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WELFARE IMPLICATIONS OF THE TAXATION OF SAVINGS*

Historically, the welfare aspects of the taxation of savings have mainly been discussed in the context of the relative merits of income and consumption (expenditure) taxation. These have long been the subject of debate, with the book by Kaldor (1955) being probably the best known contribution. In the present paper, we examine the role of the two types of taxation in the context of a model that draws both on modern analysis of optimal taxation and on the theory of economic growth. In so doing, we feel that the issues involved can be discussed in greater depth, but the cost is that some important matters have to be left out. Thus, we do not discuss the implications of alternative tax structures for short-run stabilisation policy, nor do we consider the costs of tax administration.[1] Moreover, we neglect problems of distributive justice within a given generation of individuals. Instead, we concentrate on the implications of taxation for the efficiency of resource utilisation and for the intertemporal allocation of consumption.

In popular expositions one often encounters the following efficiency argument in favour of expenditure taxation. An income tax applies to both labour earnings and interest on savings. The imposition of such a tax introduces (into an otherwise first-best world) distortions in both the labour market and the capital market. By comparison, the expenditure tax, while affecting the labour–leisure choice, is 'neutral' with respect to savings decisions, and thus appears to dominate the income tax in terms of efficiency considerations. Although still current,[2] this view is not very convincing. The theory of the second-best has taught us that one cannot evaluate alternative tax systems by simply comparing the *number* of distortions involved; it is essential to consider the magnitude of the various distortions as well as their interaction. Among other aspects, the conventional argument ignores the possibility that a tax on interest income might be desirable in order to offset the distortions introduced by a tax on labour earnings.

A more sophisticated argument, taking account of this objection, is that

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[1] For a good discussion of these and other matters with a view to practical implementation, see Andrews (1974) and Kay and King (1978).

[2] Recent examples are Meade: 'the intelligent radical would welcome [the replacement of] the progressive taxation of income by a progressive taxation of expenditure...A tax on income discriminates against private savings, whereas a tax on consumption does not do so' (1975, pp. 93–4), and Feldstein: 'income tax lowers the rate of return on savings and thus distorts everyone's choice between consuming today and saving for a higher level of consumption in the future...The consumption tax would eliminate this wasteful distortion' (1976, p. 16). (To be fair, Feldstein (1978) has given a clear account of why this argument is unconvincing.)

developed recently by Feldstein (1978) and others based on the theory of optimum taxation. Since the static general equilibrium model used in this literature (see, for example, Diamond and Mirrlees (1971), Atkinson and Stiglitz (1972) and Sandmo (1976)) can easily – although perhaps somewhat artificially – be given an intertemporal interpretation, it might seem that it would be a straightforward task to apply optimum tax theory to the problem at hand.[1] Thus Feldstein (1978) makes use of the results given in Atkinson and Stiglitz (1976). He notes that the conditions for the pure consumption tax to be optimal are unlikely to be satisfied, but suggests that 'the efficiency gain from switching completely to a progressive consumption tax may...be large even if a consumption tax is not itself the optimum optimorum' (1978, p. S49). Similarly, Bradford and Rosen (1976) refer to this result and argue that this 'illustrates the challenge implicit in the optimal tax approach to the widespread acceptance of taxation on the basis of Haig-Simons income' (p. 96).

This straightforward application of the optimum tax literature may, however, be misleading, and one of the main purposes of this paper is to show that it neglects important features of the problem. One can only apply the standard results to a dynamic model under certain conditions. In particular, it depends on the other instruments which can be employed by the government and its ability to achieve a desired intertemporal allocation. In order to bring this out, we have adopted an explicitly intertemporal model of growth with overlapping generations, a topic first investigated by Samuelson (1958) and later extended to an economy with production by Diamond (1965). This framework is described in Section I where particular attention is paid to the range of fiscal instruments at the disposal of the government.

Section II formulates the problem faced by the government in designing the structure of taxation to maximise a social welfare function defined as the discounted sum of individual lifetime welfares. This social welfare function is open to question, but is the natural analogue of that employed in static treatments of optimum taxation. Although the full dynamic path is considered, the interpretation of the results focuses on the steady state of the economy.[2] Section III discusses the case where the government can use lump-sum taxes, or debt policy, to achieve a desired intertemporal allocation. It is shown that, in this situation, the standard optimum tax results can be employed, and we discuss the implications for the choice between income and expenditure taxation. Where, however, the government is constrained, and cannot achieve a first-best level of the capital–labour ratio, the standard analysis can no longer be directly applied. Section IV describes the necessary modifications to the conditions for optimality, and illustrates with a simple Cobb–Douglas example.

The choice between tax bases depends also on the objectives pursued by the government. In particular, we need to consider the treatment of different

[1] Interestingly, little reference has been made to the discussion of the tax treatment of savings in Ramsey's original paper. On the basis of an assumed infinite elasticity of demand for saving and a finite supply elasticity, he argues 'that income-tax should be partially but not wholly remitted on savings' (1927, p. 59).

[2] In earlier versions of this paper, we confined the analysis to a choice between steady states, as in the articles by Hamada (1972), Ordover and Phelps (1975), and Ordover (1976).

generations – the issue of intertemporal distribution – and the extent to which the government respects the valuations placed by individuals on consumption at different dates. Section V considers alternative formulations of the social welfare function and the implications of non-Paretian objectives.

In examining these questions, we have built on earlier work by a number of writers. Particular reference should be made to the treatment of optimum taxation in an intertemporal setting by Diamond (1973), on whose analysis we have drawn in several respects. As in Diamond (1965), the model is one where individuals live for two periods, working in the first and being retired in the second, and there are no bequests. This framework is also adopted by Hamada (1972), who studies a model with individuals of differing abilities in an economy where there is a linear tax on wage income (but no tax on capital income); he is then able to analyse the trade-off between equity and dynamic efficiency. Pestieau (1974) introduces taxes on both labour and interest income, but he focuses primarily on the interaction between the tax structure and public investment criteria. Of the articles by Ordover and Phelps (1975) and Ordover (1976), the former is closer to our approach, but both differ from the contributions previously mentioned in adopting a Rawlsian rather than a utilitarian welfare function. Moreover, Ordover (1976) makes special assumptions about the supply of labour which in fact cause it to be independent of the rate of interest. Mitra (1975) provides a unified treatment of a range of tax policies in a dynamic context, and considers the implications of different objectives. His paper is not, however, explicitly directed to the choice between income and expenditure taxation which is our main concern here.[1]

