Growth and income distribution with the dynamics of power in labour and goods markets

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The interaction between economic growth and income distribution is examined using Kaleckian/post-Keynesian models in which there are lags in investment and in which the dynamics of income distribution between wages and profits depends on changes in power relations in both the labour market and goods market. By examining these two influences on distributional dynamics simultaneously, the relative strength of which can change over the growth process, it is shown that the growth-distributional dynamics can involve non-linearities, multiple equilibria and instability. The implications of policy-induced changes—including those in macro-economic policy and labour market and antitrust policies—on aggregate demand and distribution are examined for both wage-led and profit-led growth regimes.

Key words: Income distribution, Growth, Kaleckian/post-Keynesian models, Aggregate demand
JEL classifications: E12, O41

1. Introduction

What role do changes in labour market conditions and goods market conditions have in determining the dynamics of growth and distribution in capitalist economies? This paper examines this question using simple Kaleckian/post-Keynesian (KPK) models of growth and distribution.

The interaction between growth and distribution is, of course, a major topic of interest within the KPK tradition. A great deal of attention has been given in this tradition to the effect of a change in the distribution of income between workers and profit recipients on the rate of capital accumulation and output growth, and to the
possibilities of wage-led and profit-led growth, i.e. whether an increase in the profit share reduces or increases the rate of growth of the economy. Some attention has also been given to what determines the dynamics of income distribution, to complete the analysis of the interaction between growth and distribution. This side of the literature has focused on changes in income distribution due to changes in labour market conditions, in which a tightening of the labour market results in an increase in the wage share due to a strengthening of the bargaining power of workers vis-à-vis firms and to changes in the goods market, including those resulting from altered industrial concentration.

This paper follows the usual KPK tradition in which, at a point in time, the degree of monopoly determines the distribution of income between workers and capitalists or profit recipients, and examines how distribution, through its effects on aggregate demand, influences capital accumulation and growth. The model has the post-Keynesian feature, following the writings of Kaldor (1940) and Steindl (1952), of making desired investment depend positively on output and capacity utilisation, and in general of making desired investment depend on profitability and animal spirits. It follows Kalecki (1971) in making the degree of monopoly depend both on labour market conditions and goods market conditions involving, among others, changes in industry structure, as measured by industrial concentration, and changes in the bargaining power of workers, and in allowing for lags between investment plans and actual investment expenditures.

The paper’s contribution lies in examining how the combination of different influences on distributional dynamics and changes in their relative importance over the growth process, more specifically those emanating from alterations in the state of bargaining power between workers and firms and in goods market conditions, can lead to novel and different patterns of the interaction between growth and distribution in capitalist economies. In particular, it shows how the dynamics of growth and distribution can involve non-linearities and multiple equilibria, with important implications for policy-induced changes in aggregate demand and income distribution.

The rest of this paper proceeds as follows. Section 2 describes the structure of the basic model. Section 3 examines the short-run equilibrium and long-run dynamic properties of a basic model and some of its variants. Section 4 examines the effects of shifts in autonomous factors influencing aggregate demand and profit share dynamics. Section 5 concludes.

2. Structure of the model

We assume a simple closed economy, without government fiscal activity (except at times, informally, when explicitly noted), which produces one good with two factors of production: homogenous labour and capital, where capital is the produced good. The economy has two classes: workers who receive wages and capitalists who receive the residual income, as profits.
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Workers are assumed to spend all their income on consumption, while capitalists are assumed to save a fraction, $s$, of profits. This implies that saving, as a ratio of capital stock, is given by the standard equation:

$$S/K = s \pi u$$  \hspace{1cm} (1)$$

where $S$ is real saving, $K$ the stock of physical capital, $\pi$ the share of profits in income and $u = Y/K$ is the rate of capacity utilisation, with $Y$ the level of real income and output.

Firms set their price as a markup on labour costs using fixed coefficients and constant returns to scale technology, so that we have:

$$P = (1 + z)W a_0$$  \hspace{1cm} (2)$$

where $P$ is the price level, $z$ is the markup, $W$ is the money wage and $a_0$ is the labour required to produce a unit of output, which is given, since we abstract from technological change. Firms typically hold excess capacity, so that we assume that $u < 1/a_1$, where $a_1$ is the technologically possible minimum capital–output ratio. Unemployed labour is available in the economy at a money wage that is taken to be given, for simplicity.\(^5\) Firms are assumed to change their level of production and capacity utilisation in response to changes in the demand for goods.

Firms have given investment plans at a point in time, which, as a ratio of capital stock, we denote with $g$, so that:

$$I/K = g$$  \hspace{1cm} (3)$$

In other words, actual investment (as a ratio of the stock of capital) is predetermined by past plans, at the given level $g$. Their desired investment depends positively on the rate of capacity utilisation and the profit share, so that:

$$g^d = G(u, \pi, \gamma)$$  \hspace{1cm} (4)$$

where the partial derivatives are given by $G_u > 0$, $G_\pi > 0$ and $G_\gamma > 0$ and $\gamma$ is a shift parameter that we will usually suppress. The positive effect of the capacity utilisation rate reflects the fact that firms want to build capacity when markets are buoyant, along the lines discussed by Kaldor (1940) and Steindl (1952). The profit share enters as a determinant of profitability, which was emphasised by Robinson (1962). The profit rate is given by:

$$r = \pi u$$  \hspace{1cm} (5)$$

which shows that the profit rate is the product of the profit share and the capacity utilisation rate. If desired investment is assumed to depend on expected profits and firms use current conditions to form expectation about future conditions so that expected profits depend on the current profit rate, desired investment is seen to depend

\(^5\) Nothing is changed if money wages are assumed to change. Given the markup and the labour–output ratio, it will just result in a proportionate increase in the price level. Of course, inflation can result in changes in the distribution of income due to changes in the markup, as discussed, e.g., in Dutt (1990, 1992) and Taylor (1991), but these issues are not analysed in this paper.
positively on both the capacity utilisation rate and the profit share. Since the capacity utilisation rate has already been included as an argument in the function, the profit share is added as an additional variable. As we shall see in section 3, and as shown by Bhaduri and Marglin (1990) who proposed this form of the investment function, this formulation allows the profit share to have a positive or negative effect on desired investment, as opposed to the case in which desired investment depends only on the rate of capacity utilisation or on the rate of capacity utilisation and the rate of profit, as assumed by Steindl (1952).Actual investment is assumed to adjust to desired investment according to the adjustment equation:

$$\dot{g} = \lambda [g^d - g]$$  \hspace{1cm} (6)

where the dot denotes a time derivative and there $\lambda$ is a speed of adjustment constant. This equation captures the fact that there is a time lag in investment, as emphasised by Kalecki (1971). The markup is given at a point in time, which implies that the profit share is also given. Using equation (2), the profit share is seen to be determined by the equation:

$$\pi = \frac{z}{1 + z}$$

Over time the markup, and hence the profit share, changes according to the equation:

$$\dot{\pi} = F(u, g, \pi)$$  \hspace{1cm} (7)

where $F_u$ (the partial derivative with respect to $u$) is assumed to be negative, $F_g$ (the partial with respect to $g$) can take either sign but will be assumed to be negative or small if positive and $F_\pi$ (the partial with respect to $\pi$) will be assumed to be negative. These partial derivatives can, in principle, capture a variety of influences that reflect changes in the goods market and in the labour market. Factors in the goods market may change markups because of the decisions firms make given the degree of competition in the market, as reflected by the structure of the market (e.g. as measured by the degree of concentration), and because of changes in the structure of the market itself.

