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Interest Rate Rules Ensuring Strong Local Equilibrium Determinacy

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Interest rate rules ensuring strong local equilibrium determinacy*

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Abstract

In this paper we look for interest rate rules ensuring strong local equilibrium determinacy, *i.e.* making sure that there is a unique equilibrium starting out in the neighbourhood of the steady state and that this equilibrium remains constantly in that neighbourhood. We show in a general framework that such interest rate rules exist and in the more specific framework of the canonical New Keynesian model that they are necessarily forward-looking, that is to say that they make the nominal interest rate conditional on the private agents' expectations. We also characterize the set of such interest rate rules implementing the optimal equilibrium under discretion or under commitment (for a closed economy or a small open economy with a flexible exchange rate) or the fixed exchange rate equilibrium (for a small open economy) in this model.

Keywords: commitment, discretion, exchange rate regime, forward-looking, interest rate rules, New Keynesian model, strong local equilibrium determinacy.

JEL classification system: E52, E31, E32, E61.

Titre: Règles de taux d'intérêt assurant la détermination locale forte de l'équilibre

Résumé: Dans ce papier nous cherchons des règles de taux d'intérêt assurant la détermination locale forte de l'équilibre, c'est-à-dire telles qu'il existe un unique équilibre originaire du voisinage de l'état stationnaire et que cet équilibre reste constamment dans ce voisinage. Nous montrons dans un cadre général que de telles règles de taux d'intérêt existent et dans le cadre particulier du modèle nouveau-keynésien canonique qu'elles sont nécessairement *forward-looking*, c'est-à-dire qu'elles expriment le taux d'intérêt nominal en fonction des anticipations des agents privés. Nous caractérisons aussi l'ensemble de telles règles de taux d'intérêt implémentant l'équilibre optimal discrétionnaire ou avec *commitment* (pour une économie fermée ou une petite économie ouverte en change flexible) ou l'équilibre de change fixe (pour une petite économie ouverte) dans ce modèle.

Mots-clefs: *commitment*, discrétion, régime de change, *forward-looking*, règles de taux d'intérêt, modèle nouveau-keynésien, détermination locale forte de l'équilibre.

Classification JEL: E52, E31, E32, E61.

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Introduction

A great deal of attention has been paid these past few years to the design of interest rate rules ensuring equilibrium determinacy, that is to say precluding multiple equilibria. This issue is arguably of practical importance. For instance, Clarida, Galí and Gertler (2000) explain the relatively high American macroeconomic volatility during the pre-Volcker era by the fact that the Fed was then following an interest rate rule compatible with multiple local equilibria, which made way to endogenous fluctuations born from self-fulfilling expectations. Other authors point to the fact that Japan may have fallen into the liquidity trap because the interest rate rule followed by the Bank of Japan was compatible with multiple global equilibria.

Two research directions have been explored in order to design interest rate rules which reduce as much as possible, and ideally completely remove, equilibrium indeterminacy.

The first research direction, followed notably by Bernanke and Woodford (1997), Clarida, Galí and Gertler (1999), Woodford (2003) and Giannoni and Woodford (2003a, 2003b), focuses on what we call weak local equilibrium indeterminacy, that is to say on the possible existence of several equilibria starting out in the neighbourhood of the steady state and constantly remaining in this neighbourhood afterwards. This narrow focus enables these authors to use the log-linear approximation of their model, which is valid only in the neighbourhood of the steady state. The weak local determinacy requirement amounts to a condition *à la* Blanchard and Kahn (1980) and typically imposes an inequality constraint on the coefficients of the interest rate rule considered, as exemplified by the well-known "active Taylor rule".

