realized tax revenues, from the OECD Revenue Statistics, with estimates of the associated tax bases derived from the OECD National Accounts. These data update the tax

[17] The EU-KLEMS data are extensively documented by O'Mahony and Timmer (2009).

rates constructed by Mendoza et al (1994) and introduce some methodological improvements in their calculation, most of which are described in Carey and Tchilinguirian (2000). In essence, they involve some adjustments to the definition of the various tax bases.

5.2 Within Results

Since the change in welfare over time depends on the expected present discounted value of TFP growth, as shown by equation (57), we need to construct forecasts of future TFP. To do so, we estimate univariate time-series models using annual data for the seven countries in our data set. Our sample period runs from 1985 to 2005 for all countries.

We use the various aggregate TFP measures suggested by our theory (in log levels), and estimate simple AR processes for each country. The persistence of TFP growth is a key statistic, since it shows how the entire summation of expected productivity residuals changes as a function of the innovation in the log level of TFP. We report the persistence of TFP using simple, reduced-form forecasting equations for two different definitions of TFP, which we will use as benchmarks throughout.

The first concept is TFP in the case where we assume that government purchases are wasteful, and taxes are lump-sum. For this case, as discussed above, we construct output by aggregating consumption and investment only, but using shares that sum to $(1 - s_{c_G})$, and we do not correct the capital and labor shares for the effects of distortionary taxes. In this case, the capital and labor shares sum to one. The second case is the one where we assume government spending is optimally chosen, but needs to be financed with distortionary taxes. In this case, the output concept is the share-weighted sum (in logs) of consumption, investment and government purchases, and the capital and labor shares are corrected for both income taxes and indirect taxes (both of which reduce the after-tax shares). Note that in both cases the output concept measures absorption rather than GDP (unless the economy is closed, in which case the two concepts coincide). Thus, following our discussion in Section 4.4, both concepts (and indeed all the TFP measures that we use in this section) are appropriate for measuring welfare in both closed and open economies. In both cases, we assume that pure economic profits are zero in the steady state.

For all countries, the log level of TFP is well described by either an AR(1) or AR(2) stationary process around a linear trend. In Table (1) we report the estimation results obtained using the two definitions of TFP stated above, together with the Lagrange Multiplier test for residual first order serial correlation (shown in the last line of each panel in the table), confirming that we cannot reject the null of no serial correlation for the preferred specification for each country. For all countries, the order of the estimated AR process is invariant to the TFP measure used. In all cases, we can comfortably reject the null of a unit root in the log TFP process (after allowing for a time trend). We use the estimated AR processes to form expectations of future levels or differences of TFP, which are required to construct our welfare indexes.

We use equation (57), derived from (20), to express the average welfare change per year in each country in terms of changes in equivalent consumption. Given the time-series processes for TFP in each country, we can readily construct the first two terms in equation (57), the present value of expected TFP growth, and the change in expectations of that quantity. The third term, which depends on the change in the capital stock can also be constructed using data from EU-KLEMS. We assume that the composite discount rate, β, is common across countries and we set it equal to 0.95.[18] For the expenditure and distributional shares, we use their country specific averages over the sample period.

The results are in Table 2. We see that assumptions about fiscal policy affect the results in significant ways. We first illustrate our methods by discussing the results for the US, which are given in the last row. We then broaden our discussion to draw more general lessons from the full set of countries.

In the first column of Table 2, we construct the output data and the capital and labor shares under the assumption that government expenditure is wasteful and taxes are lump-sum. In this column, "utility-relevant output" comprises just consumption and investment, aggregated using weights that sum to less than one.[19] In this case, the average annual growth rate of welfare in the US is equivalent to a permanent annual increase in consumption of about 2.5 percent. Recall from Section 4.4 that this result applies whether we think of the US as an open or a closed economy. The same is true for all the other results in Table 2–all apply to open as well as closed economies.

Now we study the case of optimal government spending, still under the assumption that taxes are not distortionary. Thus, at the margin the consumer is indifferent between an additional unit of private consumption and an additional unit of government expenditures. In this case, output consists of consumption, investment and government purchases, aggregated using nominal expenditure shares that sum to one. In a closed economy this concept of TFP corresponds to the standard Solow residual. Note that in all cases our output concepts correspond to different measures of absorption; this is why they are relevant for both closed and open economies.

Welfare growth for the US is only slightly higher when we assume that expenditures are optimal: 2.6 versus 2.5 percent for the lump-sum tax cases. (We will see that this result is not universal within our sample of countries.) On the whole, the differing assumptions about the value of government expenditure do not change the calculated US growth rate of welfare significantly. Note, however, that this result does *not* mean that the US consumer is indifferent between wasteful and optimal government spending. Steady-state welfare is surely much lower in the case where the government wastes 20 percent of GDP. However, our results show that the difference in welfare in the two cases is almost entirely a *level* difference rather than a *growth rate* difference.

We repeat our welfare calculations under the assumption that the government raises revenue via distortionary taxes. The results are in columns 3 and 4 of Table 2. As shown above, if taxes are distortionary we need to construct the factor shares in the Solow residual using the after-tax wage and capital rental rate perceived by the household, implying that the shares will sum to less

[18] We construct our measure of β following the method of Cooley and Prescott (1995), who find $\beta = 0.947$.

[19] The weight on consumption is the nominal value of consumption divided by nominal expenditures on consumption, investment and government purchases. The weight on investment is the nominal value of investment, divided by the same denominator.

than one. We construct the new shares using the tax rates described in the previous section. The quantitative effect of this change is significant. In both of the cases we consider (wasteful spending and optimal spending), per capita welfare growth expressed in terms of consumption growth rates is *higher* by nearly half a percentage point per year. Intuitively, if taxes are distortionary then steady-state output is too low; thus, any increase in output, even with unchanged technology, is a welfare improvement. It is quantitatively important to allow for the fact that taxes are distortionary and not lump-sum. For the US, it matters more for the growth rate of welfare than whether we assume that government spending is wasteful or optimal. We take as our benchmark the case shown in the last column, where spending is optimally chosen (from the point of view of the household) and taxes are distortionary. In this case, average US welfare growth is equivalent to a growth rate of per-capita consumption of 3 percent per year.

Assumptions about fiscal policy naturally matter more in countries with a high rate of growth of government purchases per capita and with a high growth rate of factor inputs. For example, both facts are true for Spain over our sample period. The growth rate of welfare in Spain nearly doubles from the first column, where its 2.1 percent annual welfare growth rate is literally middling, to the last, where its 4 percent growth rate is the highest among all the countries in our sample. Assumptions about fiscal policy also matter significantly for Canada and Japan, and change the welfare growth rates of these countries by a full percentage point or more. In percentage terms, the change is particularly dramatic for Canada. Under three of the four scenarios, the UK leads our sample of countries in welfare growth rates; in the last case, it is basically tied with Spain.

Finally, we show the full time series of the welfare indexes for each country graphically, for our two benchmark cases of wasteful spending with lump-sum taxes and optimal spending with distortionary taxes. In Figures 1 and 2 we report the evolution over time of our welfare indexes for each country, in log deviations from their values in 1985. In Figure 1, the UK is the clear growth leader, with France and the US nearly tied in a second group, and Canada trailing badly. In Figure 2, by contrast, there are three clear groups: the UK and Spain lead, by a considerable margin; the US, France, and Japan comprise the middle group; Italy and Canada have the lowest welfare growth rates. Two countries show significant declines in growth rates, both starting in the early 1990s. The first is Japan, which in the first few years of our sample grew in line with the leading economies, Spain and the UK, and then slowly drifted down in growth rate to end the sample in the middle group, with France and the US. Similarly, Italy used to grow at the pace of the middle group, but then experienced a slowdown which, by the end of the sample, caused it to leave the middle group and form a low-growth group with Canada. Thus, our results are consistent with the general impression that Italy and Japan experienced considerable declines in economic performance over the last two decades relative to the performance in the earlier postwar period.

In Table 3, we investigate which of the two components of welfare–TFP growth or capital accumulation–contributed more to the growth rate of welfare in our sample of countries. For the purpose of this decomposition, we treat the expectation-revision term as a contribution to TFP. In order to keep the table uncluttered, we drop the case where government spending is optimal and taxes are lump sum. The first column, in which government spending is wasteful and taxes are treated as lump-sum, shows that four of the seven countries have achieved two-thirds or more of their welfare gains mostly via TFP growth. The exceptions are the three countries that are known to have had low TFP growth over our sample period: Japan, Canada, and Spain. Moving to the case of distortionary taxes raises the TFP contribution (by reducing the factor shares), as does changing the treatment of public spending as optimal rather than wasteful (which raises the growth rate of output, and thus TFP). In the case of optimal spending with distortionary taxes, all countries get a majority of their welfare growth from TFP. Only in Japan and Canada is the contribution of TFP to welfare less than 70 percent, and in most cases it is 75 percent or more.

We check the robustness of the previous results to using a more refined measure of labor input. As noted above, for the main results we use total hours worked as the measure of labor. However, if workers are heterogeneous along dimensions that affect their productivity and are paid different wages as a consequence, we should use a labor input index that recognizes this fact. This amounts to implementing our extension in Section 4.1. The results, for the case of optimal spending with distortionary taxes, are shown in Figure 3. Qualitatively, there is little change. The UK and Spain are still bunched at the top, followed by the US and France. However, with the labor index, Japan drops a little further behind these four leading economies, and ends the period with a cumulated welfare growth rate in between the US and France and the trailing economies, Italy and Canada. Overall, the results look very similar to our baseline case.

Finally, we compare the results we have just obtained using our theory-based welfare metric to those implied by standard proxies for welfare change. In Table 4, we present the average growth rates of GDP and consumption per capita for our group of countries over our sample period, as well as the average growth rate of our welfare measure under the assumption of optimal government spending and distortionary taxes. First, note the differences in magnitude. Welfare usually grows faster than do conventional measures like consumption per capita. The difference is typically in the order of one full percentage point per year, which is a striking difference in growth rates. Second, these different measures sometimes produce quite different rankings among countries. Take France, for example. Judged by consumption growth, this country comes at the very bottom of our group of seven countries, significantly behind even Canada. In terms of welfare growth, on the other hand, France comes second.

5.3 Cross Country Results

We now turn to measuring welfare differences across the countries in our sample. For each country and time period, we calculate the welfare gap between that country and the US, as defined in equation (59). Recall that this gap is the loss in welfare of a representative US consumer who is moved permanently to country i starting at time t, expressed as the log gap between the "equivalent consumption" of the consumer in the two cases. In this hypothetical move, the consumer loses the per-capita capital stock of the US, but gains the equivalent capital stock of country i. From time

t on, the consumer faces the same product and factor prices and tax rates, and receives the same lump-sum transfers and government expenditure benefits as all the other consumers in country i. In a slight abuse of language, we often use refer to the incremental equivalent consumption as "the welfare difference" or "the welfare gap."