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6 Early models, along the lines of Kalecki (1971) and Steindl (1952) as developed by Rowthorn (1982) and Dutt (1984), made investment depend on the rates of profit and capacity utilisation to show that a rise in the profit share—by redistributing income from workers who do not save to capitalists who do—reduces consumption demand, capacity utilisation and profits, and through them, investment.

7 Kalecki’s (1971) own formalisation of lags in investment is rather different and leads to a complex macrodynamical model involving a mixed difference-differential equation system. It can be argued that our formulation captures Kalecki’s essential idea that actual investment adjusts slowly when investment plans change, in a much simpler way.

8 This equation can be interpreted as a reduced-form equation that examines the dynamics of the money wage and the price level, which affects the dynamics of the real wage and hence (given labour productivity) the dynamics of the wage and profit shares. See, e.g., Taylor (1991) and Dutt (1992).

9 It can also change due to structural changes involving alterations in the relative sizes of different sectors, something that we abstract from for simplicity, given the one-sector setting adopted in this paper. Suffice it to note that by ‘concentration’, we are thinking not just of market power within a particular industry but also of the overall economic power of firms in goods markets, given that individual firms operate in a multiplicity of sectors and that markups in a particular industry are affected not just by conditions in that industry but also by the overall power of the firm within the economy.
Labour market conditions may affect the markup by changing the relative bargaining strength or power of workers and employing firms, and by other conditions that influence the ability of workers and firms to achieve their goals, such as deviations of actual distributive shares from what is considered fair according to social norms.

Two comments about these effects are in order. First, these effects may both be of a relatively short-term nature or of a long-term one, to distinguish between what can be called cyclical and long-term secular factors, but we do not enter into such distinctions involving time horizons for our purposes. Second, while much of the discussion of these issues is in terms of the effects of the relevant variable on the level of markup or profit share, our analysis considers the effects on the change in their levels. Our reason for focusing on changes is that, in our model, in the short run the profit share is given, so that it can change only over time in the long run. Thus, we can think of the effect of the relevant variables in terms of the level of profit share or markup in the ‘next’ period given its current level, or in terms of long-run equilibrium conditions in which the adjustments in the profit share or markup in the long run have been completed.

We consider first the effects of changes in the rate of capacity utilisation, \( u \). Regarding the effects in the goods market, different possible effects of changes in capacity utilisation on the change in the profit share can be expected. It is sometimes argued that if aggregate demand and the level of capacity utilisation are high, firms will actually push up markups and thereby increase the profit share. Indeed, Harrod (1936) argued that the markup on marginal cost during the boom will be procyclical. According to Harrod’s ‘law of diminishing elasticity of demand’, in monopolistically competitive markets, demand is less elastic in good times than in bad times. Harrod derived this from the idea that in prosperity when national income rises, the expected value for a consumer of searching for better opportunities among close substitutes is likely to decrease. Although all this may seem to be in accord with the intuitive idea that high demand increases the markup and the price level, there is much reason to doubt it. Two effects, one related to the goods market and the other to the labour market conditions, can indeed dominate Harrod’s effect, which leads us to assume that \( F_u < 0 \).

First, Kalecki (1971, p. 51) argued that during slumps the ratio of overhead (or fixed) costs to prime (or variable) costs increases and creates conditions for ‘tacit agreements not to reduce prices in the same proportion as prime costs’, with the converse occurring during booms (see also Kalecki, 1943, pp. 17-18). However, in a footnote he added that the tendency for the degree of monopoly to rise in the slump ‘is the basic tendency; however, in some instances the opposite process of cut-throat competition may develop in a depression’. Subsequent research, both theoretical and empirical (Bils, 1987; Morrison, 1993; Domowitz et al., 1988), has confirmed that Kalecki’s ‘basic tendency’ is the more realistic one. In terms of theory, during booms incumbent firms may feel the likelihood of entry to be higher and for this reason may reduce their markup to deter entry (Stiglitz, 1984). Oligopolistic firms may find implicit collusion more difficult when demand is relatively high, since the benefits to firms of undercutting prices and capturing a larger share of the market are likely to be higher than the possible loss from being punished

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10 We use the word ‘term’ differently from how we use the word ‘run’ to refer to short and long runs. The latter refer to logical time frames in which different sets of variables adjust, while the former refer to historical periods of calendar time. Since all the effects we discuss here refer to changes in the distribution of income into wage and markup income, since this distribution is fixed in the short run, all these effects are long-run ones by definition, although they may refer to different ‘terms’.
later, so they will set lower prices to prevent undercutting (Rotemberg and Saloner, 1986). Moreover, to the extent that higher levels of demand induce greater entry, there may also be a tendency for the degree of industrial concentration to fall, thereby reducing the tendency of the markup and the profit share to increase. There is also a great deal of evidence to suggest that markups are lower when demand is higher (see, e.g., Rotemberg and Saloner, 1986).

Second, labour market conditions also affect changes in the markup. The relative bargaining power of workers is in particular likely to be increased by increases in the level of tightness of labour markets (as in Goodwin, 1967). The tightness of labour markets can be captured using the unemployment rate or the employment rate. The employment rate can be written as:

$$e = \frac{u k}{A} \quad (8)$$

where $k = K/N$, where $N$ is the supply of labour and $A$ the output–labour ratio or the productivity of labour. To keep matters simple and not introduce an additional state variable, $k$, into our model, we will proxy the tightness of the labour market in terms of $u$, the level of capacity utilisation, given that we abstract from technological change and labour productivity growth. An increase in $u$, by increasing the employment rate, increases the bargaining power of workers and therefore reduces the markup and hence the profit share. Because of the importance this labour market effect will play in some of our subsequent analysis, the issue of using $u$ to capture labour market tightness, rather than $e$ directly, is discussed briefly in Appendix B.

The rate of growth of the economy, as captured by $g$, can also affect the markup and profit share by affecting the industrial concentration rate. It is not clear whether the effect of a higher rate of growth is to increase or reduce this rate. If growth tends to be driven by the growth of leading firms with proprietary technology who enjoy the benefits of static and dynamic scale economies, higher growth rates may well increase the concentration rate and the profit share. However, if growth tends to be driven mainly by technological diffusion, with new entrants breaking into the markets previously enjoyed by large firms, the concentration rate, markups and profit share will all fall. As noted above, we will assume that the second case of more competition with higher growth applies or, if the first case does, the effect is weak.

The level of the profit share will also affect changes in the profit share. Regarding the goods market, as the profit share falls, firms become more concerned about the fall in this share and attempt to increase the rate of change of the markup by different means at their disposal, including mergers and acquisition activity. Thus, as the profit share becomes smaller, the rate of increase in the profit share rises. It is likely that this effect is weak at higher levels of $\pi$ but becomes stronger at lower levels of $\pi$, since firms intensify their efforts to increase markups when the markup becomes very low. As the profit share increases, the pressure on firms to increase their markup falls.