The second research direction, followed notably by Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b, 2002a, 2002b, 2003) and Christiano and Rostagno (2001), considers global equilibrium indeterminacy and shows the possible existence of equilibria (converging towards a deterministic cycle, a chaotic cycle or the liquidity trap) which do not constantly remain in the neighbourhood of the steady state. These authors use their original non-linear models to study the global equilibrium dynamics. No interest rate rule has been found yet which satisfies the global determinacy requirement. The only monetary policy known to reduce global indeterminacy, advocated by Benhabib, Schmitt-Grohé and Uribe (2002a, 2002b, 2003) and Christiano and Rostagno (2001) as well as by Clarida, Galí and Gertler (1999), consists in switching from an interest rate rule ensuring weak local determinacy to another rule such as a money growth rate peg, should the endogenous variables go outside a specified range around their steady state values. Such a two-tier monetary policy raises two problems however. First, as acknowledged by Benhabib, Schmitt-Grohé and Uribe (2002a, 2002b, 2003) and Christiano and Rostagno (2001), it does not completely remove global indeterminacy as equilibria converging towards a deterministic cycle or chaotic cycle may still exist. Second, as emphasized by Christiano and Rostagno (2001), the steady state or to bring them back into this neighbourhood) raises a moral hazard problem. Besides, this focus on strong local determinacy enables us to use the log-linearized reduced form which is valid only in the neighbourhood of the steady state of the model considered.

We first consider a general framework to show that interest rate rules ensuring strong local equilibrium determinacy do exist by providing examples of such interest rate rules. In order to illustrate our point in a simple and pedagogical way, we then focus on the more specific framework of the canonical New Keynesian model, which has received much attention in the past few years, and following the existing literature we consider different cases within this framework: closed economy under discretion and under commitment, small open economy with a flexible exchange rate under discretion and under commitment, small open economy with a fixed exchange rate.

We show that interest rate rules ensuring strong local equilibrium determinacy are necessarily forward-looking in this model, that is to say that they make the nominal interest rate conditional on the private agents' expectations. Loosely speaking, the intuition is the following. Strong local equilibrium indeterminacy may arise in the form of "boom and bust" equilibria from the moment that there exist several solutions (originating in the neighbourhood of the steady state) to the log-linearized model in the absence of any terminal condition. Since the structural equations of this log-linearized model make the current values of the variables depend on their expected future values, the only way to remove strong local indeterminacy is for the interest rate rule to be forward-looking so as to disconnect the current variables from the expected future variables. To our knowledge, we thus provide a new theoretical justification for the existence of forward-looking interest rate rules, as "forward-lookingness" is not a necessary condition for an interest rate rule to ensure weak local equilibrium determinacy.

The remaining of the paper is organized as follows. Section 1 shows the existence of forward-looking interest rate rules ensuring strong local equilibrium determinacy in a general framework. Section 2 focuses on the canonical closed economy New Keynesian model, characterizing the set of interest rate rules which implement the optimal equilibrium and ensure its strong local determinacy. Section 3 focuses on the canonical small open economy New Keynesian model, characterizing the set of interest rate rules which implement the optimal equilibrium (under a flexible exchange rate regime) or the unique equilibrium (under a fixed exchange rate regime) and ensure its strong local determinacy. We then conclude and provide a technical appendix.

1 Our point in a general framework

This section makes our point in a general framework. We first present the general reduced form considered, we then discuss our strong local equilibrium determinacy requirement and we finally show that this requirement can be fulfilled by some well-chosen interest rate rules.

1.1 General reduced form

The general reduced form considered is composed of three equations on three endogenous variables and two exogenous shocks. The endogenous variables are the inflation rate Δp, the output gap y and the short-term nominal interest rate r. The exogenous shocks are the cost-push shock ε^{pc} and the demand shock ε^{is}. The equations are a Phillips curve, an IS equation and an interest rate rule, which express the present endogenous variables as finite time-invariant linear combinations of past, present and expected future endogenous variables as well as past and present exogenous shocks. They are respectively written for $t \geq t_0$:

$$\begin{aligned}\Delta p_t &= \sum_{i=1}^{N_2^{pc}} a_i^{pc} E_t\left\{\Delta p_{t+i}\right\} + \sum_{i=1}^{N_2^{pc}} b_i^{pc} E_t\left\{y_{t+i}\right\} + \\ &\quad \sum_{i=-N_1^{pc}}^{-1} a_i^{pc} \Delta p_{t+i} + \sum_{i=-N_1^{pc}}^{0} b_i^{pc} y_{t+i} + \varepsilon_t^{pc}, \end{aligned} \tag{1}$$

$$\begin{aligned}y_t &= \sum_{i=1}^{N_2^{is}} a_i^{is} E_t\left\{\Delta p_{t+i}\right\} + \sum_{i=1}^{N_2^{is}} b_i^{is} E_t\left\{y_{t+i}\right\} + \sum_{i=1}^{N_2^{is}} c_i^{is} E_t\left\{r_{t+i}\right\} + \\ &\quad \sum_{i=-N_1^{is}}^{0} a_i^{is} \Delta p_{t+i} + \sum_{i=-N_1^{is}}^{-1} b_i^{is} y_{t+i} + \sum_{i=-N_1^{is}}^{0} c_i^{is} r_{t+i} + \varepsilon_t^{is}, \end{aligned} \tag{2}$$

$$\begin{aligned}r_t &= \sum_{i=1}^{N_2} a_i E_t\left\{\Delta p_{t+i}\right\} + \sum_{i=1}^{N_2} b_i E_t\left\{y_{t+i}\right\} + \sum_{i=1}^{N_2} c_i E_t\left\{r_{t+i}\right\} + \\ &\quad \sum_{i=-N_1}^{0} a_i \Delta p_{t+i} + \sum_{i=-N_1}^{0} b_i y_{t+i} + \sum_{i=-N_1}^{-1} c_i r_{t+i} + \\ &\quad \sum_{i=-N_1}^{0} d_i \varepsilon_{t+i}^{pc} + \sum_{i=-N_1}^{0} f_i \varepsilon_{t+i}^{is}, \end{aligned} \tag{3}$$

where $\left(N_1^{pc}, N_1^{is}, N_1\right) \in \mathbb{N}^3$, $\left(N_2^{pc}, N_2^{is}, N_2\right) \in \mathbb{N}^{*3}$, $a_i^{pc} \in \mathbb{R}$ for $i \in \{-N_1^{pc}, ..., N_2^{pc}\} \setminus \{0\}$, $b_i^{pc} \in \mathbb{R}$ for $i \in \{-N_1^{pc}, ..., N_2^{pc}\}$, $\left(a_i^{is}, c_i^{is}\right) \in \mathbb{R}^2$ for $i \in \left\{-N_1^{is}, ..., N_2^{is}\right\}$, $b_i^{is} \in \mathbb{R}$ for $i \in \left\{-N_1^{is}, ..., N_2^{is}\right\} \setminus \{0\}$, $(a_i, b_i) \in \mathbb{R}^2$ for $i \in \{-N_1, ..., N_2\}$, $c_i \in \mathbb{R}$ for $i \in \{-N_1, ..., N_2\} \setminus \{0\}$ and $(d_i, f_i) \in \mathbb{R}^2$ for $i \in \{-N_1, ..., 0\}$. We adopt the convention $\sum_{i=u}^{v} \{.\} = 0$ if $u > v$ and we assume that $\exists i \in \left\{0, ..., N_2^{is}\right\}$ such that $c_i^{is} \neq 0$.