Note that these gaps are all from the point of view of a US consumer. Hence, all the shares in (59), even those used to construct output and TFP growth in country i, are the *US* shares. This naturally raises the question whether our results would be quite different if we took a different country as our baseline. We return to this issue after presenting our basic set of results.[20]

We present numerical results in Table 5. Since the size of the gap varies over time, we present the gap at the beginning of our sample, at the end of our sample, and averaged over the sample period. We present results for three cases: wasteful spending, with lump-sum and distortionary taxes, and optimal spending with distortionary taxes. These numerical magnitudes are useful references in the discussion that follows.

However, the results are easiest to understand in graphical form. We plot the welfare gap for the countries and time periods in our sample in Figures 4 and 5. Note that by definition the gap is zero for the US, since the US consumer neither gains nor loses by moving to the US at any point in time. The vertical axis shows, therefore, the gain to the US consumer of moving to any of the other countries at any point in the sample period, expressed in log points of equivalent consumption. Figure 4 shows the results for the case of wasteful spending and lump-sum taxes. Figure 5 shows the results for our benchmark case, where we allow for distortionary taxes and assume that government expenditure is optimally chosen. Since both figures show qualitatively similar results, for brevity we discuss only the benchmark case.

It is instructive to begin by focusing on the beginning and end of the sample. At the beginning of the sample, expected lifetime welfare in both France and the UK was less than 20 percent lower than in the US (gaps of 16 and 19 percent, respectively). This relatively small gap reflects both the long-run European advantage in leisure and the fact that in the mid-1980s the US was still struggling with its productivity slowdown, while TFP in the leading European economies was growing faster than in the US. Capital accumulation was also proceeding briskly in those countries. By the end of the sample, the continental European economies, Canada and Japan are generally falling behind the US, because they had not matched the pickup in TFP growth and investment experienced in the US after 1995. Italy experiences the greatest relative "reversal of fortune," ending up with a welfare gap of nearly 70 percent relative to the US. The results for France are qualitatively similar, but far less extreme. France starts with a welfare gap of 16 percent, and slowly slips further behind, ending with a gap of 21 percent. In continental Europe, only Spain shows convergence to the US in terms of welfare: it starts with a gap of 41 percent, and ends with a gap of 36 percent. However, after 1995 Spain holds steady relative to the US, but does not gain further.

[20] We conjecture that if we took a second-order approximation to the welfare gap, then the shares in our computation would be averages of the shares in the two countries, and hence bilateral comparisons would be invariant to the choice of a reference country. We leave the investigation of this hypothesis to future research.

The only economy in our sample that exhibits convergence to the US throughout our sample is the UK. Indeed, as Figure 4 shows, under the assumption of wasteful spending and lump-sum taxes, the UK overtakes the US by the end of our sample period. Table 5 shows that in more realistic cases where taxes are assumed to be distortionary the welfare level of the UK is always below that of the US, but the UK shows strong convergence, slicing two-thirds off the welfare gap in two decades in our benchmark case. This result is interesting, because the UK experienced much the same lack of TFP growth in the late 1990s and early 2000s as the major continental European economies.[21] However, the UK had very rapid productivity growth from 1985 to 1995. The other "Anglo-Saxon" country in our sample, Canada, had a welfare level about 30 percent below that of the US in 1985, but the welfare gap had grown by an additional 50 percent by the end of the sample. This result is due primarily to the differential productivity performance of the two countries: TFP in Canada actually fell during the 1990s, and rose only slowly in the early 2000s.

Perhaps the most striking comparison is between the US and Japan. Even in 1985, when its economic performance was the envy of much of the world, Japan was the least attractive country in our sample to an US consumer contemplating emigration; such a consumer would give up nearly 50 percent of his consumption permanently in order to stay in the US instead of moving to Japan. However, like the UK and Spain, Japan was closing the gap with the US until its real estate bubble burst in 1991. The relative performance of the three countries changes dramatically from that point: unlike the UK, which continues to catch up, and Spain, which holds steady, Japan begins to fall behind the US, first slowly and then more rapidly. Having closed to within 43 percent of the US welfare level in 1991, Japan ends our sample 53 percent behind. This cautionary history suggests that it would be interesting to see what the same calculations will show for the US in another 10 or 15 years, after the bursting of the US real estate bubble and the associated financial crisis.

As we did for the within-country results, we investigate whether the cross-country welfare gaps are driven mostly by the TFP gap or by differences in capital per worker. The results are in Table 6. We focus on the last column of Panel C, which gives results averaged over the full sample period for our baseline case of optimal spending with distortionary taxes. We find that for five of the six countries, TFP is responsible for the vast majority of the welfare gap relative to the US. Indeed, for Japan TFP accounts for more than 100 percent of the gap (meaning that Japan has generally had a higher level of capital per capita than the US). Thus we arrive at much the same conclusion as Hall and Jones (1999), although our definition of TFP is quite different from the one they used, and we do not focus only on steady-state differences. The exception to this rule is the UK. The average welfare gap between the US and the UK is driven about equally by TFP and by capital. Panel B shows that by the end of the sample, the UK had surpassed the US in "welfare-relevant TFP," and more than 100 percent of the gap was driven by the difference in per-capita capital between the two economies.

We now check the robustness of the preceding results along two dimensions.

[21] For discussion and a suggested explanation, see Basu, Fernald, Oulton and Srinivasan (2004).

First, as noted above, we wish to see whether our welfare rankings among countries is sensitive to the choice of the baseline country. We thus redo the preceding exercises taking France as the baseline country. France is the largest and most successful continental European economy in our sample, and by revealed preference French households place much higher weight on leisure than do US ones.[22] We summarize the results for our baseline case of optimal spending with distortionary taxes in Figure 6. For ease of comparison with the preceding cross-country figures, we still normalize the US welfare level to zero throughout, even though the comparison is done from the perspective of the French consumer and is based on French shares. Reassuringly, we see that the qualitative results are unchanged. France and the UK start closest to the US in 1985, but even they are well behind the US level of welfare. The UK converges towards the US welfare level and so, from a much lower starting point, does Spain. All the other economies, including France, fall steadily farther behind the US over time. Interestingly, from the French point of view almost all the other countries are shifted down vis-a-vis the US relative to the rankings from the US point of view. It appears that the representative French consumer believes that the US is further ahead of France than does the representative US consumer!

Second, we redo the baseline results (from the point of view of the US consumer) using an index of heterogeneous labor input. Our method demands that we construct the labor index for each country weighting the hours of different types of workers by the US shares. However, unlike the within-country case in the previous sub-section, where we used country-specific shares, this procedure yields quite different results than the baseline case using hours. Figure 7 presents the results graphically for our baseline case of optimal spending with distortionary taxes. We still find that the UK and France are closest to the US in welfare levels, but now we no longer find strong evidence of convergence for the UK; both countries steadily fall behind the US over time. Spain shows the greatest difference relative to our previous results. Instead of converging or holding steady relative to the US, Spain falls behind monotonically. This result is driven by the fact that the Spanish *growth rate* of labor input is very high for categories of workers that receive a high *share* of labor income in the US (particularly workers with a "middle" level of education, as defined by EU-KLEMS). As a check, we computed results using the index of labor input, but from the French point of view (i.e., using French shares). We find very similar results for Spain, showing that this result is likely to obtain whenever one applies disaggregated labor shares from rich countries to growth rates of labor input for middle-income (or poor) countries. For this reason, we use total hours as our baseline measure of labor input, in both the within- and cross-country cases.

Finally, as we did for the results on within-country welfare growth, we compare our welfare results to those based on traditional measures, namely PPP-adjusted GDP and consumption per capita. The results are in Table 7. Focusing on Panel B, for the final year of our sample, we see that the three measures sometimes give identical results. For example, the US is atop the world rankings by all three measures, although the gap between the US and the second-ranked country is

[22] As noted above, data limitations prevent us from including Germany in our sample, although it would be another natural baseline.

much smaller in percentage terms for welfare (6 percent) than it is for the other two variables (18 or 19 percent). On the other hand, the differences can be striking. For example, Canada, which is the second only to the US in GDP and third judged by consumption, is third from the bottom in our welfare ranking. Canada, which leads Spain by 20 percent or more in terms of consumption and GDP per capita, trails Spain by about 10 percent in our welfare comparison. Indeed, Spain is last within our group of countries in terms of the conventional metrics of consumption and GDP, but ranks fourth in welfare terms, trailing only the US, UK and France. For the other countries, the welfare measure is not so kind. Japan trails the US by only 26 percent in GDP per capita, but double that—52 percent—in terms of welfare. Similarly, Italy has more than 60 percent of the per-capita GDP of the US, but only about one-third the welfare level. On the other hand, France trails the US by 40 percent in consumption per capita, but by only half that amount in terms of a welfare. Thus, our measure clearly provides new information on welfare differences among countries.

6 Relationship to the Literature

Measuring welfare change over time and differences across countries using observable national income accounts data has been a long-standing challenge for economists. We note here the similarities and differences between our approach and ones that have been taken before. We also suggest ways in which our results might be useful in other fields of economics, where the same question arises in different contexts.

Nordhaus and Tobin (1972) originated one approach, which is to take a snapshot of the economy's flow output at a point in time and then go "beyond GDP," by adjusting GDP in various ways to make it a better flow measure of welfare. Nordhaus and Tobin found that the largest gap between flow output and flow welfare comes from the value that consumers put on leisure. Their result motivated us to add leisure to the period utility function in our model, which is standard in business-cycle analysis but not in growth theory. Nordhaus and Tobin's approach has been followed recently by Jones and Klenow (2010) who add other corrections, notably for life expectancy and inequality. However, this point-in-time approach does not take into account the link between today's choices and future consumption or leisure possibilities. For example, high consumption in the measured period might denote either permanently high welfare or low current investment. Low investment would mean that consumption must fall in the future, so its current level would not be a good indicator of long-term welfare. Our approach is to go beyond point-in-time measures of welfare and compute the expected present discounted value of consumers' entire sequence of period utility. In so doing, we also shift the focus from consumers' particular choices of one period's consumption and leisure to their intertemporal choice sets, as defined by their assets and the sequence of prices they face. This approach pays off particularly when we measure welfare differences across countries.

Our intertemporal approach echoes the methods used in the literature started by Weitzman

(1976) and analyzed in depth by Weitzman (2003), with notable contributions from many other authors.[23] This literature also relates the welfare of a representative agent to observables; for example, Weitzman (1976) linked intertemporal welfare to net domestic product (NDP). Unlike our model which allows for uncertainty about the future, this literature almost always assumes perfect foresight.[24] As we discuss later, allowing for uncertainty is important when forward-looking rules for measurement are applied to actual data. More importantly, the results in these papers are derived using a number of strong restrictions on the nature of technology (typically an aggregate production function with constant returns to scale), product market competition (always assumed to be perfect), and the allowed number of variables that are exogenous functions of time, such as technology or terms of trade (usually none, but sometimes one or two). Most of the analysis in the literature applies to a closed economy where growth is optimal.[25] Taken together, this long list of assumptions greatly limits the domain of applicability of the results.