The profit share affects the change in the profit share through the labour market as well. Kalecki (1971, p. 161) wrote:

High markups in existence will encourage strong trade unions to bargain for higher wages since they know that firms can ‘afford’ to pay them. If their demands are granted but … [the markup

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11 This specification is pursued in Dutt (1992).
is] not changed, prices also increase. This would lead to a new round of demand for higher wages and the process would go on with price levels rising. But surely an industry will not like such a process making its products more and more expensive and thus less competitive with the products of other industries. To sum up, trade-union power restrains the markups. (Kalecki, 1971, p. 161)

We take this to mean that as the markup increases, firms will be less able to pass on higher wage demands in the form of higher prices, so that the change in the markup and hence the profit share will be lower. We assume that this effect is weak for lower levels of $\pi$ but becomes stronger at high levels of $\pi$, when firms are strongly constrained by their ability to pass on higher wage costs to higher prices. This relation can also be explained in terms of norms of fairness. As the profit share increases, the distribution of income will be increasingly more unequal as compared with what most people would consider a fair distribution, and this will reduce the ability of firms to increase their markups and the profit share.\footnote{Fairness can be conceptualised either in terms of what we are calling the wage and profit shares (or, more accurately in our model, in terms of the share of markup income) or in terms of the personal income distribution. A high markup income share can therefore be perceived directly as being unfair or indirectly so in terms of its effect on the inequality of personal income distribution (where it should be remembered that managers and supervisory workers can be viewed as being paid out of markup income in the Kaleckian framework). Large departures from distributions considered fair can make it more difficult for firms to increase the markup share by increasing the price if the markup share is already high, and more difficult for workers to increase the wage share by increasing the wage if the markup share is very low.}

3. The dynamics of the economy

To analyse the dynamics of the economy, we examine its behaviour in two different runs. In the short run we assume that $K$, $g$ and $\pi$ are given, while in the long run these state variables are allowed to change.

In the short run, firms adjust the level of output, and hence capacity utilisation, according to the demand for goods. For short-run equilibrium, therefore, we can determine the rate of capacity utilisation from the goods market clearing equation:

$$Y = C + I$$

where $C$ is the real level of consumption, which can be written as:

$$I/K = S/K$$

Substituting from equations (1) and (3) we can solve for the equilibrium level of capacity utilisation:

$$u = g/s\pi$$

The equilibrium level of capacity utilisation rises with $g$ through the usual multiplier process, falls with $s$ due to the paradox of thrift and falls with the profit share because of a shift in income from workers who do not save to profit recipients who do. Aggregate demand is thus wage led in the short run.

In the long run, abstracting from depreciation, for simplicity, we have:
\[ \widehat{K} = g \]

where the circumflex denotes time rates of growth. A change in \( K \) does not affect the short-run equilibrium value of \( u \). Also, \( g \) and \( \pi \) change according to the equations (6) and (7). Substituting from equation (10), which always holds during the long run, and using equation (4), we obtain:

\[ \dot{g} = \lambda \left[ G\left( \frac{g}{s\pi}, \pi \right) - g \right] \quad (11) \]

and

\[ \dot{\pi} = F\left( \frac{g}{s\pi}, g, \pi \right) \quad (12) \]

The long-run dynamics of the simple two-dimensional system (there is no need to involve \( K \) because it does not enter these equations) given by equations (11) and (12) can be analysed using a phase diagram in \(<g, \pi>\) space.\(^\text{13}\) A more formal discussion covering the case discussed in what immediately follows and those mentioned further below is provided in Appendix A.

In Figure 1 the \( \dot{g} = 0 \) curve shows combinations of \( g \) and \( \pi \) for which \( g \) is stationary and is obtained by setting the left-hand side of equation (11) equal to zero, so that we have:

\[ g = G\left( \frac{g}{s\pi}, \pi \right) \quad (13) \]

The slope of the \( \dot{g} = 0 \) curve is given by:

\[ \frac{dg}{d\pi} = -\frac{G_{\pi} - G_{u} \frac{\dot{g}}{s\pi^2}}{G_{u} \frac{\dot{g}}{s\pi} - 1} \]

The sign of the numerator of this expression is ambiguous. If the effect of an increase in \( \pi \) on \( g \) and hence on \( \dot{g} \) is positive, we can call it the case of profit-led growth in terms of goods market adjustment (GMA), while if it is negative we can call it the case of wage-led growth in terms of GMA (we use these longer expressions to distinguish

\(^{13}\) Our analysis may be compared with the related analysis of Bhaduri (2008), which also deals with the dynamics of wage- and profit-led systems, but does not examine the effects of labour market and goods market factors on the markup and profit share by assessing changes in power relations as we do. Bhaduri's analysis is conducted by examining the dynamics of the profit share and the rate of capacity utilisation, which move in the same ‘run’. In our analysis the rate of capacity utilisation adjusts in the short run with given levels of the profit share and the growth rate of capital, while the latter two variables change over the long run, which makes it more natural to analyse long-run dynamics in the \( \pi-g \) space.
between different senses in which the expressions ‘wage-led’ and ‘profit-led’ growth can be used, discussed below). We will first consider the wage-led growth in terms of the GMA case, where \( G_\pi - G_u \frac{g}{s\pi^2} < 0 \), and return to the profit-led growth case later. If the effect of an increase in \( g \) on \( \dot{\pi} \) is negative, the denominator is negative. This condition is satisfied if \( sr > G_u \), which is the standard macroeconomic stability condition stating that a rise in output and capacity utilisation affects saving more than it does investment. We will call it the stable GMA case and assume that it holds now; we will return to the unstable GMA case, when the condition is not satisfied, briefly later.

Confining our attention to the positive quadrant, our assumptions of the wage-led GMA and stable GMA cases imply that the \( \dot{\pi} = 0 \) line has a negative slope, as shown in Figure 1, and the vertical arrows point downwards above the curve and upwards below it.

The \( \dot{\pi} = 0 \) curve shows combinations of \( g \) and \( \pi \) for which \( \pi \) is stationary. It is obtained by setting the left-hand side of equation (12) to zero. If \( F_g < 0 \), i.e. a higher rate of accumulation involves a decline in firm concentration rates and a faster fall in the markup, a rise in \( g \) necessarily reduces \( \dot{\pi} \), since in addition to this direct effect it increases \( u \) by increasing aggregate demand, tightens the labour market, increases the wage share and reduces the profit share. Thus, as shown in Figure 1, any point above (below) the curve implies that \( \pi \) is falling (rising). If \( F_g \) is positive, because faster accumulation leads to an increase in firm concentration, and this effect is strong, an increase in \( g \) may increase \( \dot{\pi} \), but we will not analyse this case. The effect of a rise in \( \pi \) on \( \dot{\pi} \) is more complex, since there are two separate effects that have to be taken into account.
account. First, there is the direct effect of the change in \( \pi \) on \( \dot{\pi} \), which is a negative effect because of both labour market and industrial concentration effects. Second, there is the indirect effect that occurs due to changes in \( u \): an increase in \( \pi \) reduces \( u \), which increases \( \dot{\pi} \) by tightening the labour market. This can be seen by differentiating equation (12) with respect to \( \pi \), which gives:

\[
\frac{d\dot{\pi}}{d\pi} = F_u - F_u \frac{g}{s\pi^2}
\]