The notation x_t represents the variable or the shock x considered at date t, while $E_t\{.\}$ stands for the expectation operator conditionally on the information available at date t, which includes the past and present variables and shocks, so that $E_t\{x_{t-k}\} = x_{t-k}$ for $k \geq 0$ and $x \in \left\{\Delta p, y, r, \varepsilon^{pc}, \varepsilon^{is}\right\}$. The variables Δp_{t_0-k}, y_{t_0-k} and r_{t_0-k} for $k \geq 1$ are assumed to be bounded. The shocks ε^{pc} and ε^{is} are assumed to follow centered stationary ARMA processes whose white noises have a bounded distribution. Finally, the Phillips curve is assumed to be "stationary-consistent" in the following sense: $\exists N^* \in \mathbb{N}$ and $\exists \left(a_i^*, b_i^*\right) \in \mathbb{R}^2$ for $i \in \{-N^*, ..., 0\}$ such that the condition

$$\forall t \geq t_0, \quad \left\{\begin{array}{l} X_t = \sum_{i=1}^{N_2^{pc}} a_i^{pc} X_{t+i} + \sum_{i=-N_1^{pc}}^{-1} a_i^{pc} X_{t+i} + \sum_{i=-N_1^{pc}}^{N_2^{pc}} b_i^{pc} Y_{t+i} \\ \sum_{i=-N^*}^{0} a_i^* X_{t+i} + \sum_{i=-N^*}^{0} b_i^* Y_{t+i} = 0 \end{array}\right.$$

implies that whatever their (finite) initial values, X_t and Y_t converge towards zero as $t \longrightarrow +\infty$. This final assumption ensures the existence of bounded sequences $(\Delta p_t)_{t_0}^{+\infty}$ and $(y_t)_{t_0}^{+\infty}$ satisfying the Phillips curve at all dates $t \geq t_0$. Note that the Phillips curve is "stationary-consistent" when $\exists i \in \{-N_1^{pc}, ..., N_2^{pc}\}$ such that $b_i^{pc} \neq 0$ and all the roots of the polynomial $Z^{N_1^{pc}+1} \sum_{i=-N_1^{pc}}^{N_2^{pc}} b_i^{pc} Z^i$ have their modulus strictly lower than one (as indeed $N^* = 0$, $a_0^* = 1$ and $b_0^* = 0$ then fit the bill), like for example in the popular canonical New Keynesian model considered in the next sections.

This reduced form is general enough to include as particular cases many reduced forms encountered in the existing literature. Concerning the Phillips curve and the IS equation, our general specification includes notably the popular canonical New Keynesian model considered in the next sections, whose non-zero coefficients are a_1^{pc}, b_0^{pc}, a_1^{is}, b_1^{is} and c_0^{is}. Criticized for their lack of empirical validity, the Phillips curve and the IS equation of this model have been extended in many ways in order to match the lagged and inertial responses of the variables observed in the data. For instance, Clarida, Galí and Gertler (1999) introduce a lagged inflation rate into the Phillips curve and a lagged output gap into the IS equation, thus adding a_{-1}^{pc} and b_{-1}^{is} to the set of non-zero parameters. Alternatively, the consideration of habit formation in consumption makes additional lags and expected leads of the output gap enter the Phillips curve and the IS equation, thus adding b_{-1}^{pc}, b_1^{pc} and b_2^{is} in Amato and Laubach (2004) and b_{-1}^{pc}, b_1^{pc}, b_{-1}^{is}, b_2^{is} in Woodford (2003, chap. 5) to the set of non-zero parameters. All these extensions of the canonical New Keynesian model are taken into account in our general framework.

Concerning the interest rate rule, our general specification includes most parametric families of interest rate rules considered in the existing literature, which are usually low-dimensional with the notable exception of the parametric family of interest rate rules considered by Giannoni and Woodford (2003a, 2003b) and Woodford (2003, chap. 8). Note that we choose to focus on interest rate rules expressing r as a finite linear combination of variables and shocks, as opposed to an infinite linear combination of variables and shocks.

In the models whose reduced form falls into our general specification, the Phillips curve and the IS equation are typically derived from the optimal behaviour of the private agents, which depends on their own expectations about the future economic situation, while the interest rate rule represents the reaction function of the central bank. As a result, all expectations featuring in these three equations should be interpreted as the private agents' expectations. We thus assume that the central bank observes these expectations (as well as the past and present exogenous shocks) when setting the nominal interest rate.