By contrast, we derive all our results based only on first-order conditions from household optimization, which allows for imperfect competition in product markets of an arbitrary type and for a vast range of production possibilities, with no assumption that they can be summarized by an aggregate production function or a convex technology set. (This makes it easy to apply our results to modern trade models, for example, since these models often assume imperfect competition with substantial producer heterogeneity.) We do not need to assume that the economy follows an optimal growth path. We are also able to allow for a wide range of shocks, including but not limited to changes in technology, tax rates, terms of trade, government purchases, the size of Marshallian spillovers, monetary policy, tariffs, and markups.[26] Crucially, we do not need to specify the sources of structural shocks to the economy. The key to the generality of our results is that we condition on observed prices and asset stocks without needing to model *why* these quantities take on the values that they do.[27] Perhaps most importantly, since we do not make assumptions about the technology or distortions prevailing in each country, we are able to compare welfare across countries that could easily have different levels of technology, taxes or product-market competition.

Our work clarifies and unifies several results in other literatures, especially international economics. Kohli (2004) shows in a static setting that terms-of-trade changes can improve welfare in

[23] A far from exhaustive list includes Asheim (1994), Arronson and Löfgren (1995), Mino (2004), Sefton and Weale (2006), Basu, Pascali, Schiantarelli and Serven (2009), and Hulten and Schreyer (2010). Reis (2005) analyses the related problem of computing a dynamic measure of inflation for a long-lived representative consumer.

[24] Arronson and Löfgren (1995) allow for stochastic population growth, and Weitzman (2003, ch. 6) considers shocks coming from stochastic depreciation of capital.

[25] Sefton and Weale (2006) and Hulten and Schreyer (2010) consider an open economy with changes in the terms of trade and Mino (2004) analyses Marshallian spillovers to R&D (all under perfect foresight).

[26] See also Sandleris and Wright (2011) for an attempt to extend the basic ideas in Basu and Fernald (2002) in order to evaluate the welfare effects of financial crises. These papers try to derive methods to measure the welfare effects of a particular shock, which requires specifying an explicit counterfactual path that the economy would have followed in the absence of the shock.

[27] We do need to forecast the present value of future TFP in order to implement our results in data. It is an open question whether specifying a complete general-equilibrium structure for the model would improve our forecasts substantially. Most macro literatures have concluded that reduced-form forecasts beat model-based ones. We decided that the possible gain in forecasting accuracy from specifying a general-equilibrium structure would not compensate for the loss of generality of our results.

open economies even when technology is constant. Kehoe and Ruhl (2008) prove a related result in a dynamic model with balanced trade: opening to trade may increase welfare, even if it does not change TFP. In these models, which assume competition and constant returns, technology is equivalent to TFP. We generalize and extend these results, and show that in a dynamic environment with unbalanced trade welfare can also change if there are changes in the quantity of net foreign assets or in their rates of return.[28] In general, we show that there is a link between observable aggregate data and welfare in an open economy, which is the objective of Bajona, Gibson, Kehoe and Ruhl (2010). While we agree with the conclusion of these authors that GDP is not a sufficient statistic for uncovering the effect of trade policy on welfare, we show that one can construct such a sufficient statistic by considering a relatively small number of other variables. Our results also shed light on the work of Arkolakis, Costinot and Rodriguez-Clare (2011). These authors show that in a class of modern trade models, which includes models with imperfect competition and micro-level productivity heterogeneity, one can construct measures of the welfare gain from trade without reference to micro data. Our results imply that this conclusion actually holds in a much larger class of models, although the exact functional form of the result in Arkolakis et al. (2011) may not. Finally, since changes in net foreign asset positions and their rates of return are extremely hard to measure, we show that one can measure welfare using data only on TFP and the capital stock, even in an open economy, provided that TFP is calculated using absorption rather than GDP as the output concept.

Our work provides a different view of a large and burgeoning literature that investigates the productivity differences across countries. As noted above, if we specialize our cross-country result to the lump sum-optimal spending case, we obtain something closely related to the results produced by the "development accounting" literature. We show that in that case, (the present value of) the log differences in TFP levels emphasized by the developing accounting literature need to be supplemented with only one additional variable, namely log level gaps in capital per person, in order to serve as a measure of welfare differences among countries.[29] This result implies immediately that estimates of TFP losses due to allocative inefficiency (e.g., Hsieh and Klenow, 2009) can be translated to estimates of welfare losses.

Our results are also related to an earlier literature on "industrial policy." and to more recent literature on the effect of "reallocation". Bhagwati, Ramaswami and Srinivasan (1969) and Bulow and Summers (1986) argue welfare would be enhanced by policies to promote growth in industries where there are rents, for example stemming from monopoly power.[30] The TFP term in our basic result, equation (20), captures this effect. When firms have market power, their output grows

[28] The result that openness does not change TFP may be fragile in models with increasing returns. If opening to trade changes factor inputs, either on impact or over time, then TFP as measured by Solow's residual will change as well, which we show has an effect on welfare even holding constant the terms of trade.

[29] As we show, what matters is the present discounted value of TFP differences. Moreover, one needs to compute TFP using different shares than the ones used by the development accounting literature, and switch to a different output concept (based on domestic absorption) in an open economy.

[30] A second-best policy might involve trade restrictions to protect such industries from foreign competition. If lump-sum taxes are available, the optimal policy is always to target the distortion directly through a tax-cum-subsidy scheme.

faster than the share-weighted sum of their inputs, even when their technology is constant. Thus, aggregate TFP rises when firms with above-average market power grow faster than average. At first this result sounds counterintuitive, since it implies that welfare is enhanced by directing more capital and labor to the most distorted sectors. However, the logic is exactly the same as the usual result that firms with the greatest monopoly power should also receive the largest unit subsidies to increase their output.

A number of recent papers suggest that a substantial fraction of output growth within countries comes from reallocation, broadly defined. However, given the different definitions of "reallocation" used in the literature, it is not clear how much reallocation matters for welfare. Our work suggests that the literature should focus on quantifying the increment to aggregate TFP growth from reallocation. Furthermore, it shows that TFP is what matters, not technical efficiency. TFP contributions can come from either higher technical efficiency, by exploiting increasing returns to scale, or by allocating inputs more efficiently across firms. When TFP and technical efficiency diverge, it is TFP that matters for welfare. We have concentrated on aggregate TFP and capital accumulation without asking how individual firms and sectors contribute to these welfare-relevant variables. Domar (1961) and Basu and Fernald (2002) show how one can decompose standard TFP based on GDP into sectoral and firm-level contributions. However, our results above show that in an open economy one should base TFP measures on absorption rather than GDP. There is as yet no parallel to Domar's (1961) decomposition for absorption or net investment, since that would require allocating the output of individual industries to particular components of final expenditure. The approach of Basu, Fernald, Fisher and Kimball (2010), based on the use of input-output tables, may be helpful in this endeavor.

Finally, our work is closely related to the program of developing sufficient statistics for welfare analysis, surveyed by Chetty (2009). We have proposed such a statistic for a representative consumer in a macroeconomic context. As Chetty notes, such measures can be used to evaluate the effects of policies. Suppose that one wishes to evaluate the effect of a policy change—for example, a change in trade policy, as in Kehoe and Ruhl (2010). The usual method is to relate the policy change to a variety of economic indicators, such as GDP, capital accumulation, or the trade balance, and then try to relate the indicators to welfare informally. Our work suggests that one can dispense with these "intermediate targets," and just directly relate the welfare outcome to a change in policy, or to some other shock.

7 Conclusions

We show that aggregate TFP, appropriately defined, and the capital stock can be used to construct sufficient statistics for the welfare of a representative consumer. To a first order approximation, the change in the consumer's welfare is measured by the expected present value of aggregate TFP growth (and its revision) and by the change in the capital stock. We also show that in order to create a proper welfare measure, TFP has to be calculated using prices faced by households rather than prices facing firms. In modern, developed economies with high rates of income and indirect taxation, the gap between household and firm TFP can be considerable. Finally, in an open economy, the change in welfare will also reflect present and future changes in the returns on net foreign assets and in the terms of trade. However, these latter terms disappear if absorption rather than GDP is used as the output concept for constructing TFP, and TFP and the initial capital stock are again sufficient statistics for measuring welfare in open economies. Most strikingly, these variables also suffice to measure welfare level differences among countries, with both variables computed as log level deviation from a reference country.

We apply these results to measuring welfare growth rates and gaps in a sample of developed countries. For reasonable assumptions about fiscal policy, we find that over our sample period the UK and Spain are the leaders in welfare growth rates. Throughout our sample period, however, the US is the world leader in welfare levels. The UK converges steadily towards US levels of welfare, and is within a few percent of catching the US by the end of our sample, 2005. At the start of our sample, several countries show evidence of convergence to the US, but by the end almost all countries are diverging away from US welfare levels. This divergence is particularly stark for Japan and Italy, which end the sample with less than half the per-capita welfare of the US.

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A Appendix: Derivations

A.1 Making the problem stationary

The representative household maximizes intertemporal utility:

$$V_t = \sum_{s=0}^{\infty} \frac{1}{(1+\rho)^s} \frac{N_{t+s}}{H} U(C_{t+s}; \overline{L} - L_{t+s}) \tag{A.1}$$

where C_{t+s} is the capita consumption of good i at time t+s, L_{t+s} are hours of work per capita, $\overline{L}$ is the time endowment, and N_{t+s} population. H is the number of households, assumed to be fixed and normalized to one from now on. Population grows at constant rate n and per capita variables at a common rate g. X_t is and index for per capita variables in the sense that their level at time t is proportional to X_t.Consider the laws of motion for N_t and for X_t:

$$N_t = N_0(1+n)^t \tag{A.2}$$

$$X_t = X_0(1+g)^t \tag{A.3}$$

and normalize $H = 1$.

We can rewrite the utility function as:

$$V_t = N_t \sum_{s=0}^{\infty} \frac{(1+n)^s}{(1+\rho)^s} U(C_{t+s}; \overline{L} - L_{t+s}) \tag{A.4}$$

For a well defined steady state in which hours of work are constant we assume that the utility function has the King Plosser and Rebelo form (1988):

$$U(C_{t+s}; L - \overline{L}_s) = \frac{1}{1-\sigma} {C_{t+s}}^{1-\sigma} \nu(\overline{L} - L_{t+s}) \tag{A.5}$$

We assume that $\nu(\overline{L} - L_{t+s})$ is an increasing and concave function of leisure, and assumed to be positive. Define $c_{t+s} = \frac{C_{t+s}}{X_{t+s}}$. We can rewrite the utility function in the following form:

$$U(c_{t+s}; L - \overline{L}_{t+s}) = \frac{1}{1-\sigma} X_{t+s}^{(1-\sigma)} {c_{t+s}}^{1-\sigma} \nu(\overline{L} - L_{t+s})$$

or

$$U(c_{t+s}; L - \overline{L}_s) = (1+g)^{s(1-\sigma)} X_t^{(1-\sigma)} \frac{1}{1-\sigma} {c_{t+s}}^{1-\sigma} \nu(\overline{L} - L_{t+s})$$

Inserting this into V_t, we get:

$$V_t = N_t X_t^{(1-\sigma)} \sum_{s=0}^{\infty} \beta^s U(c_{t+s}; \overline{L} - L_{t+s}) \tag{A.6}$$

where: $\beta = \frac{(1+n)(1+g)^{1-\sigma}}{(1+\rho)}$.