where \( F_i \) is the partial derivative of the function \( F \) with respect to \( i \). Since \( F_{\pi} < 0 \) and \( F_u < 0 \), the sign of this derivative is ambiguous. If we assume that the magnitude of the second term does not change much in the relevant range examined by the model, while \( F_s \) becomes larger in absolute value when the profit share is very low (due to the industrial concentration effect) or very high (due to the worker–firm bargaining effect), it follows that the derivative is likely to be negative at particularly low and high levels of \( \pi \) (because of the dominance of the first term) but positive at intermediate levels of \( \pi \) (because of the dominance of the second term). If these assumptions hold, then the slope of the \( \dot{\pi} = 0 \) locus, given by:

\[
\frac{dg}{d\pi} = -\frac{s\pi F_{\pi} - F_u \frac{g}{\pi}}{F_u + s\pi F_{\pi}}
\]

is negative at low and high values of \( \pi \), but positive at intermediate values of \( \pi \), because of the strength of the labour market effect of capacity utilisation. Thus there will exist \( \pi_c \) and \( \pi_d \) such that for \( \pi_c < \pi < \pi_d \), the \( \dot{\pi} = 0 \) curve will be positively sloped: starting from a position on the curve, a fall in \( \pi \) will decrease \( \dot{\pi} \), which will require a fall in \( g \) to increase \( \dot{\pi} \) and bring it back to zero. Conversely, for \( \pi < \pi_c \) and \( \pi > \pi_d \), the \( \dot{\pi} = 0 \) curve will be negatively sloped: starting from a position on the curve, a fall in \( \pi \) will increase \( \dot{\pi} \); to bring it back to zero, \( g \) must increase.

It should be noted that there may be an additional reason why, for \( \pi < \pi_c \), the \( \dot{\pi} = 0 \) curve will be negatively sloped because in that region the direct effect of the increase in \( \pi \) is strong: as we go up the curve, since \( g \) rises and \( \pi \) falls, the level of \( u \) rises, so that the labour market becomes tighter.\(^{14}\) When \( g \) is lower and \( \pi \) is higher, since labour markets are less tight, reductions in the markup due to increases in the money wage are unlikely to make firms pass on wage pressures to higher prices, so that there will be little pressure to increase the change in the markup.\(^ {15}\) However, when the wage rises when labour markets are tighter, these wage increases are likely to simultaneously affect most firms and they are likely to increase their prices in an effort to protect their shares of output.

Various possible configurations of the two curves are possible. They may not intersect at all (with the \( \dot{\pi} = 0 \) curve always lying above the \( g = 0 \) curve), only once (e.g. with the \( \dot{\pi} = 0 \) curve not sloping upwards enough to intersect the \( g = 0 \) curve again

\(^{14}\) As equation (10) shows, the level of capacity utilisation is constant along a positively sloped ray through the origin and is higher as this line rotates upwards.

\(^{15}\) In fact, if firms are profit maximisers and face a kinked demand curve, they may not change the price at all when the wage rises (see Henley, 1987).
at a low level of $\pi$), twice (e.g. with the $\dot{\pi} = 0$ not sloping down sufficiently beyond $\pi_d$ to intersect the $\dot{g} = 0$ curve again) or three times, as shown in Figure 1. The case of three intersections is particularly interesting and is worth discussing in some detail. It is obvious from the direction of the arrows in Figure 1 that the equilibrium at $E_1$, at a lower rate of growth and a higher profit share, is saddlepoint unstable. If the economy happens to start from a point on the separatrix given by $SS'$, it will converge to the equilibrium at $E_1$. But if it starts from a position to the right (left) of this separatrix, the economy may eventually end up on a growth path in which it will experience a fall (rise) in the rate of growth and a rise (fall) in the profit share until reaching equilibrium $E_3$ ($E_2$). On such a path the dynamics can be understood from the fact that the economy is wage led in terms of GMA. When the profit share increases, aggregate demand falls, capacity utilisation falls, the desired rate of accumulation falls, making the profit share increase further, so that the growth rate falls; the converse is true when the profit share falls, which increases growth and results in further falls in the profit share as the labour market tightens. This type of instability has been observed in standard KPK models of wage-led growth, which emphasise the effects of changes in labour market conditions on distribution. The equilibria at $E_2$ and $E_3$, at a higher rate of growth and a lower profit share respectively, however, are both stable equilibria to which the economy will converge without cyclical fluctuations.

We may make several comments to clarify the nature of the dynamics and consider some simple modifications. First, it is not necessary for the $\dot{\pi} = 0$ curve to have a positively sloped segment to obtain the qualitative properties just noted. Even if the curve is always negatively sloped, but is S-shaped in the sense of having a middle part in which the absolute value of the slope is smaller than when $\pi$ is very low or very high, it is possible to obtain three equilibria with the qualitative properties noted above, i.e. with two stable equilibria at high and low rates of growth and an intermediate one that is a saddlepoint.

Second, various possible trends of the growth rate and the profit share may be observed on the long-run dynamic path of the economy. For example, if the economy is moving towards the long-run equilibrium at $E_f$ from the regions between the two curves north-west or south-east of that equilibrium, an inverse relation between movements in the growth rate and the profit share will be observed, and if the economy is moving towards it from the south-west, it will experience an increase in the growth rate and a rise in the profit share. Since cycles are possible, these relations between the rate of growth and the profit share may also change over time. Thus, although we have assumed wage-led growth in terms of GMA, we cannot conclude that the wage share and the growth rate will always move in the same direction in this model. When the profit share, $\pi$, and the growth rate, $g$, move in opposite directions along a dynamic path, we can refer to it as a wage-led dynamic path, while if they move in the same direction, we can call it a profit-led dynamic path. Thus, a wage-led economy in terms of GMA can be on a profit-led dynamic path.

Third, throughout our discussion we have assumed that the economy operates with excess capacity and with unemployed labour, so that $u$ is in fact free to vary in response to changes in aggregate demand. Neglecting complications that may arise due to the disappearance of unemployed labour, we may comment briefly on what happens if the economy hits a ‘full’ capacity ceiling given by:

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16 Labour supply growth may be endogenous. Moreover, endogenous labour productivity growth of the form analysed in Dutt (2006) is also likely to prevent the depletion of unemployed labour.
Since the short-run equilibrium level of $u$ is determined by equation (10), combinations of $g$ and $\pi$ at which the economy is at full capacity are given by the equation:

$$g = (s/a_1)\pi$$

which is shown by the positively sloped straight line marked $(s/a_1)\pi$ in Figure 1. By assuming that we are always operating below the level of full-capacity utilisation, we are implying that the economy is always to the right of this line, which is where we have drawn the two long-run equilibria in the figure. What happens, however, if when experiencing, for example, increasing accumulation with a fall in the profit share, the economy hits a full-capacity ceiling? Various outcomes are possible depending on what we assume about accumulation and profit share adjustments when there is such a ‘regime’ change. One possibility is that we continue to assume that accumulation plans are always fulfilled, so that desired accumulation is given by $g^d = G(1/a_1, \pi)$, and actual accumulation evolves according to equation (6), but that we jettison equation (7) and replace it with the assumption that $\pi$ adjusts instantaneously to clear the goods market. In other words, the economy will move along the full-capacity locus, experiencing a rise in $g$ and a rise in $\pi$. This occurs because, owing to excess demand, for example, firms that are unable to increase their capacity utilisation, increase their price, thereby increasing the markup. Thus, the long-run equilibrium will occur at the intersection of the $\dot{g} = 0$ locus and the full-capacity locus. Along the full-capacity line given by the equation $g = (s/a_1)\pi$, the economy will necessarily be on a profit-led dynamic path.