1.2 Strong local determinacy

The reduced forms encountered in the existing literature which fall into our general specification result usually from the log-linearization of a model around the steady state $(\Delta p, y) = (0, 0)$ and are valid only in the neighbourhood of this steady state. If the interest rate rule is arbitrarily chosen, the model in question may typically allow for multiple equilibria which may originate inside or outside the neighbourhood of the steady state. An equilibrium is said to be strong-locally determinate when two conditions are met: first, it is the only equilibrium starting out in the neighbourhood of the steady state, and second, it remains constantly in the neighbourhood of the steady state. Strong local determinacy is therefore less constraining than global determinacy, which arises when there is only one equilibrium wherever its starting point, and more constraining than weak local determinacy, which arises when there is only one equilibrium starting out in the neighbourhood of the steady state and remaining constantly in this neighbourhood afterwards.

Like in the existing literature, for simplicity we interpret "trajectory or equilibrium in the neighbourhood of the steady state" as "trajectory or equilibrium with bounded variables Δp and y". This interpretation is not restrictive since the bound in question can be chosen arbitrarily small. Because Δp_{t_0-k} and y_{t_0-k} for $k \geq 1$ are assumed to be bounded, we focus on trajectories originating from the neighbourhood of the steady state in our framework. Weak local equilibrium indeterminacy will therefore arise from the moment that there exist several trajectories $\{\Delta p_t, y_t, r_t\}_{t_0}^{+\infty}$ satisfying (1), (2) and (3) at all dates $t \geq t_0$ and such that the corresponding sequences $(\Delta p_t)_{t_0}^{+\infty}$ and $(y_t)_{t_0}^{+\infty}$ are bounded.

We argue that strong local equilibrium determinacy will be ensured if and only if two conditions are met: first, there exists a unique trajectory $\{\Delta p_t, y_t, r_t\}_{t_0}^{+\infty}$ satisfying (1), (2) and (3) at all dates $t \geq t_0$ in the absence of any terminal condition, and second, the corresponding sequences $(\Delta p_t)_{t_0}^{+\infty}$ and $(y_t)_{t_0}^{+\infty}$ are bounded, under the assumption that the central bank is concerned with social welfare and cannot credibly commit itself to sticking to its interest rate rule whatever the welfare costs entailed (such is the case under discretion of course, but also under commitment if the commitment technology comes from reputation effects so that the central bank still weighs the *pro* and *contra* before deciding whether to stick to its interest rate rule).

Indeed, consider a trajectory $\{\Delta p_t, y_t, r_t\}_{t_0}^{+\infty}$ satisfying (1), (2) and (3) at all dates $t \geq t_0$. If this trajectory remains constantly in the neighbourhood of the steady state (*i.e.* if $(\Delta p_t)_{t_0}^{+\infty}$ and $(y_t)_{t_0}^{+\infty}$ are bounded), then it qualifies as an equilibrium since it is a local solution of the locally log-linearized model. If this trajectory leaves the neighbourhood of the steady state (*i.e.* if $(\Delta p_t)_{t_0}^{+\infty}$ or $(y_t)_{t_0}^{+\infty}$ is not bounded), then equilibria can be found whose initial development coincides with the initial development of this trajectory. Indeed, this initial development will trigger a reaction from the social-welfare-concerned central bank which will eventually abandon its interest rate rule in order to keep the variables within the neighbourhood of the steady state, so that the resulting path will end up being bounded and hence not violating the transversality condition typically imposed by the model. This "stabilization of last resort" raises a moral hazard problem, as private agents, rightly expecting the reaction of the central bank, can settle on an initially diverging path even though this path would not be an equilibrium if the central bank were compelled to stick to its interest rate rule.