A.2 Budget constraint

Start from the usual budget constraint:

$$P_t^I K_t + B_t = (1-\delta)\, P_t^I K_{t-1} + (1+i_t)\, B_{t-1} + P_t^L L_t N_t + P_t^K K_{t-1} + \Pi_t - P_t^C C_t N_t \tag{A.7}$$

Divide both sides by $P_t^I X_t N_t$ to get:

$$\begin{aligned}\frac{K_t}{X_t N_t} + \frac{B_t}{P_t^I X_t N_t} &= (1-\delta)\frac{K_{t-1}}{X_{t-1}N_{t-1}}\frac{X_{t-1}N_{t-1}}{X_t N_t} + (1+i_t)\frac{B_{t-1}}{P_{t-1}^I X_{t-1}N_{t-1}}\frac{P_{t-1}^I}{P_t^I}\frac{X_{t-1}N_{t-1}}{X_t N_t} \\ &+ \frac{P_t^L}{P_t^I}\frac{L_t N_t}{X_t N_t} + \frac{P_t^K}{P_t^I}\frac{K_{t-1}}{X_{t-1}N_{t-1}}\frac{X_{t-1}N_{t-1}}{X_t N_t} + \frac{\Pi_t}{P_t^I X_t N_t} - \frac{P_t^C}{P_t^I}\frac{C_t N_t}{X_t N_t}\end{aligned}$$

Define: $k_t = \frac{K_t}{X_t N_t}$, $b_t = \frac{B_t}{P_t^I X_t N_t}$, $p_t^K = \frac{P_t^K}{P_t^I}$, $p_t^L = \frac{P_t^L}{P_t^I X_t}$, $p_t^C = \frac{P_t^C}{P_t^I}$, $(1+r_t) = \frac{(1+i_t)}{(1+\pi_t)}$, $\pi_t = \frac{\Pi_t}{P_t^I X_t N_t}$. The budget constraint can be rewritten as:

$$k_t + b_t = \frac{(1-\delta) + p_t^K}{(1+g)(1+n)} k_{t-1} + \frac{(1+r_t)}{(1+g)(1+n)} b_{t-1} + p_t^L L_t + \pi_t - p_t^C c_t \tag{A.8}$$

A.3 Optimality conditions

The representative household maximizes normalized intertemporal utility $v_t = \frac{V_t}{N_t X_t^{(1-\sigma)}}$. The Lagrangean for this problem is:

$$\begin{aligned}\Lambda_t &= E_t \sum_{s=0}^{\infty} \beta^s \{U(c_{t+s}; \overline{L} - L_{t+s}) \\ &+\lambda_{t+s}\left(-k_{t+s} - b_{t+s} + \frac{(1-\delta) + p_{t+s}^K}{(1+g)(1+n)} k_{t+s-1} + \frac{(1+r_{t+s})}{(1+g)(1+n)} b_{t+s-1} + p_{t+s}^L L_{t+s} + \pi_{t+s} - p_{t+s}^C c_{t+s}\right)\}\end{aligned}$$

The FOCs are:

$$U_{c_t} - \lambda_t p_t^C = 0 \tag{A.9}$$

$$U_{L_t} + \lambda_t p_t^L = 0 \tag{A.10}$$

$$-\lambda_t + \beta E_t \frac{(1-\delta) + p_{t+1}^K}{(1+g)(1+n)} \lambda_{t+1} = 0 \tag{A.11}$$

$$-\lambda_t + \beta \frac{1}{(1+g)(1+n)} E_t (1+r_{t+1}) \lambda_{t+1} = 0 \tag{A.12}$$

A.4 Approximation around SS

Define with $\widehat{x} = \log x_t - \log x$ the log deviation from the steady state of a variable (x is the steady state value of x_t). Taking a first order approximation in logs of the Lagrangean (which equals the value function along the optimal path), one obtains:

$$\begin{aligned}
v_t - v &= E_t[\sum_{s=0}^{\infty} \beta^s (U_c c \widehat{c}_{t+s} + U_L L \widehat{L}_{t+s} \\
&\quad + \lambda p^L L \widehat{L}_{i,t+s} - \lambda p^C c \widehat{c}_{t+s} - \lambda k \widehat{k}_{t+s} - \lambda b \widehat{b}_{t+s}) \\
&\quad + \sum_{s=0}^{\infty} \beta^{s+1} (\lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t+s} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t+s}) \\
&\quad + \sum_{s=0}^{\infty} \beta^s \widehat{\lambda}_{t+s} \, (-k - b + \frac{(1-\delta) + p^K}{(1+g)(1+n)} k + \frac{(1+r)}{(1+g)(1+n)} b \\
&\quad + p^L L + \pi - p^C c)] \\
&\quad + \sum_{s=0}^{\infty} \beta^s (\lambda p^L L \widehat{p}^L_{t+s} + \frac{p^K k}{(1+g)(1+n)} \, \widehat{p}^K_{t+s} - p^C_t c \widehat{p}_{t+s} + \pi \widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s}) \\
&\quad + \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1}
\end{aligned} \tag{A.13}$$

Using the first order conditions, the first four lines equal zero and, therefore, we get:

$$\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^L L \widehat{p}^L_{t+s} + \frac{p^K k}{(1+g)(1+n)} \, \widehat{p}^K_{t+s} - p^C c \widehat{p}_{t+s} + \pi \widehat{\pi}_{t+s} + \frac{rb}{(1+g)(1+n)} \widehat{r}_{t+s}) \right] \\
&\quad + \lambda \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} + \lambda \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1}
\end{aligned} \tag{A.14}$$

Now log linearize the budget constraint:

$$\begin{aligned}
& k \ \widehat{k}_t + b \widehat{b}_t - \frac{(1-\delta) + p^K}{(1+g)(1+n)} k \widehat{k}_{t-1} - \frac{(1+r)}{(1+g)(1+n)} b \widehat{b}_{t-1} - p^L L \widehat{L}_t - p^L L \widehat{p}^L_t - \frac{p^K k}{(1+g)(1+n)} \widehat{p}^K_t \\
& - \frac{rb}{(1+g)(1+n)} \widehat{r}_t - \pi \widehat{\pi}_t + p^C c \widehat{c}_t + p^C c \widehat{p}_t = 0
\end{aligned} \tag{A.15}$$

Using this result and the steady state version of the FOC for capital in (A.14) gives us:

$$\begin{aligned} v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda [p^C c\widehat{c}_{t+s} + k\widehat{k}_{t+s} - \frac{(1-\delta)+p^K}{(1+g)(1+n)} k\widehat{k}_{t+s-1} + b\widehat{b}_{t+s} - \frac{(1+r)}{(1+g)(1+n)} b\widehat{b}_{t+s-1} \\ & \quad -p^L L\widehat{L}_{t+s}] + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} + \lambda \frac{1+r}{(1+g)(1+n)} b\widehat{b}_{t-1} \end{aligned} \tag{A.16}$$

Rearranging the terms, we get:

$$\begin{aligned} v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c\widehat{c}_{t+s} + k\widehat{k}_{t+s} - \frac{(1-\delta)+p^K}{(1+g)(1+n)} k\widehat{k}_{t+s-1} - p^L L\widehat{L}_{t+s} \right] \\ & \quad + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} + \lambda \sum_{s=0}^{\infty} \beta^s \left[b\widehat{b}_{t+s} - \beta \frac{(1+r)}{(1+g)(1+n)} b\widehat{b}_{t+s} \right] \end{aligned} \tag{A.17}$$

Using the FOC and the trasversality condition for bonds, the equation above becomes:

$$\begin{aligned} v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c\widehat{c}_{t+s} + k\widehat{k}_{t+s} - \frac{(1-\delta)+p^K}{(1+g)(1+n)} k\widehat{k}_{t+s-1} - p^L L\widehat{L}_{t+s} \right] \\ & \quad + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} \end{aligned} \tag{A.18}$$

Notice that the law of motion of capital: $K_t = (1-\delta)K_{t-1} + I_t$, can be rewritten as: $\frac{K_t}{X_t N_t} = (1-\delta)\frac{K_{t-1}}{X_{t-1}N_{t-1}}\frac{X_{t-1}N_{t-1}}{X_t N_t} + \frac{I_t}{X_t N_t}$ which after some algebra becomes:

$$k_t = \frac{(1-\delta)}{(1+g)(1+n)} k_{t-1} + i_t \tag{A.19}$$

Differentiating it around the steady state, we get:

$$k\widehat{k}_t = \frac{(1-\delta)}{(1+g)(1+n)} k\widehat{k}_{t-1} + i\widehat{i}_t \tag{A.20}$$

Inserting this equation into equation (A.17) we get:

$$v_t = v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C c\widehat{c}_{t+s} + i\widehat{i}_{t+s} - p^L L \ \widehat{L}_{t+s} - \frac{p^K k}{(1+g)(1+n)} \ \widehat{k}_{t+s-1} \right] + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} \tag{A.21}$$

This is equation (11) in the main text.

A.5 Connecting the level of productivity to the level of welfare

Define value added (for normalized variables in deviation from steady state) as:

$$\widehat{y}_t = \log y_t - \log y = \frac{P^C CN}{P^Y Y}\widehat{c}_{it} + \frac{P^I I}{P^Y Y}\widehat{i}_t = s_c\widehat{c}_t + s_i\widehat{i}_t \tag{A.22}$$

Inserting this equation into (A.21), and noticing that $\frac{p^K k}{p^Y y(1+g)(1+n)}$is the SS value of $s_{K,t} \equiv \frac{P_t^K K_{t-1}}{P_t^Y Y_t}$ we get:

$$v_t = v + E_t\sum_{s=0}^{\infty}\beta^s\lambda p^Y y\left[\widehat{y}_t - s_L\ \widehat{L}_{t+s} - s_K\ \widehat{k}_{t+s-1}\right] + \lambda\frac{1}{\beta}k\widehat{k}_{t-1} \tag{A.23}$$

Using the definition of the normalized variable, this can be rewritten as:

$$\begin{aligned} v_t &= v + \left(\lambda p^Y y\right)E_t\sum_{s=0}^{\infty}\beta^s\left[(\log\frac{Y_{t+s}}{N_{t+s}X_{t+s}} - \log y) - s_L(\log L_{t+s} - \log L) - s_K(\log\frac{K_{t+s-1}}{N_{t+s-1}X_{t+s-1}} - \log\ k)\right] \\ &\quad +\lambda\frac{1}{\beta}k\widehat{k}_{t-1} \end{aligned} \tag{A.24}$$

where $\log Y_t = \frac{P^C CN}{P^Y Y}\log(C_t N_t) + \frac{P^I I}{P^Y Y}\log I_t$.The previous equation can be rewritten as:

$$\frac{v_t - v}{\lambda p^Y y} = E_t\sum_{s=0}^{\infty}\beta^s\left[\log\frac{Y_{t+s}}{N_{t+s}} - s_L\log\ L_{t+s} - s_K\log\frac{K_{t+s-1}}{N_{t+s-1}}\right] + \frac{1}{\beta}\frac{k}{p^Y y}\log\frac{K_{t-1}}{N_{t-1}} - f(t) \tag{A.25}$$

where:

$$\begin{aligned} f(t) &= \frac{1}{1-\beta}\left[\log y - s_L\log L - s_K\log k + \frac{\beta}{(1-\beta)}\left[g(1-s_K) + n(1-s_L-s_K)\right]\right] \\ &\quad +\frac{1}{1-\beta}\left[(1-s_K)\log X_t\right] + \frac{s_K}{1-\beta}g \\ &\quad +\frac{1}{\beta}\frac{k}{p^Y y}(\log k + \log X_{t-1}) \end{aligned} \tag{A.26}$$