I. THE FRAMEWORK

As noted at the outset, we rule out any consideration of redistribution within a generation, which is taken to consist of identical individuals. The preferences of a particular generation are represented by the utility function of a representative member of the generation born at time i (referred to as generation i):

$$U = U(C_1^i,\, C_2^{i+1}, L^i), \tag{1}$$

where C_1^i, C_2^{i+1} are consumption in the first and second periods, respectively, of the individual's life and L^i is labour hours per worker supplied in period i. In the second period, individuals are retired and their consumption is constrained by savings carried over from the first period. The number of individuals in each generation is $(1+n)$ times that in the previous one, so there are $N_0(1+n)^i$ ($\equiv N^i$) workers at time i. In the absence of taxes the budget constraint of a representative member of the generation starting life at i would be

$$C_1^i + \frac{1}{1+r^{i+1}} C_2^{i+1} = w^i L^i, \tag{2}$$

[1] The reader is also referred to Ordover and Phelps (1979), which we received after the analysis of this paper was essentially completed.

where w^i is the wage rate at time i and r^{i+1} is the next-period rate of interest. The individual is assumed to have perfect foresight regarding r^{i+1} (and regarding taxes).[1]

The economy is assumed to be perfectly competitive, with firms maximising profits:

$$Y^i - w^i L^i N^i - r^i K^i, \tag{3}$$

where Y^i, K^i denote output and capital, respectively, subject to a constant returns to scale production function (assumed unchanging over time, although Harrod-neutral technical progress could readily be introduced),

$$Y^i = L^i N^i f(k^i/L^i) \quad \text{where} \quad f' > 0, f'' < 0. \tag{4}$$

The capital stock per worker is denoted by k^i, so that k^i/L^i is the capital per worker hour. We assume for convenience that there is no depreciation of capital. In the absence of taxation, and government debt, the supply of capital is determined by the level of savings for retirement of the preceding generation:

$$K^{i+1} = (w^i L^i - C_1^i)\, N^i \tag{5}$$

or

$$k^{i+1} = \frac{w^i L^i - C_1^i}{1+n}. \tag{6}$$

We now introduce taxation, and begin with a full statement of the range of policy instruments considered. The literature on optimum taxation has demonstrated how the results may be very sensitive to the assumptions made about the types of taxation which are possible (Atkinson and Stiglitz, 1976). We confine our attention to linear taxes, but otherwise allow for a tax on labour income (t_w), a tax on interest income (t_r), a tax on consumption (t_C) and lump-sum taxes in periods 1 and 2 (T_1 and T_2). Each of these may be varied at each date, so that a superscript i should be added. In addition, the government issues one period debt D^i per worker, which bears the same interest as other capital.

The individual budget constraint now becomes:

$$C_1^i(1+t_C^i) + \frac{C_2^{i+1}(1+t_C^{i+1}) + T_2^{i+1}}{1+r^{i+1}(1-t_r^{i+1})} = w^i(1-t_w^i)\, L^i - T_1^i. \tag{7}$$

It may be noted that we are assuming that all those alive at a particular date pay the same indirect tax, but that the lump-sum taxes may be differentiated by generation (as where there is a state pension scheme). The per capita level of savings by generation i is now:

$$w^i(1-t_w^i)\, L^i - T_1^i - C_1^i(1+t_C^i) \equiv A^i. \tag{8}$$

The interest income tax paid (in period $i+1$) is therefore per worker in period $i+1$:

$$t_r^{i+1} r^{i+1} A^i/(1+n). \tag{9}$$

[1] Here, and on the production side, we are abstracting from the problems introduced by uncertainty. The effects of income and expenditure taxes on risk-taking need further investigation.

The government is assumed to have a revenue constraint which requires it to raise an amount G per worker in each period, expressed in units of consumption, in addition to financing the repayment of existing debt, net of the issue of new debt. At time i this may be written:

$$t_w^i w^i L^i + t_C^i \left(C_1^i + \frac{C_2^i}{1+n}\right) + t_r^i r^i \frac{A^{i-1}}{1+n} + T_1^i + \frac{T_2^i}{1+n} = G + \left(\frac{1+r^i}{1+n}\right) D^{i-1} - D^i. \quad (10)$$

The government may be seen as choosing the tax rates, and debt policy, subject to this revenue constraint and the capital market equilibrium condition, which now becomes:

$$(1+n)\,k^{i+1} = A^i - D^i, \quad (11)$$

where the right hand side is private saving net of the holding of government debt. It is, however, more straightforward to work with the aggregate production constraint:

$$Y^i = L^i N^i f(k^i/L^i) = N^i C_1^i + N^{i-1} C_2^i + N^i G + K^{i+1} - K^i \quad (12)$$

or, dividing by N^i,

$$L^i f(k^i/L^i) = C_1^i + \frac{C_2^i}{1+n} + G + (1+n)\,k^{i+1} - k^i. \quad (13)$$

(It may be checked that (13) and (11), together with the individual budget constraint, imply the revenue constraint (10).)

This formulation allows for a wide range of tax and debt policies, but not all of them are independent. In particular, debt policy can be shown to be equivalent in this model to use of the lump-sum taxes T_1 and T_2, as has been noted by Diamond (1973, p. 222) and Bierwag, Grove and Khang (1969). This may be seen by considering a unit rise in T_1^i, coupled with a fall in T_2^{i+1} equal to:

$$\frac{1}{1 + r^{i+1}(1 - t_r^{i+1})}.$$

This leaves unchanged the present value of lump-sum tax payments by the individual in generation i (see the budget constraint (7)). If at the same time D^i is reduced by 1 unit, this leaves the capital market condition unchanged (see equations (8) and (11)). If we now turn to the government revenue constraint, the one unit changes in T_1^i and D^i ensure that it is unchanged in period i; and the change in T_2^{i+1} is similarly offset in period $(i+1)$ by the fall in D^i (allowing for the effect via A^i). From this point on, we therefore drop any explicit reference in the model to government debt ($D^i = 0$ for all i). It should, however, be noted that when we refer to restrictions on the use of lump-sum taxes, these also apply to debt policy.

Finally, we may simplify the representation of the impact of taxation by introducing the new variables:

$$\omega^i \equiv w^i(1 - t_w^i), \quad (14a)$$

which is the net of tax wage rate,

$$Z^i \equiv T_1^i + \frac{T_2^i}{1 + r^{i+1}(1 - t_r^{i+1})}, \quad (14b)$$

which is the present value of lump-sum tax payments made by generation i, and

$$p^{i+1} \equiv \frac{1}{1+r^{i+1}(1-t_r^{i+1})}\left(\frac{1+t_C^{i+1}}{1+t_C^i}\right), \tag{14c}$$

which is the 'price' to generation i of second period consumption relative to that in the first. The individual budget constraint for generation t is then:

$$(1+t_C^i)(C_1^i+p^{i+1}C_2^{i+1}) = \omega^i L^i - Z^i. \tag{15}$$

The behaviour of generation i (C_1^i, C_2^{i+1}, L^i) may therefore be treated as a function of t_C^i, p^{i+1}, ω^i and Z^i. Moreover, it is invariant with respect to (positive) proportional changes in $(1+t_C^i)$, ω^i and Z^i. The aggregate production constraint is unaffected, since the policy variables do not enter directly.