Such "boom and bust" equilibria exist only under the assumption that the credible threat of the central bank to act as a "stabilizer of last resort" in a finite time horizon is not dissuasive, that is to say under the assumption that this threat is not enough to nip any initially diverging equilibrium in the bud. The existing literature sometimes adopts the opposite assumption, like Clarida, Galí and Gertler (1999, p. 1701), but does not specify how this threat could work in a dissuasive way. As will become clear in the next subsection, one way to make this threat dissuasive could be to specify the "stabilization of last resort" as a switch to a well-defined interest rate rule of the kind which ensures strong local equilibrium determinacy. But the central bank could then just as well follow such an interest rate rule from the start.

1.3 Interest rate rules

Appendix A shows that interest rate rules ensuring strong local equilibrium determinacy do exist by providing examples of such interest rate rules. For instance, **appendix A** shows that if $c_i^{is} = 0$ for $i \in \{1, ..., N_2^{is}\}$ and if $\exists i \in \{1, ..., N_2^{pc}\}$ such that $(a_i^{pc}, b_i^{pc}) \neq (0,0)$, so that we can define $H \equiv \max\left[i \in \{1, ..., N_2^{pc}\} \text{ such that } (a_i^{pc}, b_i^{pc}) \neq (0,0)\right]$, then there exist $N \in \mathbb{N}$ with $N \geq H$ and $(A_i, B_i) \in \mathbb{R}^2$ for $i \in \{-N, ..., -H\}$ such that the interest rate rule

$$\begin{aligned} r_t = {} & \frac{1}{c_0^{is}} \left(y_t - \sum_{i=1}^{N_2^{is}} a_i^{is} E_t\{\Delta p_{t+i}\} - \sum_{i=1}^{N_2^{is}} b_i^{is} E_t\{y_{t+i}\} - \right. \\ & \left. \sum_{i=-N_1^{is}}^{0} a_i^{is} \Delta p_{t+i} - \sum_{i=-N_1^{is}}^{-1} b_i^{is} y_{t+i} - \sum_{i=-N_1^{is}}^{-1} c_i^{is} r_{t+i} \right) + \\ & \left(\Delta p_{t-H} - \sum_{i=1-H}^{0} a_{H+i}^{pc} \Delta p_{t+i} - \sum_{i=1-H}^{0} b_{H+i}^{pc} y_{t+i} - \right. \\ & \left. \sum_{i=-N}^{-H} A_i \Delta p_{t+i} - \sum_{i=-N}^{-H} B_i y_{t+i} \right) + \sum_{i=-N}^{0} d_i \varepsilon_{t+i}^{pc} + \sum_{i=-N}^{0} f_i \varepsilon_{t+i}^{is} \end{aligned}$$

ensures strong local equilibrium determinacy. As explained in **appendix A**, the forward-looking part of this interest rate rule is carefully chosen so as to insulate the current variables from the forward-looking part of the IS equation, while its backward-looking part is designed to insulate (in the future) the current variables from the forward-looking part of the Phillips curve. As a consequence, this interest rate rule in effect disconnects the current situation from the expectations about the future situation, thus making sure that there exists a unique trajectory $\{\Delta p_t, y_t, r_t\}_{t_0}^{+\infty}$ satisfying (1), (2) and (3) at all dates $t \geq t_0$ in the absence of any terminal condition. By contrast, if the interest rate rule were arbitrarily chosen, then strong local equilibrium indeterminacy would typically arise because the current values of the variables would depend in a non-controlled way on their expected future values in the absence of any terminal condition.