Define aggregate productivity (in log level) as: $\log PR_t = \log\frac{Y_t}{N_t} - s_L\log L_t - s_K\log\frac{K_{t-1}}{N_{t-1}}$. Notice that we are using a definition with constant shares. Then:

$$f(t) = E_t\sum_{s=0}^{\infty}\beta^s(\log PR_{t+s})_{SS} + \frac{1}{\beta}\frac{k}{p^Y y}\log\left(\frac{K_{t-1}}{N_{t-1}}\right)_{SS} \tag{A.27}$$

where we have used the fact that $\log\left(PR_{t+s}\right)_{SS} = \log\left(\frac{Y_t}{N_t}\right)_{SS} - s_L\log L - s_K\log\left(\frac{K_{t-1}}{N_{t-1}}\right)_{SS}$ equals $\log\left(X_t\left(1+g\right)^s y\right) - s_L\log L - s_K\log\left(X_t\left(1+g\right)^{s-1}k\right)$. Inserting the equation above into (A.25), we obtain:

$$\frac{v_t - v}{\lambda p^Y y} = E_t \sum_{s=0}^{\infty} \beta^s \left[\log PR_{t+s} - (\log PR_{t+s})_{SS}\right] + \frac{1}{\beta}\frac{k}{p^Y y}\left[\log \frac{K_{t-1}}{N_{t-1}} - \log\left(\frac{K_{t-1}}{N_{t-1}}\right)_{SS}\right] \tag{A.28}$$

Assume $v_t > 0$ ($\sigma < 1$). Since $\frac{v_t - v}{v} \simeq \log v_t - \log v$, this result can be also re-written, to a first order approximation, as:

$$\frac{v}{\lambda p^Y y}(\log v_t - \log v) = E_t \sum_{s=0}^{\infty} \beta^s \left[\log PR_{t+s} - (\log PR_{t+s})_{SS}\right] + \frac{1}{\beta}\frac{k}{p^Y y}\left[\log \frac{K_{t-1}}{N_{t-1}} - \log\left(\frac{K_{t-1}}{N_{t-1}}\right)_{SS}\right] \tag{A.29}$$

which can be also expressed in terms of log-deviation from the steady state of intertemporal per-capita utility as:

$$\frac{v}{\lambda p^Y y}(\log \frac{V_t}{N_t} - \log\left(\frac{V_t}{N_t}\right)_{SS}) = E_t \sum_{s=0}^{\infty} \beta^s \log\left[\log PR_{t+s} - (\log PR_{t+s})_{SS}\right] + \frac{1}{\beta}\frac{k}{p^Y y}\left[\log \frac{K_{t-1}}{N_{t-1}} - \log\left(\frac{K_{t-1}}{N_{t-1}}\right)_{SS}\right] \tag{A.30}$$

where $\log\left(\frac{V_t}{N_t}\right)_{SS}$ denotes the log value of per-capita intertemporal utility along the balanced-growth path.

If $\sigma > 1$ so that $v_t < 0$ (recall we are assuming $\nu(\overline{L} - L) > 0$), the R.H.S. of (A.30) should equal instead $\frac{v}{\lambda p^Y y}(\log\left(-\frac{V_t}{N_t}\right) - \log\left(-\frac{V_t}{N_t}\right)_{SS})$, which is positive for positive deviations from the steady state.

In order to interpret $\frac{v}{\lambda p^y y}$, notice that by applying the envelope theorem on the household maximization problem, we get:

$$\frac{\partial v_t}{\partial y_t} = \lambda p_t^y \tag{A.31}$$

Moreover, using the definition of v_t and y_t, together with the fact that N_t and X_t are deterministic functions of time, it follows that:

$$\begin{aligned}
\frac{\partial v_t}{\partial y_t} &= \frac{\partial \frac{V_t}{N_t X_t^{1-\sigma}}}{\partial y_t} = \frac{\frac{1}{N_t X_t^{1-\sigma}} \partial V_t}{\partial y_t} = \frac{1}{N_t X_t^{1-\sigma}} \frac{1}{\frac{\partial y_t}{\partial V_t}} \\
&= \frac{1}{N_t X_t^{1-\sigma}} \frac{1}{\frac{\partial \frac{Y_t}{N_t X_t}}{\partial V_t}} = \frac{1}{N_t X_t^{1-\sigma}} \frac{1}{\frac{1}{N_t X_t}\frac{\partial Y_t}{\partial V_t}} = \frac{1}{X_t^{-\sigma}} \frac{\partial V_t}{\partial Y_t}
\end{aligned} \tag{A.32}$$

Taking the result above together with equation (A.31), we obtain that:

$$\lambda p_t^y = \frac{1}{X_t^{-\sigma}} \frac{\partial V_t}{\partial Y_t} \tag{A.33}$$

which, together with the definitions of v_t and y_t implies that:

$$\frac{v_t}{\lambda_t p_t^y y_t} = \frac{\frac{V_t}{N_t X_t^{1-\sigma}}}{\frac{1}{X_t^{-\sigma}} \frac{\partial V_t}{\partial Y_t} \frac{Y_t}{N_t X_t}} = \frac{V_t}{Y_t} \frac{\partial Y_t}{\partial V_t}$$

In other words, $\frac{v}{\lambda p^y y}$ equals the reciprocal of the elasticity of utility with respect to income evaluated along the balanced growth path. Notice that we can re-express this concept in per capita terms since: $\frac{V_t}{Y_t} \frac{\partial Y_t}{\partial V_t} = \frac{\frac{V_t}{N_t}}{\frac{Y_t}{N_t}} \frac{\partial Y_t}{\partial V_t} = \frac{\frac{V_t}{N_t}}{\frac{Y_t}{N_t}} \frac{\partial \frac{Y_t}{N_t}}{\partial \frac{V_t}{N_t}}$. Therefore, $\frac{v}{\lambda p^y y}$ can be also interpreted as the percentage increase in per-capita income necessary to generate a one percentage point increase in lifetime per-capita utility.

Alternatively, welfare can expressed in terms of "equivalent consumption" per capita, denoted by C_t^*. C_t^* is defined as the level of consumption per capita at time t that, if it grows at the steady state rate g from t onward, with leisure set at its steady state level, delivers the same intertemporal utility per capita as the actual stream of consumption and leisure, More precisely, C_t^* satisfies:

$$\begin{aligned} \frac{V_t}{N_t} &= \sum_{s=0}^{\infty} \frac{(1+n)^s}{(1+\rho)^s} \left(C_t^* (1+g)^s\right)^{1-\sigma} \nu(\overline{L} - L)) \\ &= \frac{1}{(1-\sigma)(1-\beta)} C_t^{*1-\sigma} \nu(\overline{L} - L) \end{aligned} \tag{A.34}$$

Per-capita utility on the steady state growth path can be written as:

$$\begin{aligned} \left(\frac{V_t}{N_t}\right)_{SS} &= \sum_{s=0}^{\infty} \frac{(1+n)^s}{(1+\rho)^s} \left((C_t)_{SS} (1+g)^s\right)^{1-\sigma} \nu(\overline{L} - L)) \\ &= \frac{1}{(1-\sigma)(1-\beta)} (C_t)_{SS}^{1-\sigma} \nu(\overline{L} - L) \end{aligned} \tag{A.35}$$

The subscript SS denotes the steady state values of time varying variables. By taking the log-difference between equation (A.34) and (A.35) (for $\sigma < 1$), we get:

$$(\log \frac{V_t}{N_t} - \log \left(\frac{V_t}{N_t}\right)_{SS}) = (1-\sigma)(\log C_t^* - \log (C_t)_{SS}) \tag{A.36}$$

Using the definition of v and the F.O.C. for consumption, it follows that $\frac{v}{\lambda} = \frac{cp^c}{(1-\beta)(1-\sigma)}$. This, together with equation (A.29) implies that:

$$\begin{aligned} \log C_t^* &= \log (C_t)_{SS} + \frac{(1-\beta)}{s_C} E_t \sum_{s=0}^{\infty} \beta^s \left[\log PR_{t+s} - (\log PR_{t+s})_{SS}\right] \\ &\quad + \frac{(1-\beta)}{s_C} \frac{1}{\beta} \frac{k}{p^Y y} \left[\log \frac{K_{t-1}}{N_{t-1}} - \log \left(\frac{K_{t-1}}{N_{t-1}}\right)_{SS}\right] \end{aligned} \tag{A.37}$$

This holds for σ smaller or greater than one.

A.6 Connecting the aggregate Solow residual with the change in welfare

Take the difference between the expected level of intertemporal utility v_t defined in (A.21) and v_{t-1}.

$$\begin{aligned}\Delta v_t &= E_t\sum_{s=0}^{\infty}\beta^s\lambda\left[p^C c\log c_{t+s}+i\log i_{t+s}-p^L L\log L_{t+s}-\frac{p^K k}{(1+g)(1+n)}\log\ k_{t+s-1}\right]\\ &-E_{t-1}\sum_{s=0}^{\infty}\beta^s\lambda\left[p^C c\log c_{t+s-1}+i\log i_{t+s-1}-p^L L\log L_{t+s-1}-\frac{p^K k}{(1+g)(1+n)}\log\ k_{t+s-2}\right]\\ &+\lambda\frac{1}{\beta}k\Delta\log k_{t-1} \end{aligned} \tag{A.38}$$

The right hand side, after adding and subtracting, for each variable x_{t+s}, $E_t x_{t+s}$, can be written as:

$$\begin{aligned}\Delta v_t &= E_t\sum_{s=0}^{\infty}\beta^s\lambda[p^C c\Delta\log c_{t+s}+i\Delta\log i_t-p^L L\Delta\log L_{t+s}-\frac{p^K k}{(1+g)(1+n)}\Delta\log\ k_{t+s-1}]\\ &+\sum_{s=0}^{\infty}\beta^s\lambda[p^C c\left(E_t\log c_{t+s}-E_{t-1}\log c_{t+s}\right)+i\left(E_t\log i_{t+s}-E_{t-1}\log i_{t+s}\right)\\ &-p^L L E_t(\log L_{t+s}-E_{t-1}\log L_{t+s})-\frac{p^K k}{(1+g)(1+n)}\left(E_t\log k_{t+s-1}-E_{t-1}\log k_{t+s-1}\right)]\\ &+\lambda\frac{1}{\beta}k\Delta\log k_{t-1}\end{aligned} \tag{A.39}$$