With this reformulation, the choice between income and expenditure taxation may be seen in terms of the choice of p^{i+1}. If p^{i+1} is greater than $1/(1+r^{i+1})$, then the tax system departs from a pure expenditure tax in the direction of taxing second period consumption more heavily. This may be achieved either by a rising rate of indirect taxation over time or by the taxation of interest income ($t_r^{i+1} > 0$). For a pure income tax, $t_r^{i+1} = t_w^{i+1}$ (and t_C zero). In what follows, particular attention is paid to the relationship between p^{i+1} and $1/(1+r^{i+1})$.

II. FORMULATION OF THE GOVERNMENT'S PROBLEM

At this stage we assume that the government's objective is to maximise the discounted sum of individual lifetime welfares. The choice of discount factor allows some flexibility in the formulation, but the assumption that governments respect individual lifetime valuations is a restrictive one, and in section V we consider an alternative treatment.

In order to formalise this, we introduce the indirect utility function giving lifetime welfare for generation i as a function of ω^i, t_C^i, p^{i+1} and Z^i. This is denoted by:

$$V^i = V^i(\omega^i, t_C^i, p^{i+1}, Z^i). \tag{16}$$

If α^i is the private marginal utility of income, then the derivatives of V^i are given by:

$$\left.\begin{aligned} \frac{\partial V^i}{\partial \omega^i} &= \alpha^i L^i, \quad \frac{\partial V^i}{\partial t_C^i} = -\alpha^i(C_1^i+p^{i+1}C_2^{i+1}), \\ \frac{\partial V^i}{\partial p^{i+1}} &= -\alpha^i(1+t_C^i)\,C_2^{i+1}, \quad \frac{\partial V^i}{\partial Z^i} = -\alpha^i. \end{aligned}\right\} \tag{17}$$

The government's objective at time j may be written as maximising:

$$\sum_{i=j}^{\infty} (\gamma)^i V^i, \tag{18}$$

where we assume $0 < \gamma < 1$. (The welfare of generation $j-1$, in the second period of their lives at j, is taken as given.) This objective may be interpreted in several ways. If the government is concerned with discounted *total* utility, then $\gamma = (1+n)/(1+\delta)$, n being the rate of population growth and δ the discount factor, with $\delta > n$. If, as in much of the optimum growth literature, the objective is the discounted sum of *average* utility, then $\gamma = 1/(1+\delta)$.[1]

The government is assumed to have inherited at time j a capital stock per worker, k^j, and the policy parameters set in the preceding period (ω^{j-1}, t_C^{j-1}, p^j, Z^{j-1}, T_1^{j-1}). (It may be noted that these determine the welfare of generation $j-1$.) We can then introduce the state valuation function $\Gamma(k^j, \omega^{j-1}, t_C^{j-1}, p^j, Z^{j-1}, T_1^{j-1})$ to represent the maximal level of social welfare (discounted to time j) obtainable given these initial conditions. The government maximises by choosing k^{j+1}, ω^j, t_C^j, p^{j+1}, Z^j and T_1^j subject to the constraints. The latter may be written in the form:

$$k^j + L^j f(k^j/L^j) = C_1^j + \frac{C_2^j}{1+n} + G + A^j \tag{19}$$

(eliminating k^{j+1} from (11) and (13)), or

$$k^j + L^j[f(k^j/L^j) - \omega^j] = \frac{C_2^j}{1+n} + G - T_1^j - t_C^j C_1^j \tag{20}$$

(using the definition of A^j from equation (8)), and

$$k^{j+1} = L^j\left[f(k^j/L^j) - C_1^j - \frac{C_2^j}{1+n} - G + k^j\right] \Big/ (1+n) \tag{21}$$

(rearranging (13)). If we now introduce the multiplier λ^j for the constraint (20), we can apply the principle of optimality of dynamic programming (this is the same method as employed by Diamond (1973)). In view of the stationarity of the problem,

$$\begin{aligned}\Gamma(j) &\equiv \Gamma(k^j, \omega^{j-1}, t_C^{j-1}, p^j, Z^{j-1}, T_1^{j-1}) \\ &= \max\left\{V^j + \lambda^j\left[k^j + L^j(f-\omega^j) - \frac{C_2^j}{1+n} - G + T_1^j + t_C^j C_1^j\right] + \gamma\Gamma(j+1)\right\},\end{aligned} \tag{22}$$

where k^{j+1} in $\Gamma(j+1)$ is given by equation (21).

The simplest case to consider is that where there is no restriction on lump-sum taxation, so that T_1^j can be varied freely (or equivalently the government can employ debt policy). Since T_1^j does not affect the maximum attainable level of welfare, for a given k^{j+1}, the derivative $(\partial\Gamma(j+1)/\partial T_1^j)$ of the valuation function is zero, and the necessary condition for optimality is simply that $\lambda^j = 0$. This accords with intuition in that the government now has sufficient instruments to achieve the desired intertemporal allocation. Where this is the case, by varying T_1^j the government can ensure that a level of k^{j+1} which is feasible according to the production constraint can be achieved by individual savings decisions.[2]

Where $\lambda^j = 0$, the problem may be simplified by setting $t_C^j = 0$ all j. We have earlier noted that consumer behaviour was invariant with respect to (positive) proportional changes in $(1+t_C^j)$, ω^j and Z^j; and from (21) it is clear that k^{j+1} is unaffected. We therefore adopt this normalisation. The necessary conditions for

[1] Although this formulation has been the more popular, its rationale may be questioned (see, for example, Dasgupta (1969)).

[2] There may be constraints on the range within which T_1^j may be varied. Here, as elsewhere, we are not attempting to give a full characterisation of the optimum policy.

optimality may be set out in terms of the choice of ω^j, t^j_C, p^{j+1} and Z^j (k^{j+1} is eliminated using equation (21)). The first-order conditions for maximisation are:

$$-V^j_\omega = \gamma\Gamma_2(j+1) + \frac{\gamma}{1+n}\Gamma_1(j+1)\left[\left(f-\frac{f'k^j}{L^j}\right)L^j_\omega - C^j_{1\omega}\right], \qquad (23a)$$

where Γ_i denotes the derivatives with respect to the ith argument, and L^j_x, C^j_{1x} denote the derivative with respect to x,

$$-V^j_p = \gamma\Gamma_4(j+1) + \frac{\gamma}{1+n}\Gamma_1(j+1)\left[\left(f-\frac{f'k^j}{L^j}\right)L^j_p - C^j_{1p}\right] \qquad (23b)$$

and
$$-V^j_Z = \gamma\Gamma_5(j+1) + \frac{\gamma}{1+n}\Gamma_1(j+1)\left[\left(f-\frac{f'k^j}{L^j}\right)L^j_Z - C^j_{1Z}\right]. \qquad (23c)$$