Fourth, if the $\dot{\pi} = 0$ and $\dot{g} = 0$ curves intersect only once, or do not intersect at all, the dynamics will be different. If they intersect only once, then there may be no stable equilibrium $E_2$ and the equilibrium will be saddlepoint unstable. If the economy is on a wage-led growth path, it will eventually hit full-capacity utilisation and the outcome can be analysed as discussed in the previous paragraph. If the $\dot{\pi} = 0$ locus lies everywhere above the $\dot{g} = 0$ locus, there will be no long-run equilibrium. In this case the economy will (eventually) find itself on a dynamic path of declining accumulation and worsening income distribution.

Fifth, if the economy exhibits the case of a profit-led growth in terms of GMA, an increase in $\pi$ operating through the indirect capacity utilisation effect and direct profit share effect will increase desired investment. The $\dot{g} = 0$ curve will be upward rising since, starting from an initial equality between actual and desired investment, an increase in the level of desired investment resulting from the increase in the profit share will require an increase in $g$ to restore the equality between actual and desired investment. Several configurations of the two curves are possible, of which two are shown in Figure 2. In the case shown in Figure 2A, the curves intersect thrice, yielding three

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17 This occurs when $E_2$ lies to the left of the $(s/a_1)\pi$ locus.
18 Alternative outcomes are possible if it is assumed that when desired investment exceeds actual investment firms are unable to increase their rate of investment, so that $g$ becomes stationary at full capacity and the dynamics of $\pi$ continue to be given by equation (7), or if it is assumed that there is partial and immediate adjustment of both $g$ and $\pi$. The implications of these possibilities are not explored here because they are not central to the concerns of this paper.
long-run equilibria, in which $E_1$ is saddlepoint unstable and $E_2$ and $E_3$ are stable. In the case shown in Figure 2B there is only one equilibrium, at $E$, around which the economy will experience cycles that may be locally stable or unstable, depending on the sign of the trace of the dynamic system’s Jacobian matrix:

$$\text{Tr} = \lambda \left( \frac{G_u}{s\pi} - 1 \right) + F_{\pi} - F_u \frac{\dot{g}}{s\pi^2}$$

If the trace is negative, the system is locally stable; if the trace is positive, the system is unstable. Limit cycles become a possibility in either case.

Finally, if we have an unstable GMA process with $s\pi < G_u$, so that the macroeconomic stability condition – which requires that saving is more responsive to output than investment – is violated, an increase in $g$ increases $\dot{g}$. In the case of wage-led goods market-adjustment growth, the $\dot{g} = 0$ curve is positively sloped because an increase in $g$ raises $\dot{g}$, which requires an increase in $\pi$ to reduce $\dot{g}$. In the profit-led case the curve is negatively sloped, because a fall in $\pi$ is required to reduce $\dot{g}$ after it is increased by the increase in $g$. In both cases, because of the unstable dynamics for $g$, the vertical arrows point upwards above the curve and downwards below it, taking the economy further way from it. In the wage-led case, if there is only one intersection for the positively sloped $\dot{g} = 0$ curve and a negatively sloped $\dot{\pi} = 0$ curve, the equilibrium at it will be saddlepoint unstable. If there are three equilibria—for which we require the $\dot{\pi} = 0$ curve to have a positively sloped segment—we will have a high equilibrium with high $g$ and high $\pi$, a low equilibrium with low $g$ and low $\pi$, both of which are saddlepoint unstable, and an intermediate equilibrium that is unstable. The dynamics are more interesting for the profit-led case, with a negatively sloped $\dot{g} = 0$ line, as shown in Figure 3. In the case in which the $\dot{\pi} = 0$ curve is entirely downward sloping, if it is flatter than the $\dot{g} = 0$ curve (as shown in Figure 3A), the economy will experience cycles around the long equilibrium at $E$, which may be locally stable or unstable, depending on the sign of the trace given by equation (14). If it is negative, the equilibrium will be locally stable; if it is positive, the equilibrium will be locally unstable. In either case
we may have stable limit cycles. In the case in which the \( \dot{\pi} = 0 \) curve has an upward-rising segment we may have three equilibria, as shown in Figure 3B. It can be seen that the two long-run equilibria and \( E_1 \) and \( E_3 \) are saddlepoint unstable, whereas since

\[
F_g - F_u \frac{\dot{g}}{s\pi^2} > 0,
\]

the equilibrium at \( E_2 \) is necessarily unstable but the economy exhibits cycles around it. If the economy starts within the area between the two separatix that lead to the two equilibria at \( E_1 \) and \( E_3 \) (shown by the dashed lines), it will never leave it. There will thus exist at least one stable limit cycle around \( E_2 \). In these cases an unstable GMA system can be stabilised by the endogeneity of \( \pi \).

Fig. 3. Dynamics with unstable goods market adjustment. (A) Negatively sloped \( \dot{\pi} = 0 \) curve. (B) \( \dot{\pi} = 0 \) curve with positively-sloped segment.

4. Effects of autonomous changes in aggregate demand and distribution

In this section we discuss the effects of parametric changes involving autonomous changes in aggregate demand, which shift the \( \dot{g} = 0 \) curve, and autonomous changes in distribution, which shift the \( \dot{\pi} = 0 \) curve. We will mostly consider the basic case of a stable GMA that is wage led in terms of GMA and comment briefly on some other cases.

First, consider the effects of an exogenous increase in aggregate demand as represented by an increase in the parameter \( \gamma \) introduced in equation (4) and subsequently suppressed. An increase in the parameter can be interpreted as an autonomous increase in animal spirits or as fiscal expansion (although we do not explicitly consider fiscal policy and government deficits and debt) or monetary expansion (which increases investment demand). In the short run, with \( g \) and \( \pi \) given, there is no effect on the level of capacity utilisation. However, the desired level of accumulation increases, \( \dot{g} \) increases and, as can be checked from equation (11), the \( \dot{g} = 0 \) curve shifts up. If we are in a situation with three equilibria, as is shown in Figure 1, the upward shift in this curve will imply that if we start from the initial long-run equilibrium at \( E_3 \), if the new stable long-run equilibrium occurs at a position with excess capacity then the equilibrium growth rate
will be higher and the equilibrium profit share lower. Alternatively, if the economy hits the full-capacity barrier, it will experience a higher rate of accumulation and the equilibrium profit share may go up or down. If, instead, we start from the low equilibrium E_3, it can be seen that a small upward shift in the \( \dot{g} = 0 \) curve will increase \( g \) and reduce \( \pi \) in the long run, provided that the shift is less than what is shown by the dotted line in Figure 4. In both cases we find that the increase in equilibrium \( g \) occurs at the same time that equilibrium \( \pi \) falls. Since \( g \) and \( \pi \) move in opposite directions, we can refer to this as wage-led growth, but in the sense of dynamic equilibria. Thus, wage-led growth in terms of GMA is associated with wage-led growth in terms of dynamic equilibria.