Define value added growth (at constant shares) as:[31]

$$\Delta\log y_t=\frac{p^C c}{p^Y y}\Delta\log c_{t+s}+\frac{i}{p^Y y}\Delta\log i_t \tag{A.40}$$

Using the fact that nominal value added $P_tY_t=P_t^C C_t N_t+P_t^I I_t$, it is also true that:

$$\Delta\log Y_t=\frac{P^C CN}{P^Y Y}\Delta\log(C_t N_t)+\frac{P^I I}{P^Y Y}\Delta\log I_t \tag{A.41}$$

Now, insert this into equation (A.39) and factor out $p^Y y$ to obtain:

[31] Here we are departing slightly from convention, as value added is usually calculated with time varying shares.

$$
\begin{aligned}
\Delta v_t \quad = \quad & \lambda p^Y y E_t \sum_{s=0}^{\infty} \beta^s [\Delta \log y_t - s_L \Delta \log L_{t+s} - s_K \Delta \log \ k_{t+s-1}] \qquad \text{(A.42)}\\
& + \sum_{s=0}^{\infty} \beta^s \lambda [(E_t \log y_{t+s} - E_{t-1} \log y_{t+s}) \\
& - s_L E_t (\log L_{t+s} - E_{t-1} \log L_{t+s}) - s_K \left(E_t \log k_{t+s-1} - E_{t-1} \log k_{t+s-1}\right)] \\
& + \lambda \frac{1}{\beta} k \Delta \log k_{t-1}
\end{aligned}
$$

Using the fact that:

$$\Delta \log y_t = \Delta \log \frac{Y_t}{N_t} - g$$

$$\Delta \log k_t = \Delta \log \frac{K_t}{N_t} - g$$

and dividing both terms by $\lambda p^Y y$ we can rewrite equation (A.42) as:

$$
\begin{aligned}
\frac{\Delta v_t}{\lambda p^Y y} \quad = \quad & E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} \\
& + \sum_{s=0}^{\infty} \beta^s \left[E_t \log PR_{t+s} - E_{t-1} \log PR_{t+s} \right] \\
& + \frac{1}{\beta} \frac{k}{p^Y y} \Delta \log \frac{K_{t-1}}{N_{t-1}} - f_1 \qquad \text{(A.43)}
\end{aligned}
$$

where $E_t \log PR_{t+s} - E_{t-1} \log PR_{t+s}$ represents the revision in expectations of the level of the productivity residual (normalized by population and X_t) based on the new information received between t-1 and t and:

$$f_1 = \frac{1}{(1-\beta)} g(1 - s_K) + \frac{1}{\beta} \frac{k}{p^Y y} g \qquad \text{(A.44)}$$

Since $\frac{\Delta v_t}{v} \simeq \Delta \ln v_t$ (for $v_t > 0$ $(\sigma > 1)$), equation (A.43) can be re-written, to a first order approximation, as:

$$
\begin{aligned}
\frac{v}{\lambda p^Y y}\Delta \log v_t &= \frac{v}{\lambda p^Y y}\left[\Delta \log \frac{V_t}{N_t} - (1-\sigma)g\right] \\
&= E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} \\
&\quad + \sum_{s=0}^{\infty} \beta^s \left[E_t \log PR_{t+s} - E_{t-1} \log PR_{t+s}\right] \\
&\quad + \frac{1}{\beta}\frac{k}{p^Y y}\Delta \log \frac{K_{t-1}}{N_{t-1}} - f_1
\end{aligned} \tag{A.45}
$$

Using the fact that $\frac{v}{\lambda} = \frac{cp^c}{(1-\beta)(1-\sigma)}$, this result can be also expressed in terms of log-change of per capita intertemporal utility as following:

$$
\begin{aligned}
\frac{v}{\lambda p^Y y}\Delta \log \frac{V_t}{N_t} &= E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} \\
&\quad + \sum_{s=0}^{\infty} \beta^s \left[E_t \log PR_{t+s} - E_{t-1} \log PR_{t+s}\right] \\
&\quad + \frac{1}{\beta}\frac{k}{p^Y y}\Delta \log \frac{K_{t-1}}{N_{t-1}} \\
&\quad + \frac{s_C}{1-\beta}(1-\sigma)g - f_1
\end{aligned} \tag{A.46}
$$

For $\sigma > 1$ so that $v_t < 0$ the R.H.S. of (A.46) should equal, instead, $\frac{v}{\lambda p^Y y}\Delta \ln\left(-\frac{V_t}{N_t}\right)$. Most importantly, using equations (A.11) and (A.19) evaluated in steady state, one can easily show that $(\frac{s_C}{(1-\beta)} - \frac{1}{(1-\beta)}(1-s_K) - \frac{1}{\beta}\frac{k}{p^Y y})g = 0$, so that the last line in the equation above equals zero. This yields equation (17) in the main text.

Alternatively, we can measure changes in welfare in terms of changes in equivalent per-capita consumption. Taking the log-change over time of equation (A.34), we have:

$$
\Delta \log C_t^* = \frac{1-\beta}{s_C}\frac{v}{\lambda p^Y y}\Delta \ln \frac{V_t}{N_t} \tag{A.47}
$$

where, again, we used the fact that: $\frac{v}{\lambda} = \frac{cp^c}{(1-\beta)(1-\sigma)}$. Using (A.46), we obtain:

$$
\begin{aligned}
\Delta \log C_t^* &= \frac{1-\beta}{s_C} E_t \sum_{s=0}^{\infty} \beta^s \Delta \log PR_{t+s} \\
&\quad + \frac{1-\beta}{s_C} \sum_{s=0}^{\infty} \beta^s \left[E_t \log PR_{t+s} - E_{t-1} \log PR_{t+s}\right] \\
&\quad + \frac{1-\beta}{s_C}\frac{1}{\beta}\frac{k}{p^Y y}\Delta \log \frac{K_{t-1}}{N_{t-1}}
\end{aligned} \tag{A.48}
$$

This is equation (20) in the main text.

A.7 Cross-country comparison

Start from equation (22) in the text:

$$\begin{aligned}\frac{v^{us}}{\lambda^{us}p^{Y,us}y^{us}}(\ln v_t^i - \ln v^{us}) \quad = \quad & E_t \sum_{s=0}^{\infty} \beta^s [s_c^{us}(\ln c_{t+s}^i - \ln c^{us}) + s_i^{us}(\ln i_{t+s}^i - \ln i^{us}) \\ & -s_L^{us}\ (\ln L_{t+s}^i - \ln L^{us}) - s_K^{us}(\ln k_{t+s-1}^i - \ln k^{us})] \\ & +\frac{1}{\beta}\frac{k^{us}}{p^{Y,us}y^{us}}(\ln k_{t-1}^i - \ln k^{us})\end{aligned} \tag{A.49}$$

This can be re-written as:

$$\begin{aligned}\frac{v^{us}}{\lambda^{us}p^{Y,us}y^{us}}\left[\ln\frac{V_t^i}{N_t^i} - \ln\left(\frac{V_t^{us}}{N_t^{us}}\right)_{SS}\right] \quad = \quad & E_t \sum_{s=0}^{\infty} \beta^s [s_c^{us}(\ln C_{t+s}^i - \ln\left(C_{t+s}^{us}\right)_{SS}) + s_i^{us}(\ln\frac{I_{t+s}^i}{N_{t+s}^i} - \ln\left(\frac{I^{us}}{N_{t+s}^{us}}\right)_{SS}) \\ & -s_L^{us}\ (\ln L_{t+s}^i - \ln L^{us}) - s_K^{us}\left(\log\frac{K_{t+s-1}^i}{N_{t+s-1}^i} - \log\left(\frac{K_{t+s-1}^{us}}{N_{t+s-1}^{us}}\right)_{SS}\right)] \\ & +\frac{1}{\beta}\frac{k^{us}}{p^{Y,us}y^{us}}\left(\log\frac{K_{t-1}^i}{N_{t-1}^i} - \log\left(\frac{K_{t-1}^{us}}{N_{t-1}^{us}}\right)_{SS}\right) \\ & +\left[\frac{s_c^{us}}{(1-\beta)} + \frac{1-s_K^{us}}{(1-\beta)} + \frac{1}{\beta}\right]\left(\ln X_{t+s}^i - \ln X_{t+s}^{us}\right)\end{aligned} \tag{A.50}$$

where we have use the fact that $\frac{v}{\lambda} = \frac{cp^c}{(1-\beta)(1-\sigma)}$. Using equations (A.11) and (A.19) evaluated in US steady state: $\frac{s_c^{us}}{(1-\beta)} - \frac{1}{(1-\beta)}(1-s_K) - \frac{1}{\beta}\frac{k}{p^Y y} = 0$. This implies that the last line in the equation above equals zero. Define the productivity term obtained using US shares as $\overline{PR}_{t+s}^i = s_c^{us}\ln C_{t+s}^i + s_i^{us}\ln\frac{I_{t+s}^i}{N_{t+s}} - s_L^{us}\ \ln L_{t+s}^i - s_K^{us}\ln\frac{K_{t+s-1}^i}{N_{t+s=1}}$. Using this modified measure of productivity, where input shares are common across countries, the equation above can be re-written as:

$$\begin{aligned}\frac{v^{us}}{\lambda^{us}p^{Y,us}y^{us}}\left[\ln\frac{V_t^i}{N_t^i} - \ln\left(\frac{V_t^{us}}{N_t^{us}}\right)_{SS}\right] \quad = \quad & E_t \sum_{s=0}^{\infty} \beta^s \left[\log\overline{PR}_{t+s}^i - \log(PR_{t+s}^{us})_{SS}\right] \\ & +\frac{1}{\beta}\frac{k^{us}}{p^{Y,us}y^{us}}\left(\log\frac{K_{t-1}^i}{N_{t-1}^i} - \log\left(\frac{K_{t-1}^{us}}{N_{t-1}^{us}}\right)_{SS}\right)\end{aligned} \tag{A.51}$$

This is equation (23) in the main text.

A.8 Distortionary taxes

We now allow for distortionary taxes on capital, labor and financial assets, and for indirect taxes on consumption and investment (at rates τ_t^K,τ_t^K,τ_t^R,τ_t^C,τ_t^I respectively). The numeraire is now $\left(1+\tau_t^I\right)P_t^I$.