The second set of equations are those obtained by differentiating the recursion relation (22) with respect to the state variables k^j, ω^{j-1}, p^j and Z^{j-1}:

$$\Gamma_1(j) = \frac{\gamma}{1+n}\Gamma_1(j+1)(1+f'), \qquad (24a)$$

$$\Gamma_2(j) = \frac{\gamma}{1+n}\Gamma_1(j+1)\left(-\frac{C^j_{2\omega}}{1+n}\right), \qquad (24b)$$

$$\Gamma_4(j) = \frac{\gamma}{1+n}\Gamma_1(j+1)\left(-\frac{C^j_{2p}}{1+n}\right), \qquad (24c)$$

$$\Gamma_5(j) = \frac{\gamma}{1+n}\Gamma_1(j+1)\left(-\frac{C^j_{2Z}}{1+n}\right). \qquad (24d)$$

In the next section we turn to the interpretation of these results. Throughout this discussion we assume both that an optimum policy exists and that it converges to a steady state.[1]

III. INTERPRETATION OF THE RESULTS: FIRST-BEST INTERTEMPORAL ALLOCATION

This section examines the interpretation of the results obtained where T_1 is freely variable (so that $\lambda^j = 0$ all j), focusing on the steady-state properties. (The properties of the approach to steady state can readily be deduced.) In this steady state, the condition (24a) yields $\Gamma_1(j) = \Gamma_1(j+1)$ or

$$1+f' = \frac{1+n}{\gamma}. \qquad (25)$$

Where the objective function takes the total utility form (i.e. $\gamma = (1+n)/(1+\delta)$), this means that $f' = \delta$. The intertemporal allocation is such that the rate of return equals the rate of discount applied to different generations (as in Diamond (1973) and Pestieau (1974)). Where the objective is of the average utility form (i.e. $\gamma = 1/(1+\delta)$), this means that $1+f' = (1+n)(1+\delta)$, which is often referred to as the 'modified golden rule' (e.g. Cass and Shell (1976) and Dixit (1976)).

[1] For discussion of a fully-controlled economy, and references to the literature, see for example Cass and Shell (1976).

In the steady state, the values of Γ_i/Γ_1 are given by (24b–d). Substituting first into (23c), we obtain, dropping the time superscript and using the properties of the indirect utility function (17):

$$\left(\frac{\alpha}{\Gamma_1}\right)\left(\frac{1+n}{\gamma}\right) = \frac{\gamma}{1+n}(-C_{2Z}) + (wL_Z - C_{1Z}). \tag{26c}$$

From the individual budget constraint (differentiating with respect to Z),

$$C_{1Z} + pC_{2Z} = w(1-t_w)\,L_Z - 1. \tag{27}$$

Hence (26c) yields the condition

$$\mu \equiv \left(\frac{1+n}{\gamma}\right)\left(\frac{\alpha}{\Gamma_1}\right) - t_w w L_Z - \tau C_{2Z} = 1, \tag{28}$$

where
$$\tau \equiv p - \frac{1}{1+f'} = \frac{1}{1+r(1-t_r)} - \frac{1}{1+f'} = \frac{pf't_r}{1+f'} \tag{29}$$

and we have used the steady-state condition (25).

The condition (28) has a straightforward interpretation. The left hand side (μ) measures the benefit, denominated in terms of government revenue, of a unit increase in lump-sum income, or equivalently a unit reduction in the lump-sum tax. The second and third terms are the change in revenue arising from the income effect; the first term is the private marginal utility of income divided by the 'shadow' price of revenue (i.e. the effect of an easing of the revenue target). This has been referred to in the optimum tax literature as the 'net social marginal valuation of income' (Atkinson and Stiglitz, 1980). Where the government can employ lump-sum taxes, then the necessary condition is that $\mu = 1$; moreover, as in the standard literature, this implies that there is no recourse to distortionary income or expenditure taxes. This may be seen from (23a) and (23b). Making use of the steady-state conditions for Γ_2/Γ_1 and Γ_4/Γ_1, and the properties of the indirect utility function,

$$-\left(\frac{\alpha}{\Gamma_1}\right)\left(\frac{1+n}{\gamma}\right)L = \left(\frac{-C_{2\omega}}{1+f'}\right) + (wL_\omega - C_{1\omega}), \tag{26a}$$

$$\left(\frac{\alpha}{\Gamma_1}\right)\left(\frac{1+n}{\gamma}\right)C_2 = \left(\frac{-C_{2p}}{1+f'}\right) + (wL_p - C_{1p}). \tag{26b}$$

Using the budget constraint, and the Slutsky relationships,[1] these may be rewritten:

$$-t_w w S_{LL} - \tau S_{2L} = (\mu - 1)L, \tag{30a}$$

$$t_w w S_{L2} + \tau S_{22} = (\mu - 1)C_2. \tag{30b}$$

Thus if the government can use lump-sum taxes freely ($\mu = 1$), the right hand side of both equations is zero.

[1] I.e. where I denotes lump-sum income,

$$\frac{\partial L}{\partial \omega} = L\frac{\partial L}{\partial I} + S_{LL}, \quad \frac{\partial L}{\partial p} = -C_2\frac{\partial L}{\partial I} + S_{L2}, \quad \frac{\partial C_2}{\partial \omega} = L\frac{\partial C_2}{\partial I} + S_{2L}, \quad \frac{\partial C_2}{\partial p} = -C_2\frac{\partial C_2}{\partial I} + S_{22}.$$

In the model as formulated, there is no apparent reason why lump-sum taxes cannot be employed. As argued in Atkinson and Stiglitz (1976), ideally one should build into the analysis the reasons why in reality governments are not happy to rely principally on lump-sum taxes, particularly on account of distributional objectives. Our purpose here is to focus on the relative merits of income and expenditure taxation, and in view of this we simply assume that there is a limit to the use of lump-sum taxation – for example because of its adverse intra-temporal distributional consequences.

Where the government cannot employ lump-sum taxation, we have the standard Ramsey results. We are therefore in a situation where the argument of Feldstein and others applies. If we define the compensated elasticities

$$\left.\begin{aligned} \sigma_{LL} &= \frac{\omega}{L} S_{LL}, \quad \sigma_{2L} = \frac{\omega}{C_2} S_{2L}, \\ \sigma_{L2} &= \frac{p}{L} S_{L2}, \quad \sigma_{22} = \frac{p}{C_2} S_{22}, \end{aligned}\right\} \tag{31}$$

and eliminate μ from (30a) and (30b), we obtain (using the fact that $S_{2L} = -S_{L2}$)

$$\frac{t_w}{1-t_w}(\sigma_{LL} - \sigma_{2L}) = \frac{\tau}{p}(\sigma_{L2} - \sigma_{22}). \tag{32}$$

As noted by Diamond (1973) and Pestieau (1974), this is the application to the dynamic case of the results obtained by Corlett and Hague (1953).