If there is initially no long-run equilibrium because the curves do not intersect and the economy is on a path with declining growth and rising profit share, the increase in autonomous demand can make the curves intersect and stabilise the economy. If the economy is at a position to the right of the separatrix, the increase in autonomous demand will shift the separatrix to the right and possibly move the economy from a path of declining growth to one of increasing growth.

An interesting implication of the model is that if we start from a high-growth equilibrium such as E_2, a fall in the autonomous aggregate demand (e.g. due to an exogenous fall in animal spirits of a large magnitude) can move the economy to a low-growth equilibrium such as E_3, with a dramatic fall in the growth rate and rise in the profit share, or even go off in an unstable downward spiral (if there is no equilibrium such as E_3). Then, an increase in the aggregate demand through expansionary policy that shifts the curve up by the same amount may leave the economy at a low-level equilibrium with a high profit share, such as E_3.

Fig. 4. Effect of an autonomous increase in aggregate demand.
This property of the model can be shown using Figure 5, which plots long-run equilibrium values of \( \pi \) for different values of \( \gamma \) (with equilibrium values of \( g \) not shown). Since a rise in \( \gamma \) shifts the \( \dot{g} = 0 \) curve up, this equilibrium locus takes a shape similar to that of the \( \dot{\pi} = 0 \) curve, and the values \( \gamma_c \) and \( \gamma_d \) of \( \gamma \) which produce equilibrium values of \( \pi \) are given by \( \pi_c \) and \( \pi_d \) in Figure 1. We have seen above, from our discussion of Figure 1, that the dynamics on branches \( SP \) and \( QR \) are stable, while those on the branch \( PQ \) are unstable. It is apparent from this that there is hysteresis in \( g \) and \( \pi \) as \( \gamma \) varies. If we start from a value of \( \gamma \) higher than \( \gamma_c \) and reduce \( \gamma \) to positions that are greater than \( \gamma_d \), the economy will stay on the segment \( SP \), with low levels of \( \pi \) and high levels of \( g \). If \( \gamma \) falls below \( \gamma_d \), however, the economy will jump to the segment \( RQ \) and have a high level of \( \pi \) and low level of \( g \). If, now, \( \gamma \) increases to \( \gamma_c \), it will remain on the segment \( RQ \). Moreover, for small increases (decreases) in \( \gamma \) from below (above) \( \gamma_c \) (\( \gamma_d \)), there will be large changes in \( \pi \) and \( g \).

We now comment briefly on some of the other cases discussed in section 3. If the economy is profit led in terms of GMA and the \( \dot{g} = 0 \) curve is positively sloped, an exogenous increase in aggregate demand will still shift the curve upwards. In the case of three long-run equilibria (as shown in Figure 2A), if we are initially at stable long-run equilibria such as \( E_2 \) or \( E_3 \), with a ‘small’ increase in \( \gamma \), the equilibrium value of \( g \) will rise and that of \( \pi \) will fall when the \( \dot{g} = 0 \) curve shifts up. Profit-led growth in terms of GMA is therefore associated with wage-led growth in terms of dynamic equilibria. However, a sufficiently large increase in \( \gamma \) can result in the disappearance of the high growth equilibrium at \( E_1 \) and the economy will be pushed to the long-run equilibrium at lower growth, in the segment around \( E_2 \), and the effect may be to reduce \( g \) and \( \pi \). The expansion in capital accumulation and employment growth will, in this case, increase the rate of increase in wages, reduce the profit share and, because of the lower profit share, reduce desired investment and growth. A reduction in \( \gamma \) will, in this case, increase both \( g \) and \( \pi \) and...

\[ \text{Fig. 5. Multiple equilibria and hysteresis in the dynamic system.} \]

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\[ \text{19 Our analysis may be compared with the related analysis of Nicolis and Prigogine, who state that ‘[a] system with multiple steady states is, therefore, endowed with an intrinsic excitability’ (1997, p. 173).} \]
we will have profit-led growth in terms of dynamic equilibria. In the case of a single equilibrium at E (or a limit cycle around it), as in Figure 2B, the effect of an increase in $\gamma$ is to move the new long-run equilibrium with lower $g$ and lower $\pi$ (or move the centre of the limit cycle to such a position), although an increase in $g$ cannot be ruled out if the $\dot{\pi} = 0$ curve is always negatively sloped. In the case of unstable GMA with $s_{eq} < G_{eq}$, we will have the situations depicted in Figure 3. In this case, expansionary policy or an increase in autonomous investment shifts the $\dot{g} = 0$ curve downwards, because at points initially on it, $g$ must be rising with the increase in desired investment and a fall in $g$ is required to reduce aggregate demand and restore goods market equilibrium. In this case, as seen above, stability or limit cycles can be obtained in the profit-led GMA case. In the case shown in Figure 3A, the long-run equilibrium value of $g$ will rise and that of $\pi$ will fall as the economy moves up the $\dot{\pi} = 0$ curve. This implies that if the equilibrium is not globally unstable, either the economy will actually move to a new long-run equilibrium with a higher growth rate and a lower profit share or it will experience limit cycles around a new equilibrium with a higher growth rate and lower profit share. Thus the economy will be wage led in terms of dynamic equilibria. In the case shown in Figure 3B, if the economy does not head off on paths exhibiting unstable dynamics, it will experience limit cycles around a new equilibrium with a lower profit share and a lower growth rate, despite aggregate demand expansion; although with a $\dot{\pi} = 0$ curve that is monotonically negatively sloped, the effect on the growth rate will be positive.

Second, we consider the effect of an exogenous change in the determinants of $\dot{\pi}$, i.e. changes other than those due to changes in $u$, $g$ and $\pi$. Such a change could be due to changes in government policies that, for example, weaken or strengthen the power of unions and weaken or strengthen anti-trust legislation and implementation, and make goods or labour markets more ‘flexible’. We can distinguish between two aspects of such changes, although in reality a policy (or other) change may involve both effects simultaneously. One aspect is that they may change the relative power of workers and firms, so that, for example, if the relative power of firms is increased (by weakening unions or by weakening antitrust policy) at a given situation, the rate of change in $\pi$ will increase. In this case of an increase in the relative power of firms, the effect will make $\dot{\pi} > 0$ for all situations in which initially we have $\dot{\pi} = 0$. The other aspect is greater flexibility in labour markets, so that distribution becomes more responsive to labour market conditions, i.e. when there is a decrease (increase) in employment and capacity utilisation, the profit share will rise (fall) more quickly. We will interpret this to mean that $F_u$ falls or becomes more negative.