The household budget constraint is now:

$$k_t + b_t = \frac{(1-\delta)+p_t^K\left(1-\tau_t^K\right)}{(1+g)(1+n)}k_{t-1} + \frac{\left(1+r_t\left(1-\tau_t^R\right)\right)}{(1+g)(1+n)}b_{t-1} + p_t^L\left(1-\tau_t^L\right)L_t + \pi_t - p_t^C\left(1+\tau_t^C\right)c_t \tag{A.52}$$

Log-linearizing it one obtains:

$$\begin{aligned} 0 \quad = \quad & k\,\widehat{k}_t - \frac{(1-\delta)+p^K\left(1-\tau^K\right)}{(1+g)(1+n)}k\widehat{k}_{t-1} - \frac{p^K\left(1-\tau^K\right)k}{(1+g)(1+n)}\widehat{p}_t^K + \frac{p^K\tau^K k}{(1+g)(1+n)}\widehat{\tau}_t^K \\ & -p^L\left(1-\tau^L\right)L\widehat{L}_t - p^L\left(1-\tau^L\right)L\widehat{p}_t^L + p^L\tau^L L\widehat{\tau}_t^L \\ & +p^C\left(1+\tau^C\right)c\widehat{c}_t + p^C\left(1+\tau^C\right)c\widehat{p}_t^C + p^C\tau^C c\widehat{\tau}_t^C - \pi\widehat{\pi}_t \\ & +b\widehat{b}_t - \frac{\left(1+r\left(1-\tau^R\right)\right)b}{(1+g)(1+n)}\widehat{b}_{t-1} - \frac{r\left(1-\tau^R\right)b}{(1+g)(1+n)}\widehat{r}_t - \frac{r\tau^R b}{(1+g)(1+n)}\widehat{\tau}_t^R \end{aligned} \tag{A.53}$$

Log-linearizing the maximization problem as before, we get:

$$\begin{aligned} v_t \quad = \quad & v + E_t\sum_{s=0}^{\infty}\beta^s\lambda\left[p^L\left(1-\tau^L\right)L\widehat{p}_{t+s}^L + \frac{p^K\left(1-\tau^K\right)k\,\widehat{p}_{t+s}^K}{(1+g)(1+n)} + \frac{r\left(1-\tau^R\right)b\widehat{r}_{t+s}}{(1+g)(1+n)} + \pi\widehat{\pi}_{t+s} - p^C\left(1+\tau^C\right)c\widehat{p}_{t+s}\right] \\ & -E_t\sum_{s=0}^{\infty}\beta^s\lambda\left[p^L\tau^L L\widehat{\tau}_{t+s}^L + \frac{p^K\tau^K k}{(1+g)(1+n)}\,\widehat{\tau}_{t+s}^K + \frac{r\tau^R b}{(1+g)(1+n)}\,\widehat{\tau}_{t+s}^R - p^C\tau^C c\widehat{\tau}_{t+s}^C\right] \\ & +\lambda\frac{(1-\delta)+p^K\left(1-\tau^K\right)}{(1+g)(1+n)}k\widehat{k}_{t-1} + \lambda\frac{\left(1+r\left(1-\tau^R\right)\right)}{(1+g)(1+n)}b\widehat{b}_{t-1} \end{aligned} \tag{A.54}$$

Using the log-linearized budget constraint in the equation above, we obtain:

$$\begin{aligned} v_t - v \quad = \quad & \lambda E_t\sum_{s=0}^{\infty}\beta^s[\left(1+\tau^C\right)p^C c\widehat{c}_{t+s} + k\,\widehat{k}_{t+s} - \frac{(1-\delta)+p^K\left(1-\tau^K\right)}{(1+g)(1+n)}k\widehat{k}_{t+s-1} - \left(1-\tau^L\right)\,p^L L\widehat{L}_{t+s} \\ & +b\widehat{b}_{t+s} - \frac{\left(1+r\left(1-\tau^R\right)\right)}{(1+g)(1+n)}b\widehat{b}_{t+s-1}] + \\ & +\lambda\frac{(1-\delta)+p^K\left(1-\tau^K\right)}{(1+g)(1+n)}k\widehat{k}_{t-1} + \lambda\frac{\left(1+r\left(1-\tau^R\right)\right)}{(1+g)(1+n)}b\widehat{b}_{t-1} \end{aligned} \tag{A.55}$$

Rearranging the terms, we get:

$$
\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[\left(1+\tau^C\right) p^C c\widehat{c}_{t+s} + k\ \widehat{k}_{t+s} - \frac{(1-\delta) + p^K \left(1-\tau^K\right)}{(1+g)(1+n)} k\widehat{k}_{t+s-1} - \left(1-\tau^L\right)\ p^L L\widehat{L}_{t+s} \right] \\
&\quad + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} + \lambda \sum_{s=0}^{\infty} \beta^s \left[b\widehat{b}_{t+s} - \beta \frac{\left(1 + r\left(1-\tau^R\right)\right)}{(1+g)(1+n)} b\widehat{b}_{t+s} \right] \qquad \text{(A.56)}
\end{aligned}
$$

Using the FOC for bonds evaluated in steady state, the last term equals zero. Finally, using the fact that:

$$
k\widehat{k}_{t+s} - \frac{(1-\delta)}{(1+g)(1+n)} k\widehat{k}_{t+s-1} = i\widehat{i}_{t+s} \qquad \text{(A.57)}
$$

we obtain:

$$
\begin{aligned}
v_t &= v + E_t \sum_{s=0}^{\infty} \beta^s \lambda \left[p^C \left(1+\tau^C\right) c\widehat{c}_{t+s} + i\widehat{i}_{t+s} -\ p^L \left(1-\tau^L\right) L\widehat{L}_{t+s} - \frac{p^K \left(1-\tau^K\right)}{(1+g)(1+n)} k\widehat{k}_{t+s-1} \right] \\
&\quad + \lambda \frac{1}{\beta} k\widehat{k}_{t-1} \qquad \text{(A.58)}
\end{aligned}
$$

which is equation (35) in the paper.

Table 1: Log Productivity

Dependent variable: $logPR_t$

CASE 1: Wasteful Government. Lump-Sum Taxes

	Canada	France	Italy	Japan	Spain	UK	USA
log PR(t-1)	0.459	0.967	0.835	1.424	1.174	0.962	0.798
	(0.195)	(0.210)	(0.153)	(0.186)	(0.199)	(0.200)	(0.135)
log PR(t-2)		-0.287		-0.582	-0.393	-0.449	
		(0.204)		(0.187)	(0.195)	(0.179)	
LM1(Prob>chi2)	0.191	0.130	0.444	0.960	0.338	0.450	0.124

CASE 2: Optimal Government. Distortionary Taxes

	Canada	France	Italy	Japan	Spain	UK	USA
log PR(t-1)	0.689	1.107	0.874	1.468	1.388	1.104	0.801
	(0.167)	(0.202)	(0.119)	(0.184)	(0.172)	(0.196)	(0.135)
log PR(t-2)		-0.393		-0.591	-0.623	-0.491	
		(0.197)		(0.186)	(0.172)	(0.182)	
LM1(Prob>chi2)	0.157	0.274	0.166	0.717	0.309	0.820	0.084

Notes: Time period: 1985-2005.

Table 2: Annual Average Log Change in Per-Capita Equivalent Consumption

	Wasteful Spending Lump-Sum Taxes	Optimal Spending Lump-Sum Taxes	Wasteful Spending Distortionary Taxes	Optimal Spending Distortionary Taxes
Canada	0.013	0.014	0.021	0.023
France	0.026	0.031	0.026	0.031
Italy	0.018	0.020	0.021	0.023
Japan	0.018	0.025	0.023	0.030
Spain	0.021	0.030	0.030	0.040
UK	0.032	0.036	0.036	0.039
USA	0.025	0.026	0.029	0.030

Notes: Time period: 1985-2005.

Table 3: Components of the Annual Log-Change in Per-Capita Equivalent Consumption

	Wasteful Spending Lump-Sum Taxes		Wasteful Spending Distortionary Taxes		Optimal Spending Distortionary Taxes	
	Fraction due to:		Fraction due to:		Fraction due to:	
	TFP	*Capital*	*TFP*	*Capital*	*TFP*	*Capital*
Canada	0.445	0.555	0.658	0.342	0.690	0.310
France	0.830	0.170	0.827	0.173	0.857	0.143
Italy	0.659	0.341	0.707	0.293	0.724	0.276
Japan	0.429	0.571	0.559	0.441	0.661	0.339
Spain	0.512	0.488	0.663	0.337	0.747	0.253
UK	0.816	0.184	0.833	0.167	0.848	0.152
USA	0.830	0.170	0.852	0.148	0.858	0.142

Notes: Time period: 1985-2005. TFP includes both expected present value and expectation revision.

Table 4: Annual Average Log-Change in Per-capita Consumption, GDP and Equivalent Consumption

	Consumption	GDP	Equivalent Consumption Opt Gov, Dist Tax
Canada	0.016	0.016	0.023
France	0.016	0.016	0.031
Italy	0.016	0.017	0.023
Japan	0.019	0.018	0.030
Spain	0.027	0.027	0.040
UK	0.024	0.030	0.039
USA	0.020	0.022	0.030

Notes: Time period: 1985-2005.

Table 5: Welfare Gap Relative to USA: 1985, 2005 and Average

	Wasteful Spending Lump-Sum Taxes	Wasteful Spending Distortionary Taxes	Optimal Spending Distortionary Taxes
PANEL A: 1985			
Canada	-0.256	-0.294	-0.295
France	-0.069	-0.176	-0.165
Italy	-0.368	-0.420	-0.437
Japan	-0.511	-0.488	-0.471
Spain	-0.327	-0.396	-0.414
UK	-0.096	-0.182	-0.189
USA	0.000	0.000	0.000
PANEL B: 2005			
Canada	-0.407	-0.455	-0.451
France	-0.078	-0.240	-0.213
Italy	-0.569	-0.641	-0.664
Japan	-0.540	-0.582	-0.526
Spain	-0.396	-0.405	-0.362
UK	0.034	-0.068	-0.059
USA	0.000	0.000	0.000
PANEL C: average 1985-2005			
Canada	-0.328	-0.372	-0.370
France	-0.065	-0.201	-0.181
Italy	-0.445	-0.507	-0.525
Japan	-0.498	-0.505	-0.468
Spain	-0.348	-0.389	-0.376
UK	-0.026	-0.120	-0.120
USA	0.000	0.000	0.000

Table 6: Components of Welfare Gap Relative to USA: 1985, 2005 and Average

	Wasteful Spending Lump-Sum Taxes		Wasteful Spending Distortionary Taxes		Optimal Spending Distortionary Taxes	
	Fraction due to:		Fraction due to:		Fraction due to:	
	TFP	*Capital*	*TFP*	*Capital*	*TFP*	*Capital*
PANEL A: 1985						
Canada	0.936	0.064	0.945	0.055	0.945	0.055
France	0.968	0.032	0.988	0.012	0.987	0.013
Italy	1.007	-0.007	1.006	-0.006	1.006	-0.006
Japan	1.091	-0.091	1.096	-0.096	1.099	-0.099
Spain	0.855	0.145	0.880	0.120	0.885	0.115
UK	0.379	0.621	0.672	0.328	0.683	0.317
PANEL B: 2005						
Canada	0.900	0.100	0.911	0.089	0.910	0.090
France	0.421	0.579	0.811	0.189	0.788	0.212
Italy	0.972	0.028	0.975	0.025	0.976	0.024
Japan	1.065	-0.065	1.060	-0.060	1.067	-0.067
Spain	0.898	0.102	0.900	0.100	0.888	0.112
UK	-	-	-0.079	1.079	-0.233	1.233
PANEL C: average 1985-2005						
Canada	0.915	0.085	0.925	0.075	0.925	0.075
France	0.666	0.334	0.891	0.109	0.879	0.121
Italy	0.999	0.001	0.999	0.001	0.999	0.001
Japan	1.115	-0.115	1.113	-0.113	1.122	-0.122
Spain	0.887	0.113	0.899	0.101	0.895	0.105
UK	-1.513	2.513	0.448	0.552	0.443	0.557