In seeking to use this analysis to make an efficiency argument for expenditure taxation, people have adopted two approaches. First, one can seek qualitative statements. This is illustrated by the results of Atkinson and Stiglitz (1972) for the case where U is directly additive. In that situation, unitary expenditure elasticities imply that the optimal tax rate τ is zero, hence there should be no taxation of savings. This can be shown from equation (32) by noting that direct additivity implies that the substitution terms can be expressed in terms of income derivatives (see, for example, Houthakker, 1960, p. 248) and then making use of the unitary elasticity condition $(1/C_1)(dC_1/dI) = (1/C_2)(dC_2/dI)$.[1] This assumption of unitary expenditure elasticities would therefore justify the expenditure tax, but there is no strong reason to believe that it is valid.

The second approach is to make use of empirical estimates of the parameters σ_{ij}. This faces the difficulty that there is considerable disagreement about key parameters, and that in some cases there is virtually no empirical evidence at all.

[1] The term $\sigma_{LL} - \sigma_{2L}$ is proportional to

$$\chi \equiv \frac{1}{L}\frac{dL}{dI} + \frac{1}{\omega L} - \frac{1}{C_2}\frac{dC_2}{dI}.$$

From the budget constraint

$$C_1(1/C_1)(dC_1/dI) + pC_2(1/C_2)(dC_2/dI) = 1 + \omega L(1/L)(dL/dI).$$

Hence the unitary elasticity condition (and $I = 0$, $Z = 0$) implies $\chi = 0$. See also Sandmo (1974) for a similar analysis.

This applies particularly to the cross-elasticities, σ_{L2} and σ_{2L}, very little being known, for example, about the elasticity of labour supply with respect to the interest rate.[1]

Moreover, experimentation with possible values shows that the results may be highly sensitive. If we start with $\sigma_{L2} = \sigma_{2L} = 0$, then $\sigma_{LL} = 0{\cdot}3$ and $\sigma_{22} = -1{\cdot}5$ would imply a positive tax on savings. There is, however, no reason why it should equal the income tax rate: e.g. with $t_w = \frac{1}{3}$, $r = 1$, it is less than the equivalent income tax.[2] On the other hand, if the elasticity of labour supply were larger,[3] and that of savings lower, this could imply a higher rate of tax on capital income – a surcharge on investment income – for all likely values of the tax rates. For example, if $\sigma_{LL} = 1{\cdot}5$ and $\sigma_{22} = -0{\cdot}5$, savings would be taxed more heavily where t_w is less than $\frac{2}{3}$. At the same time, there is no reason to suppose that the cross-elasticities are zero, and the conclusions reached can depend sensitively on their value. If σ_{2L} were $0{\cdot}3$, τ would fall to zero, whereas $\sigma_{2L} = -0{\cdot}3$ would double the optimum tax rate on investment income. It is unfortunate that the conclusions appear to rest on the values of elasticities which have typically been ignored in empirical work. Even if we could agree on estimates of the own-elasticities, there would remain a considerable range of estimates of the tax rates.

It should be clear from the foregoing that it is difficult to make a strong case *either* for the expenditure tax *or* for taxing interest income at the same rate as wage income. It is in fact expecting too much to hope that one could derive such concrete conclusions. What the optimum tax literature can do is to indicate some of the factors influencing the design of tax structure. For example, the condition (32) depends on the direct compensated elasticities in the way intuition suggests: the tax on savings is more likely to raise welfare, the larger is the compensated elasticity of labour supply (σ_{LL}) relative to that of future consumption ($-\sigma_{22}$). It may also suggest some qualitative propositions. For example, the fact that σ_{2L} and σ_{L2} are of opposite signs means that a sufficient condition for an increase in τ above zero to be optimal is that $\sigma_{2L} \leqslant 0$ (the compensated supply of labour must decrease with the interest rate).[4] However, it is in our view a misapplication of the optimum taxation literature to suggest that it provides a clearcut answer to the choice between income and consumption bases. This is even more true when there are restrictions on the government's ability to achieve a desired inter-temporal allocation – a case to which we now turn.

[1] As noted earlier, in Ordover's (1976) model this cross-elasticity is identically zero. The reason is that variations in effective labour supply in his model come about through the choice of how much of the first period to devote to education, and since there is no labour supply in the second period of an individual's life, the interest rate becomes irrelevant to the educational decision. While the model is perfectly logical, this implication may seem somewhat paradoxical, because one might have expected the introduction of investment in education to be the most interesting way of introducing a relationship between labour supply and the rate of interest.

[2] It should be noted that the relevant time period is a generation. With other values of t_w and r, the rate may exceed the equivalent income tax.

[3] The labour supply decision should be interpreted broadly to include participation, retirement, emigration, etc.

[4] This is the analogue, in terms of compensated demand functions, of what Ordover and Phelps (1975) call the anti-Hicks–Lucas case. (Their failure to refer to it as the anti-Hicks–Lucas-*Rapping* case is particularly strange in a paper on justice.)

IV. CONSTRAINTS ON INTERTEMPORAL ALLOCATION

In the previous section it was assumed that the government could use the timing of lump-sum taxation (i.e. vary T_1 for a given Z), or debt policy, to achieve the desired intertemporal allocation. Thus, with the total utility formulation of the objective, the steady state capital stock satisfies the 'first-best' condition $f' = \delta$. On the other hand, the assumption that the government possesses this degree of control is crucial to the results. In particular, if the government cannot achieve the desired level of capital by varying lump-sum taxes, then the standard optimum tax results no longer necessarily apply.

The introduction of such constraints raises a number of questions. What is it that prevents the government from levying differential lump-sum taxes? If there are constraints on using the income tax for this purpose (e.g. that the tax exemption must be identical for all), what is to stop the generations from being treated differently under a social security scheme (e.g. Samuelson, 1975)? If tax policy cannot be used, what are the limitations on the use of debt policy? In a full treatment, these issues need to be addressed. Our purpose here, however, is not to explore these questions, but rather to draw attention to some of the possible implications of such constraints. *If* the government is constrained in this way, how do the results need to be modified?