Concentrating on the case of wage-led GMA and stable GMA, an increase in the relative power of firms will shift the $\dot{\pi} = 0$ curve upwards, as shown in Figure 6. If we are initially at equilibria such as $E_2$ or $E_3$, a ‘small’ upward movement of the line will reduce the equilibrium level of $g$ and raise that of $\pi$. The increase in the profit share, due to wage-led GMA, reduces $g$ and increases $\pi$, so that we have wage-led growth in terms of dynamic equilibria. A sufficiently large upward shift of the curve can move the economy quite dramatically from high $g$ and low $\pi$ at a high-growth equilibrium, such as the one at $E_2$, to a low-growth equilibrium, such as the one at $E_3$. Comparing

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20 Another aspect of labour market flexibility is the greater ease of hiring and firing. Our model does not have any long-term labour contracts and cannot address this aspect of flexibility. See, however, Dutt et al. (2011) for an analysis of a KPK model with long-term labour contracts that examines this issue.
the two situations just described, the following can be noted. In the first case (when the shift in the $\dot{\pi} = 0$ schedule does not remove the high-growth intersection, such as the one at $E_2$, shown by a move to a position lower than the one indicated by the dotted line in Figure 6) a small increase in autonomous demand (through fiscal expansion) can increase growth and increase the wage share. In the second case, such a policy-induced improvement in growth and distribution requires a much larger expansion in aggregate demand, and is consequently more difficult to achieve, given uncertainties in the effects of demand-induced expansions.

Deregulation in the labour market, which changes the responsiveness of distribution to alterations in aggregate demand and employment, can also affect the relative bargaining power of workers or firms at any particular position. To show the effect of a pure change in flexibility, we consider an increase in the slope of the $\dot{\pi} = 0$ line without a shift in the relative bargaining power of workers or firms at an initial equilibrium ($E_1$) in Figure 7, where the $\dot{\pi} = 0$ curve is shown by the solid negatively sloped line. Now, if there is an exogenous shift that deregulates the labour market in the way just discussed, the slope of the $\dot{\pi} = 0$ curve will become larger, such that the curve will swivel to a position like the one shown by the dashed line, where the curve now has a positive slope in its middle region. The equilibrium at $E_1$ can become unstable, as is shown in Figure 7, and three equilibria may appear. Which equilibrium the economy will be attracted to will depend on the initial direction in which the economy moves. If there is an initial reduction in $g$ and $u$, which is often the situation in which political support for labour market liberalisation increases, the economy is likely to move to $E_3$, with a lower equilibrium rate of growth and a higher profit share.$^{21}$

$^{21}$ Deregulation of the labour market, instead of eventually entailing higher wage share and growth—as authors such as Blanchard and Giavazzi (2003) assert—is on the contrary likely to lead to a lower wage share and low growth rate.
In the profit-led GMA case, an increase in the bargaining position of firms, which shifts up the $\pi = 0$ curve, will increase the long-run equilibrium values of $g$ if we are in the multiple-equilibria or single-equilibrium stable cases shown in Figure 2, but by increasing the long-run equilibrium $\pi$. We therefore have the case of profit-led dynamic equilibria. But in the multiple-equilibrium case, if the low $\pi$ equilibrium disappears, the increase in $g$ will occur only if there is a large increase in $\pi$. If at very high levels of profit share the economy comes to have wage-led GMA (so that the $\dot{g} = 0$ curve becomes negatively sloped), it cannot be guaranteed that the eventual long-run equilibrium growth rate will be higher than at the initial equilibrium. Labour market deregulation can twist the $\pi = 0$ curve from a position shown by E in Figure 2B to one shown in Figure 2A as well as increasing the chances of instability and possibly lower growth and a higher wage share (such as at the equilibrium shown by E2).

5. Conclusion

This paper has developed a simple KPK model of growth and distribution, for which it has examined the short- and long-run dynamics of the economy by considering slow movements in investment due to the existence of lags and slow movements in the profit share due to both goods market and labour market effects. The main contribution of the paper has been to allow for the existence of different reasons for distributional changes, based on changes in the relative bargaining power of workers and firms and changes in industrial concentration, and for the possibility that the relative strength of these effects will change depending on the growth rate and the state of income distribution. Our analysis, emphasising the feedback effects involving the interaction between the labour and goods markets, has yielded mathematical non-linearities in the dynamic system involving growth and distribution. Our examination of this system has allowed us to analyse the possibility of multiple equilibria and instability. It has also provided us with a method for analysing the implications of various exogenous changes, including those in aggregate demand due to autonomous shifts in business confidence and animal spirits
and fiscal and monetary policies, and in distribution due to autonomous shifts in bargaining power between workers and firms and due to deregulation and increases in the flexibility of labour markets. The main conclusions of our analysis are as follows.

(i) If the desired investment rate of firms is increased by a rise in the wage share (or what we have called wage-led GMA)—due to the positive effect of a redistribution of income to workers, who have a higher propensity to consume than profit recipients, which outweighs any direct depressive effect the profit share has on desired accumulation—an expansion in aggregate demand will lead to improvements in the rate of growth of the economy and an increase in the wage share, and a weakening in the bargaining position of workers and/or greater labour market flexibility can be expected to reduce the rate of growth of the economy and reduce the wage share.

(ii) These results will occur for small exogenous changes. For larger exogenous shifts, the adverse effects can be very large and restoring growth and distribution through policy changes may be more difficult to achieve, unlike the case for small changes.

(iii) If desired investment is increased by a rise in the profit share (what we have called profit-led GMA), it is still likely that an exogenous rise in aggregate demand will increase growth and the wage share. However, now an increase in the rate of growth will require a decrease in the bargaining position of workers and the resulting fall in the wage share may be significant. If large increases in the profit share make the economy have wage-led GMA, the growth rate may fall in the long run.

(iv) Unstable GMA due to the fact that investment may respond to changes in capacity utilisation more than does saving need not necessarily be destabilising for the economy if one examines the dynamics of the profit share, which can stabilise the overall dynamics of the economy.

(v) In the case of wage-led GMA, a rise in the wage share may lead to a rise in the rate of growth, which will increase the wage share, etc.; this growth path may be self-limiting but not necessarily self-defeating. The self-limiting property is found by the fact that the behaviour of firms and capitalists operating in goods and labour markets is likely to limit the increase in the wage share. However, the gains to workers will be permanent, in the sense that the wage share does not get reversed by this process. A reversal may occur, of course, if there is a political backlash that reduces the power of workers due to changes exogenous to the normal dynamics of the economy—which shifts the \( \pi = 0 \) curve upwards—but it is important to keep the distinction between self-limiting economic dynamics and self-defeating political and ideological changes, which may not be inevitable.

(vi) There are a number of different senses in which the terms ‘wage-led’ and ‘profit-led’ growth may be used. We distinguished between: wage-led GMA which relate to the effect of an exogenous change in the profit share on desired investment; GMA without taking into account endogenous changes in distribution at all; wage-led dynamics along a disequilibrium path of the economy; and the relation between growth and distribution when comparing dynamic equilibria due to exogenous changes in aggregate demand on the one hand and in distribution on the other. We found that we can observe wage-led growth in one sense but not necessarily in another. Hence, one should in general be wary of drawing
conclusions on the nature of GMA by observing actual changes in rates of growth and distribution in real economies, as has been attempted in some empirical studies examining whether demand and growth are wage- or profit-led. This is especially the case if the curve showing the relationship between the profit share and growth is non-monotonic.\footnote{In the course of writing this paper we have become aware of interesting empirical papers by Nikiforos and Foley (2012) and Tavani et al. (2011), which provide some evidence that the distributive schedule—which is related to the curve showing stationary levels of the profit share—is not monotonic. It is beyond the scope of the present paper to relate its implications to empirical trends.}

We end by noting that our analysis has been conducted with the use of a simple theoretical model of growth and distribution that has abstracted from many important features of real economies. Two such features involve financial and open-economy considerations, which have been entirely neglected from our analysis, both of which have received a fair amount of attention in KPK modelling. Financial issues—which are dealt with in the paper using a simple accommodationist endogenous or horizontalist money view—can be examined by allowing investment and the markup to depend on financial factors. Open-economy issues require the analysis of exports, imports and international capital flows. Financial and open-economy issues are, in fact, closely related to some of the specific issues examined in our model. For example, changes in the profit share are likely to be affected by financial factors, both because of the pricing decisions of firms, but also because changes in industrial concentration may depend on financial considerations. Further, open-economy factors may affect the profit share not only because competitiveness is related to the availability of imports, but also because foreign investment may affect changes in concentration rates. If our simpler analysis without such complications is found to be of interest, it would be fruitful to extend it to deal with them.