Note that for certain specifications of the forward-looking part of the IS equation, the interest rate rules ensuring strong local equilibrium determinacy given in **appendix A** are forward-looking, that is to say that they make the nominal interest rate conditional on the private agents' expectations. Interestingly, there exist specifications of the Phillips curve and the IS equation such that all interest rate rules ensuring strong local equilibrium determinacy are forward-looking. Such is the case, as will be shown, of the canonical New Keynesian model considered in the next sections. This result provides what is to our knowledge a new theoretical justification for the adoption of forward-looking interest rate rules. Indeed, the existing literature focuses on interest rate rules ensuring weak local equilibrium determinacy, and as acknowledged by Woodford (2003) and Giannoni and Woodford (2003a, 2003b) for instance, "forward-lookingness" is not a necessary condition for an interest rate rule to ensure weak local equilibrium determinacy, at least under the rational expectations assumption. Under the assumption that the private agents follow adaptative learning rules instead of having rational expectations, Evans and Honkapohja (2002, 2003) show that some forward-looking interest rate rules do ensure the weak local determinacy and the stability of the equilibrium under discretion and under commitment, contrary to some non-forward-looking interest rate rules, but they do not show that "forward-lookingness" is a necessary condition for an interest rate rule to ensure the weak local determinacy and the stability of the equilibrium, so that whether adaptative learning provides a theoretical justification for basing the interest rate rule on the private agents' expectations is still open to question.

2 Application to a closed economy model

This section applies our point to the canonical New Keynesian model of a closed economy. We first present the reduced form of this model, we then determine analytically the optimal equilibrium under discretion and under commitment, and we finally characterize the set of the interest rate rules implementing this equilibrium and ensuring its strong local determinacy.

2.1 Reduced form of the model

The canonical New Keynesian model of a closed economy, used notably by Clarida, Galí and Gertler (1999) and Woodford (2003), is an intertemporal general equilibrium model which manages to combine a simple reduced (log-linearized around the steady state) form with sound microfoundations. This reduced form is composed of a Phillips curve, an IS equation and a loss function (for the social planner) or an interest rate rule (for the central bank). In this subsection, we present the Phillips curve and the IS equation.

The Phillips curve, derived from the firms' profits maximization, is written:

$$\Delta p_t = \beta E_t \{\Delta p_{t+1}\} + \gamma y_t + \varepsilon_t^{pc}, \tag{4}$$

where Δp_t denotes the inflation rate and y_t the output gap (namely the deviation of the logarithm of the real output level from its flexible-price value) at date t, while β and γ are two parameters such that $0 < \beta < 1$ and $\gamma > 0$. This Phillips curve is forward-looking because of the assumption of price-setting *à la* Calvo, as firms know that the price they choose today will remain effective for more than one period on average. As put forward by Clarida, Galí and Gertler (1999), the exogenous cost-push shock ε_t^{pc} occurring at date t may be interpreted as the consequence of frictions in the wage contracting process or as the consequence of pricing errors. For simplicity, it is assumed to be identically and independently distributed with mean zero.

The IS equation, derived from the representative household's utility maximization, is written:

$$y_t = E_t \{y_{t+1}\} - \eta (r_t - E_t \{\Delta p_{t+1}\}) + \varepsilon_t^{is}, \tag{5}$$

where r_t denotes the short-term nominal interest rate (expressed as deviations from its steady state value) at date t, while η is a strictly positive parameter. This IS equation is forward-looking due to the usual intertemporal substitution effect. The exogenous demand shock ε_t^{is} occurring at date t may be interpreted as an unexpected exogenous public spending. For simplicity, it is assumed to be identically and independently distributed with mean zero. As in all frameworks with infinitely-lived utility-maximizing agents, there is also a transversality condition attached to the optimization programme of the representative household, which will be satisfied by the optimal equilibria considered in the next subsection.

2.2 Determination of the optimal equilibrium

In this subsection, we assume the existence of a social planner which chooses the values of Δp, y and r minimizing the loss function

$$L_t = E_t \left\{ \sum_{k=0}^{+\infty} \delta^k \left[(\Delta p_{t+k})^2 + \lambda (y_{t+k})^2 \right] \right\} \tag{6}$$

subject to the structural equations (4) and (5). As shown by Woodford (2003, chap. 6), such a loss function corresponds to (the opposite of) the second-order approximation of the representative household's utility function in the neighbourhood of the steady state when first-order terms are offset by structural policies. Parameters δ and λ are then related to the structural parameters of the model, and in particular $\delta = \beta$. In what follows we assume more generally that $\lambda > 0$ and $0 < \delta < 1$, and that δ is sufficiently close to β in a sense to be further specified below. We consider two alternative cases, depending on whether the social planner minimizes L_t at each date t or once and for all.