In order to illustrate the effects of constraints on the intertemporal allocation, we assume that there is no lump-sum taxation ($T_1^i = T_2^i = Z^i = 0$ all i), no debt ($D^i = 0$ all i) and no indirect taxation ($t_C^i = 0$ all i).[1] The maximisation problem may be reformulated as (using Ω in place of Γ):

$$\Omega(j) = \Omega(k^j, \omega^{j-1}, p^j)$$
$$= \max\left\{V^j + \lambda^j\left[k^j + L^j(f-\omega^j) - \frac{C_2^j}{1+n} - G\right] + \gamma\Omega(j+1)\right\}. \tag{33}$$

The necessary conditions are:

$$(-V_\omega^j) = \lambda^j[(f-\omega^j - f'k^j/L^j)L_\omega^j - L^j] + \gamma\Omega_2(j+1)$$
$$+\left(\frac{\gamma}{1+n}\right)\Omega_1(j+1)\,(w^jL_\omega^j - C_{1\omega}^j) \tag{34a}$$

and

$$(-V_p^j) = \lambda^j[(f-\omega^j - f'k^j/L^j)\,L_p^j] + \gamma\Omega_3(j+1)$$
$$+\left(\frac{\gamma}{1+n}\right)\Omega_1(j+1)\,(w^jL_p^j - C_{1p}^j). \tag{34b}$$

The recursion equation yields:

$$\Omega_1(j) = [1+f'(k^j)]\left[\lambda^j + \frac{\gamma}{1+n}\Omega_1(j+1)\right], \tag{35a}$$

$$\Omega_2(j) = \left(-\frac{C_{2\omega}^j}{1+n}\right)\left[\lambda^j + \frac{\gamma}{1+n}\Omega_1(j+1)\right], \tag{35b}$$

[1] In this case the normalisation of tax rates is not an arbitrary matter. From the individual budget constraint (15), an equal proportionate rise in $(1+t_C^i)$ and ω^i leaves the individual unaffected (with $Z^i = 0$), but the level of private savings is changed, and hence affects (19). Put another way, the government is no longer indifferent about the timing of receipts.

$$\Omega_3(j) = \left(-\frac{C^j_{2p}}{1+n}\right)\left[\lambda^j + \frac{\gamma}{1+n}\Omega_1(j+1)\right]. \tag{35c}$$

In steady state, equation (35*a*) gives (dropping the time superscript):

$$\frac{\lambda}{\Omega_1} = \frac{1}{1+f'} - \frac{\gamma}{1+n} \tag{36}$$

and from (35*b*) and (35*c*) we can solve for Ω_2/Ω_1 and Ω_3/Ω_1 as functions of λ/Ω_1. Substituting into (34*a*) and (34*b*), and using the Slutsky equations, yields:

$$-t_w w S_{LL} - \frac{\gamma(1+f')}{1+n}\tau S_{2L} = (\mu^* - 1)L, \tag{37a}$$

$$t_w w S_{L2} + \frac{\gamma(1+f')}{1+n}\tau S_{22} = \left[\mu^* - 1 + (1+f')\frac{\lambda}{\Omega_1}\right]C_2, \tag{37b}$$

where
$$\mu^* \equiv \frac{\alpha(1+f')}{\Omega_1} - t_w w L_Z - \frac{\gamma(1+f')}{1+n}\tau C_{2Z}. \tag{37c}$$

If $(1+f')$ were equal to $(1+n)/\gamma$, as in the 'first-best' condition (25), then these equations would reduce to the standard Ramsey formula. In order to see what happens where the first-best cannot be attained, let us consider first the case where the objective is of the total utility form, so that $\gamma/(1+n) = 1/(1+\delta)$. If we then define

$$\theta \equiv -\frac{\lambda}{\Omega_1}(1+f') = \frac{1+f'}{1+\delta} - 1, \tag{38}$$

this will be positive (negative) where the capital stock is less than (greater than) the 'first-best' level: i.e. as f' is greater (less) than δ. Substituting into (37*a*) and (37*b*), and eliminating μ^*, we obtain

$$\frac{t_w}{1-t_w}(\sigma_{LL} - \sigma_{2L}) = \frac{\tau}{p}(\sigma_{L2} - \sigma_{22})(1+\theta) - \theta. \tag{39}$$

This is the analogue of condition (32), and reduces to this when $\theta = 0$.[1] If instead the objective is formulated in terms of average utility, with $\gamma = 1/(1+\delta)$, then θ is positive where $(1+f') > (1+n)(1+\delta)$. Where $\delta \to 0$, this gives the 'golden rule' condition: θ is positive (negative) where the capital stock is below (above) the golden rule. (Here, since $\gamma = 1$, a different argument is necessary to characterise the optimum.)

The equation (39) does not allow us to draw direct conclusions about the effect of constraints, since θ is itself a variable of the problem. To get a better understanding of the nature of the solution, it is useful to work out a specific example. Suppose that the individual utility functions are Cobb–Douglas:

$$U = a_1 \log C_1 + a_2 \log C_2 + a_3 \log(1-L), \tag{40}$$

[1] One interpretation of the relationship between (39) and (32) is that the former contains an additive term to correct for the 'externality' arising from the change in savings when the economy is not at the 'first-best'. For a general analysis of the additivity property of optimal taxes when there are externalities, see Sandmo (1975).

where $a_1+a_2+a_3 = 1$. This example is chosen because it is simple and because we know that in this case the standard optimal taxation framework would involve $\tau = 0$, since the expenditure elasticities with respect to labour income are all equal to one. The compensated elasticities in the Cobb–Douglas case are:[1]

$$\left.\begin{aligned}\sigma_{LL} = \sigma_{2L} = a_3, \quad \sigma_{22} = a_2 - 1,\\ \sigma_{L2} = -a_2 a_3/(1-a_3).\end{aligned}\right\} \tag{41}$$

Substituting these values into (39), we obtain

$$\frac{\tau}{p} = \left(\frac{\theta}{1+\theta}\right)\left(1+\frac{a_2}{a_1}\right). \tag{42}$$

(This confirms that where $\theta = 0$ there should be no tax on savings.) Suppose that the objective function is of the total utility form. From the capital market equation (8) and the production constraint (20), and assuming that the production function is also Cobb–Douglas ($f = B(k/L)^{\eta}$), we can then calculate that:

$$\left(1/\eta+\frac{a_2}{a_1}\right)(f'-\delta) = \left(\frac{1-\eta}{\eta}\right)(f'_0-\delta)+\frac{G}{k}, \tag{43}$$

where f'_0 denotes the return to capital at the no-tax equilibrium.[2] We may also note that the after-tax interest rate at the optimum is (using (38) and (42)):

$$r(1-t_r) = r-(1+r)\frac{\tau}{p} = \delta-(f'-\delta)\frac{a_2}{a_1}. \tag{44}$$

From this we may conclude that if in the absence of taxation the capital stock is less than its 'first-best' level, and if $G \geqslant 0$, then at the optimum $f' > \delta$ and hence $\theta > 0$. In other words, the gap between f' and δ may be reduced by the optimum choice of tax policy but not totally eliminated. This in turn implies that there will be a positive level of taxation on savings. Suppose, for example, that in the no-tax situation, $f' = 2{\cdot}0$, $\delta = 1{\cdot}2$, and that $a_1 = a_2$, $\eta = \frac{1}{3}$. Then with zero revenue requirement, the optimum f' may be seen from equation (43) to be 1·6. This implies $\theta = 0{\cdot}18$ and an optimum tax rate on capital income of 50 %. The after-tax interest rate faced by the individual is 0·4 of the no-tax value.