**Bibliography**


Appendix A: Long dynamics

The Jacobian matrix of the system given by equations (11) and (12) is:

\[
J = \begin{pmatrix}
\frac{\partial \dot{g}}{\partial g} & \frac{\partial \ddot{g}}{\partial g} \\
\frac{\partial \dot{\pi}}{\partial g} & \frac{\partial \ddot{\pi}}{\partial g}
\end{pmatrix} = \begin{pmatrix}
\lambda \left( \frac{G_u}{s} - 1 \right) & \lambda \left( -\frac{G_u}{s} \frac{g}{s^2} + G_\pi \right) \\
F_u \frac{1}{s^2} + F_\pi & F_\pi - F_u \frac{g}{s^2}
\end{pmatrix}
\]

The necessary and sufficient conditions for local stability are:

\[
\text{Det} = \left[ \lambda \left( \frac{G_u}{s\pi} - 1 \right) \right] \left[ F_\pi - F_u \frac{g}{s^2} \right] - \left[ \lambda \left( -\frac{G_u}{s\pi^2} + G_\pi \right) \right] \left[ F_u \frac{1}{s\pi} + F_\pi \right] > 0
\]

\[
\text{Trace} = \lambda \left( \frac{G_u}{s\pi} - 1 \right) + F_\pi - F_u \frac{g}{s^2} < 0
\]
Growth and income distribution: dynamics of power

The condition for stable GMA is \( \left( \frac{G_u}{s\pi} - 1 \right) < 0 \), the condition for wage-led GMA is

\[
\left( -G_u \frac{g}{s\pi^2} + G_u \right) < 0 \text{ and } \frac{F_u}{s\pi} - \frac{g}{s\pi^2} \text{ can become positive if the absolute value of } F_u \text{ is large; it is negative otherwise.}
\]

We assume that \( \left[ \frac{F_u}{s\pi} + F_u \right] < 0 \) even if \( F_u > 0 \).

Appendix B. Dynamics with the employment rate

We assumed earlier, in section 2, that we can measure the state of tightness of the labour market using the rate of capacity utilisation. Alternatively, we could introduce the rate of employment directly to measure this state, which would mean that our \( F \) function would be written as:

\[
\pi = F(e, \pi)
\]

where \( e \) is given by equation (10) in the main text and where \( F_e < 0 \), i.e. an increase in the employment rate causes the rate of change in the profit share to fall. For simplicity, we remove \( u \) and \( g \) as arguments in this function. The equation for \( g \) can be the same as before, that is, equation (11). However, now we have introduced an additional variable into our model, \( k \). If we assume that \( N \) is given at a point in time and grows over time at the rate \( n \), we can write the dynamic equation for \( k \) as:

\[
\dot{k} = (g - n)k
\]

One way to proceed with the analysis is to assume, as in many other growth models, that \( n \) is exogenously given, and using equations (11), and the two new dynamic equations just mentioned, substituting from (8) and (10) which gives the short-run equilibrium value of \( u \), to analyse the properties of the three-variable dynamic system involving \( g, k \) and \( \pi \). The Jacobian matrix of this system is given by:

\[
J = \begin{bmatrix}
\lambda \left( \frac{G_u}{s\pi} - 1 \right) & \lambda \left( G_u - \frac{G_u g}{s\pi^2} \right) & 0 \\
\frac{F_u k}{sA\pi} & \frac{F_u - F_u g k}{sA\pi^2} & \frac{F_u g}{sA\pi} \\
k & \frac{F_u g}{sA\pi} & \frac{F_u g}{sA\pi}
\end{bmatrix}
\]

We can analyse the local stability properties of long-run equilibria for this model, at which \( g = n \), using the Routh–Hurwitz conditions that are necessary and sufficient for local stability. The stability conditions for this model can be shown to be:

I. \( \text{Trace } (J) = \lambda \left( \frac{G_u}{s\pi} - 1 \right) + F_u - \frac{F_u g k}{sA\pi^2} < 0 \)

II. \( \text{Det } (J) + \text{Det } (J) + \text{Det } (J) = \lambda \left[ -F_u + \frac{F_u k}{sA\pi} \left( \frac{g}{\pi} - G_u \right) \right] > 0 \)
III. \( \text{Det}(\mathcal{J}) = \lambda \left( G_{e} - \frac{G_{e}g}{s \pi^2} \right) k \frac{F_{e}g}{sA\pi} < 0 \)

IV. \( \text{Det}(\mathcal{J}) - \text{Trace}(\mathcal{J}) \left[ \text{Det}(\mathcal{J}_1) + \text{Det}(\mathcal{J}_2) + \text{Det}(\mathcal{J}_3) \right] < 0 \)

where \( \text{Det}(\mathcal{J}) \) refers to the \( 2 \times 2 \) determinant obtained from \( \mathcal{J} \) by excluding its \( i \)th row and column. Condition III can be satisfied only under the assumption that growth is profit led, which we assume for this model. The other conditions are more likely to be satisfied if the absolute value of \( F_{e} \) is larger than the absolute value of \( F_{e} \) and if \( \lambda \) is small. If the absolute value of \( F_{e} \) is large at high and low values of \( \pi \) but low for intermediate values of \( \pi \), then long-run equilibria at intermediate levels of \( \pi \) are more likely to be unstable, as can be seen from conditions I and III (given, as is likely, that \( \frac{g}{\pi} > G_{e} \)), as in the model of the text of the paper.

This approach, with a given rate of growth of labour supply to which the rate of growth of capital adjusts in long-run equilibrium, is not very much in keeping with the heterodox demand-led growth approaches in the KPK tradition. This is partly because of the endogeneity of labour productivity growth, but also because of endogenous changes in labour supply due to reasons that have been discussed in the literature on ‘hysteresis’ in labour markets. If endogenous labour supply effects are considered and if \( N \) changes endogenously with \( K \), then we can take \( k \) to be approximately constant over time, which makes \( u \) change proportionately with \( e \), as shown by equation (8). If \( A \) is also endogenous, then the proportional relationship between \( e \) and \( k \) will not hold, but growth will still be determined by demand parameters in the long run. This endogeneity—which leads to increases in \( A \) if labour markets are tighter—will tend to reduce the tendency towards instability, by making effective labour supply respond to labour demand and hence reduce the tendency of demand increases to squeeze profits, but need not reverse it. Thus our instability result can still hold.