Table 7: Per-Capita GDP, Consumption and Equivalent Consumption relative to USA: 1985, 2005 and Average

	Consumption	*GDP*	Equivalent Consumption Opt Gov, Dist Tax
PANEL A: 1985			
Canada	-0.179	-0.106	-0.295
France	-0.285	-0.250	-0.165
Italy	-0.378	-0.287	-0.437
Japan	-0.434	-0.211	-0.471
Spain	-0.624	-0.570	-0.414
UK	-0.306	-0.304	-0.189
USA	0.000	0.000	0.000
PANEL B: 2005			
Canada	-0.324	-0.177	-0.451
France	-0.401	-0.317	-0.213
Italy	-0.501	-0.370	-0.664
Japan	-0.456	-0.261	-0.526
Spain	-0.527	-0.419	-0.362
UK	-0.190	-0.219	-0.059
USA	0.000	0.000	0.000
PANEL C: average 1985-2005			
Canada	-0.248	-0.163	-0.373
France	-0.338	-0.265	-0.181
Italy	-0.396	-0.278	-0.529
Japan	-0.391	-0.176	-0.468
Spain	-0.537	-0.447	-0.375
UK	-0.220	-0.251	-0.116
USA	0.000	0.000	0.000

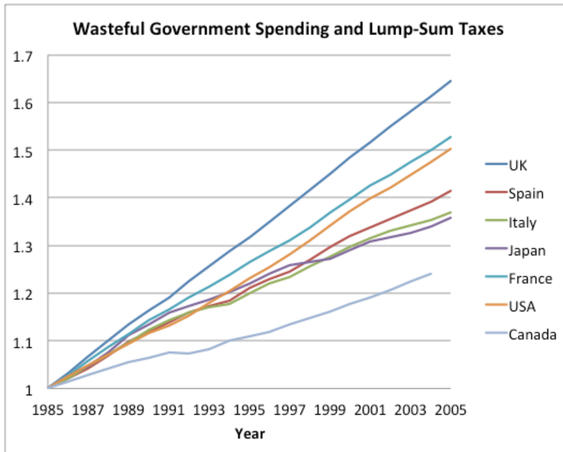

Figure 1: Within-country welfare comparisons: log equivalent permanent consumption

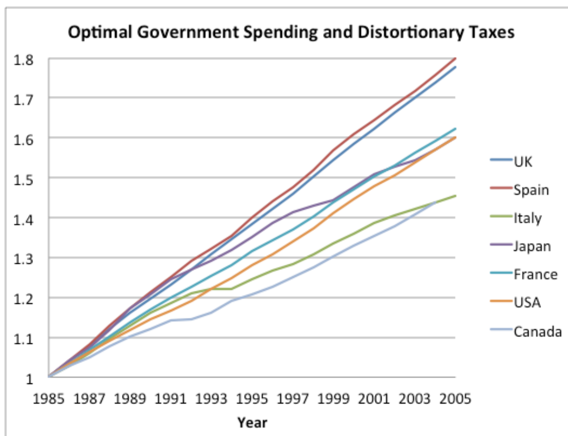

Figure 2: Within-country welfare comparisons: log equivalent permanent consumption

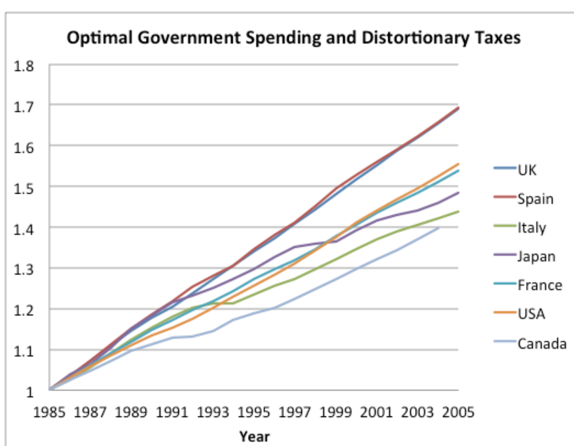

Figure 3: Within-country welfare comparisons: log equivalent permanent consumption (computed using the EU-KLEMS labor service index)

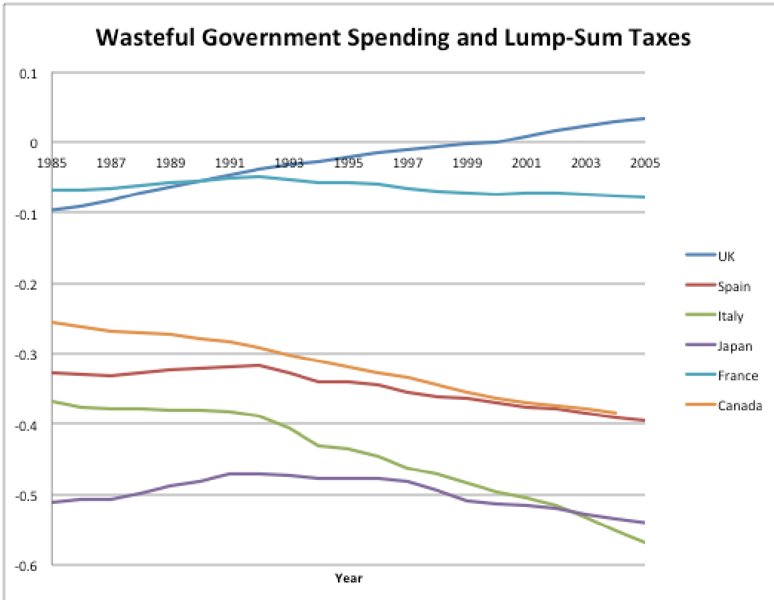

Figure 4: Cross-country welfare comparisons: log equivalent permanent consumption gap (vis-a-vis the U.S.)

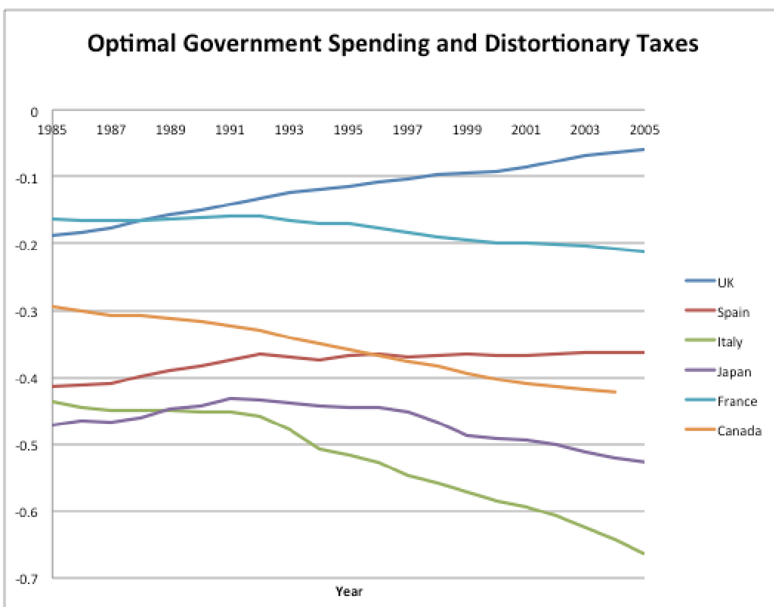

Figure 5: Cross-country welfare comparisons: log equivalent permanent consumption gap (vis-a-vis the U.S.)

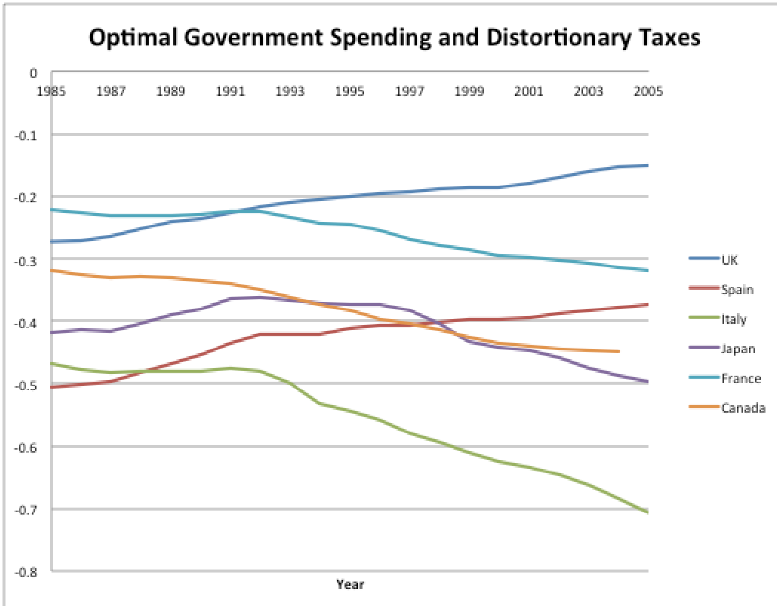

Figure 6: Cross-country welfare comparisons: log equivalent permanent consumption gap (vis-a-vis the U.S.). French preferences.

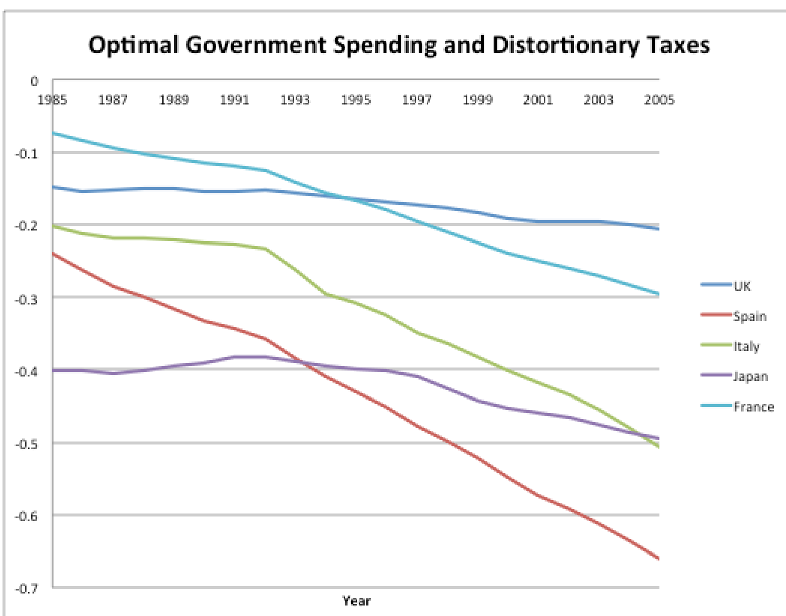

Figure 7: Cross-country welfare comparisons: log equivalent permanent consumption gap (vis-a-vis the U.S.) computed using the EU-KLEMS labor service index.