The Cobb–Douglas case is illustrated in Fig. 1, where we have plotted the levels of C_1 and C_2 (L being a constant). In the absence of taxation, the feasible frontier follows the curve through P and GR. Each point on the curve is associated with a particular level of k, and k rises, f' falls, as we move up the curve. The point GR is that where $f' = n$, or the golden rule, but there is no reason to expect the competitive equilibrium to be at this point (Diamond (1965)). In Fig. 1, we have shown the no-tax competitive equilibrium as occurring at P, where $f'_0 > n$. On the assumption that $f'_0 > \delta > n$, the optimal steady state solution involves a rise in k (to point Q) and a reallocation between generations (to point Q^*).

[1] The maximisation of U subject to the individual budget constraint yields

$$L = a_1+a_2; \quad C_1 = a_1\omega; \quad pC_2 = a_2\omega.$$

[2] The no tax equilibrium has $r = \eta(1-a_3)(1+n)/[a_2(1-\eta)]$. This is the same as the example given by Diamond (1965, p. 1135) except that he does not allow for any labour response.

It may appear paradoxical that where the no-tax capital stock falls short of the 'first-best' level there should be a positive tax rate on capital (in the Cobb–Douglas example). The reason for intervention is, however, the level of capital formation and – with the particular savings function – private savings are increased by a switch from t_w to τ.[1] This is a special feature of the particular example, but the important point is that, where the government is constrained in achieving the desired intertemporal allocation, it is the absolute effect on savings which is relevant. The essential element is the *uncompensated* response, rather than the *compensated* effects which have received most attention in the optimum tax literature.

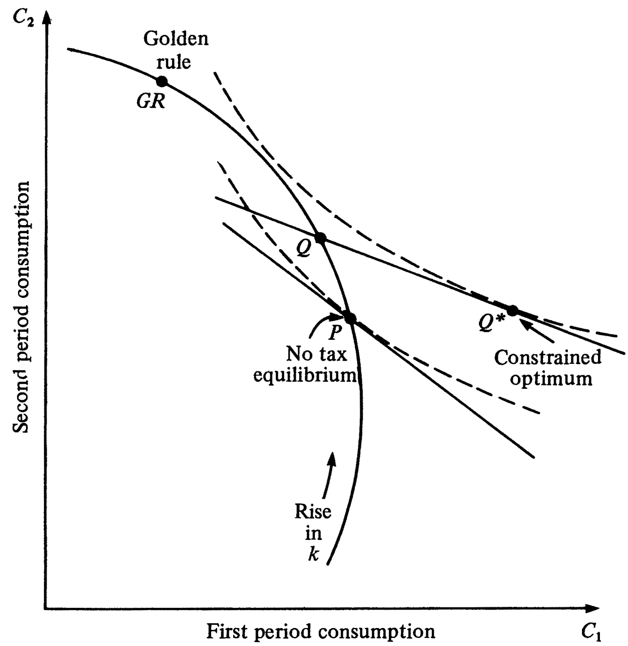

Fig. 1. Cobb–Douglas example: steady states.

V. ALTERNATIVE FORMULATIONS OF THE GOVERNMENT'S OBJECTIVE

In the previous section we have seen that the standard optimum tax formulae need to be modified where the government does not have sufficient instruments to ensure the intertemporal allocation condition:

$$1+f' = \frac{1+n}{\gamma} \equiv 1+\epsilon. \tag{45}$$

[1] The level of private savings is

$$A = w(1-t_w)L - C_1 = a_2 w(1-t_w)$$

and hence rises if τ is increased and t_w reduced.

Thus the nature of the solution, and the desirability of income or expenditure taxation, depends on the instruments at the disposal of the government. It also depends on the objective pursued by the government, and in this section we explore some of the implications of alternative goals.

The first aspect considered is the effect of variation in γ. This may be illustrated by the Cobb–Douglas example used in the previous section. Defining ϵ as in equation (45), the optimum steady state value of f' can be obtained from equation (43) by replacing δ by ϵ. The optimum tax rate on capital income may in turn be obtained (see (44)):

$$t_r = \left(1+\frac{a_2}{a_1}\right)\left(\frac{r-\epsilon}{\epsilon}\right) = \frac{(a_1+a_2)\,[(1-\eta)\,(r_0-\epsilon)+\eta(G/k)]}{\epsilon(a_1+\eta a_2)}. \tag{46}$$

It follows that the tax rate is lower, the larger is ϵ. A rise in the discount rate, reducing γ and hence raising ϵ, leads to lower taxation on capital income. (As noted before, this is a product of the particular savings function implied by the Cobb–Douglas utility function.)

The consequences of the replacement of the total utility objective by that of average utility may therefore be seen directly. The latter implies that

$$\epsilon = (1+n)\,(1+\delta)-1,$$

which exceeds δ where $n > 0$, and the tax rate on capital income is lower with the average utility objective. On the other hand, if we reduce δ, while maintaining the average utility assumption, then the tax rate rises; and in the limiting 'golden rule' case, with $\delta = 0$, the value of ϵ is lower (we assumed $\delta > n$ in the earlier discussion). The 'golden rule' solution involves a higher level of taxation, and of capital formation.

Secondly, we may relate our analysis to the debate between Samuelson (1958), Lerner (1959) and others concerning the formulation of government objectives and intertemporal allocation. In order to see some of the issues involved, let us consider the case where the utility function may be written:

$$u_1(C_1, L) + u_2(C_2). \tag{47}$$

The debate has largely been carried out in terms of comparing steady state paths. In this context, Samuelson took as the social utility function the welfare of a representative generation: i.e. (47). In our terms, this is equivalent to the case where $\gamma = 1$, and it leads, as Diamond (1965) has shown for a production economy, to the golden rule solution. Alternatively, one can consider the welfare of those alive at a representative instant, weighted by their numbers and discounted according to their generation: i.e.

$$u_1(C_1, L) + \left(\frac{1+n}{1+\delta}\right) u_2(C_2). \tag{48}$$

This may be seen as corresponding to the Lerner alternative formulation (see also Asimakopoulos (1968)), although Lerner himself took $\delta = 0$. This alternative is given in our case by $\gamma = (1+n)/(1+\delta)$, and hence leads to a lower level of taxation on capital income where $\delta > n$.

To this point we have assumed that the discounting applies to generations; it can, however, be argued that the discounting should apply to calendar time. The distinction between these is discussed by Mitra (1975) who refers to the former as discounting for the remoteness of future generations in time and the latter as allowing for the probability of extinction. With the latter approach, there is the distinct possibility that private and social judgments will differ. In order to explore this, let us suppose that the government attaches a weight h to the second period component of the utility function (u_2). Where the government applies a value of h greater than 1, it is in effect attaching a higher probability to survival than the individual. Alternatively, it may be seen as acting less myopically.[1]

The implications of $h \neq 1$ may be seen in the case where the government is unconstrained in its intertemporal allocation (as in section III), but is constrained not to employ Z^i. We need to replace the derivatives of the indirect utility function by:

$$\frac{\partial V}{\partial \omega} = +\alpha L + \alpha(h-1)p\frac{\partial C_2}{\partial \omega}, \tag{49a}$$

$$\frac{\partial V}{\partial p} = -\alpha C_2 + \alpha(h-1)p\frac{\partial C_2}{\partial p}. \tag{49b}$$

Using the Slutsky equations and definitions of the elasticities in equations (31), the first-order condition can be shown to be:

$$\frac{\tau}{p} + (h-1)\left(\frac{\alpha}{\Gamma_1}\right)\left(\frac{1+n}{\gamma}\right) = \left(\frac{t_w}{1-t_w}\right)\frac{\sigma_{LL}-\sigma_{2L}}{\sigma_{L2}-\sigma_{22}}. \tag{50}$$

If, for example, the right hand side is zero (as in the Cobb–Douglas case), then $h > 1$ implies that the optimum tax on savings is now negative. Where the government attaches more weight than the individual to future consumption, savings are subsidised rather than exempt (as they would be if $h = 1$). The magnitude of the corrective subsidy depends on the extent of the difference between private and social valuations, and on the 'cost' of raising revenue, as measured by the ratio of the private marginal utility of income (α) to the 'shadow' price of revenue ($\gamma\Gamma_1/(1+n)$).

The consequences may be illustrated by the Cobb–Douglas example. In that case it may be shown that the optimum tax is given by:

$$\frac{\tau}{p} = 1-h. \tag{51}$$

From this simple result, we may see that a 20 % difference in the weight attached by the government implies a 20 % subsidy on capital income.

The non-Paretian objective function, where the government does not respect individual preferences, provides a further departure from the standard optimum tax formulae, although in this case the modification takes a quite intuitive form.

[1] This is related to the concept of Allais optimality (see Malinvaud, 1972, ch. 10), in that individual preferences *within* a period are respected.

VI. CONCLUSIONS

In the course of this paper we have examined how the standard optimum tax results may need to be modified when applied to intertemporal problems, particularly the choice between income and expenditure taxation. The main results are summarised in Table 1 which shows the conditions for optimality and the results in the special Cobb–Douglas case.

Table 1

Summary of Main Results

First-order conditions	Cobb–Douglas utility and production functions (where $G = 0$)
First-best intertemporal allocation (section III)	
$\frac{t_w}{1-t_w}(\sigma_{LL}-\sigma_{2L}) = \frac{\tau}{p}(\sigma_{L2}-\sigma_{22})$	$t_r = 0$
equation (32)	
Constraints on intertemporal allocation (sections IV and V)	
$\frac{t_w}{1-t_w}(\sigma_{LL}-\sigma_{2L}) = \frac{\tau}{p}(\sigma_{L2}-\sigma_{22})(1+\theta)-\theta$	$\frac{\tau}{p} = \left(\frac{\theta}{1+\theta}\right)\left(\frac{a_1+a_2}{a_1}\right)$
equation (39)	equation (42)
where objective is sum of utilities	
$\theta = \frac{r-\delta}{1+\delta}$,	$t_r = (1-\eta)\frac{(r_0-\delta)}{\delta}\left(\frac{a_1+a_2}{a_1+\eta a_2}\right)$
where objective is 'average' utility	
$\theta = \frac{r-n-\delta-n\delta}{(1+n)(1+\delta)}$	$t_r = (1-\eta)\frac{(r_0-\epsilon)}{\epsilon}\left(\frac{a_1+a_2}{a_1+\eta a_2}\right)$
	where $\epsilon = (1+n)(1+\delta)-1$ equation (46)
Non-Paretian objective (section V) (where first-best intertemporal allocation)	
$\frac{t_w}{1-t_w}(\sigma_{LL}-\sigma_{2L}) = \left[\frac{\tau}{p}+(h-1)\frac{\alpha}{\Gamma_1}\left(\frac{1+n}{\gamma}\right)\right](\sigma_{L2}-\sigma_{22})$	$\frac{\tau}{p} = 1-h$
equation (50)	equation (51)

The main lesson is that it is difficult to argue on the basis of existing results for the welfare superiority of either an expenditure tax or a pure income tax. Even in the case where the standard optimum tax results may be directly applied (section III), there is no strong reason to suppose that the exemption of saving is desirable on efficiency grounds. There are situations, including the Cobb–Douglas example (more generally, where there are unitary expenditure elasticities), such that $t_r = 0$ satisfies the first-order conditions, but the existing empirical evidence does not allow us to draw firm conclusions. It is indeed the case that the calculated tax results may depend crucially on parameters, such as the interest elasticity of labour supply, which have typically been disregarded in empirical studies.

When the government is constrained in the instruments it can employ to achieve a desired intertemporal allocation, the results need to be modified. As is illustrated by the Cobb–Douglas example, if the non-intervention capital stock differs from its 'first-best' level there may be a case for taxing or subsidising capital income. In that example, if the capital stock is below the 'first-best', a tax on capital income raises welfare, bringing the capital closer to the full optimum but not closing the gap entirely. The optimum tax on capital depends on the response of savings and on the nature of government objectives. For example, in the Cobb–Douglas case, the optimum tax on capital income is a declining function of the social discount rate. Moreover, where the government's valuation departs from that of the individual – as may quite easily happen in a dynamic context – this provides a further modification of the results.

The analysis does not therefore lead to clearcut policy conclusions; rather the lessons to be drawn are about the *nature* of the arguments which can be made in this field. We have for example tried to bring out the interdependence between different policy instruments, and the way in which the case for different tax bases depends on the range of measures at the disposal of the government to achieve a desired intertemporal allocation. We have shown how the results may need to be modified where these are constrained, and how they are affected by different formulations of the social welfare function. In part these conclusions are quite intuitive: for example, the correction for divergences between private and social valuations (equation (50)). Others are less obvious: for example, the fact that it is the uncompensated rather than compensated response of savings which is relevant (in section IV).

The analysis has been limited in several important respects. In particular, we have been concerned with intertemporal allocation, which is essentially a question of intergenerational equity, but we have not addressed the issue of intragenerational redistribution. This is important both in its own right and also because of its implications for the policy instruments which can be employed (particularly the limitations on the use of poll taxes). A natural extension of the model is to assume that individuals have the same preferences, but differ in their earning abilities; this is the formulation of Mirrlees (1971), taken over in an intertemporal context by Hamada (1972), Ordover and Phelps (1975) and Ordover (1976). These authors allow in effect for differences in wage rates; we should also take account of differences in rates of return which may arise in an 'imperfect' capital market. This can be modelled in a number of different ways,[1]

[1] There are two approaches that seem to be worth pursuing. In the first, the rate of interest r_i is simply individual i's marginal productivity of capital as derived from his personal production function. Thus, there is really no capital market at all, and each individual's future consumption is constrained by the return on his real investment undertaken in the first period. In the second interpretation there is a capital market, but because of imperfect information, transactions costs etc. consumers do not have the same degree of access and therefore receive different rates of return.

but is an important phenomenon which should be incorporated, along with uncertainty, into the intertemporal treatment of the design of taxation.

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