We first determine the optimal equilibrium under discretion, that is to say the equilibrium obtained when the social planner re-optimizes at each period, in other words when at each date $t \geq t_0$ the social planner chooses Δp_t, y_t and r_t so as to minimize L_t subject to the structural equations. As shown in **appendix B**, the resulting outcome, usually named discretionary equilibrium, or time-consistent plan, or non-reputational solution, is the following one for $t \geq t_0$:

$$\Delta p_t = \frac{\lambda}{\gamma^2 + \lambda} \varepsilon_t^{pc}, \quad y_t = \frac{-\gamma}{\gamma^2 + \lambda} \varepsilon_t^{pc} \text{ and } r_t = \frac{1}{\eta} \varepsilon_t^{is} + \frac{\gamma}{(\gamma^2 + \lambda)\eta} \varepsilon_t^{pc}.$$

These results, which hold under the assumption that δ is sufficiently close to β for the inequality $\delta \left(\gamma^2 + \lambda\right)^2 \geq \beta^2 \lambda^2$ to be satisfied, are discussed in details by Clarida, Galí and Gertler (1999). In brief, they indicate that demand shocks ε^{is} are entirely countered by monetary policy and have therefore no impact on the representative household's welfare, as output gap stabilization and inflation stabilization are then mutually compatible, while on the contrary cost-push shocks ε^{pc} are not entirely countered by monetary policy, as the central bank then faces a trade-off between a higher inflation rate and a lower output gap. In both cases (ε^{pc} or ε^{is}), the effect of the shock is one-shot, that is to say that the responses of Δp, y and r are not inertial.

We then determine the optimal equilibrium under commitment, that is to say the equilibrium obtained when the social planner optimizes once and for all, in other words when at date t_0 the social planner chooses the state-contingent values of Δp_{t_0+n}, y_{t_0+n} and r_{t_0+n} for all $n \geq 0$ so as to minimize L_{t_0}. To that aim, we specify the variables as (possibly not time-invariant) linear combinations of the complete history of the exogenous disturbances:

$$\begin{aligned} \Delta p_{t_0+n} &= \sum_{k=0}^{+\infty} \left(a_n^{n-k} \varepsilon_{t_0+n-k}^{pc} + b_n^{n-k} \varepsilon_{t_0+n-k}^{is} \right), \\ y_{t_0+n} &= \sum_{k=0}^{+\infty} \left(c_n^{n-k} \varepsilon_{t_0+n-k}^{pc} + d_n^{n-k} \varepsilon_{t_0+n-k}^{is} \right) \\ \text{and } r_{t_0+n} &= \sum_{k=0}^{+\infty} \left(e_n^{n-k} \varepsilon_{t_0+n-k}^{pc} + f_n^{n-k} \varepsilon_{t_0+n-k}^{is} \right) \end{aligned}$$

for $n \geq 0$, and we determine these linear combinations which minimize the loss function (6) subject to the structural equations (4) and (5). Note that we depart from the existing literature in three important ways.

First, we assume that shocks ε^{pc} and ε^{is} are serially uncorrelated, which enables us to obtain a simple analytical expression for the variables in equilibrium as functions of the exogenous shocks only. By contrast, the existing literature typically considers serially correlated shocks and consequently does not determine analytically the variables in equilibrium as functions of the exogenous shocks only. Indeed, these analytical results would rest on the general analytical expression of the roots of a polynomial whose degree is strictly higher than two under the assumption of serially correlated shocks, and this expression either is little exploitable (when the degree of the polynomial is three or four) or does not exist (when the degree of the polynomial is strictly higher than four).

Second, we optimize over the class of linear combinations of shocks which are possibly not time-invariant. By contrast, the existing literature considers only time-invariant linear equilibrium candidates, whether implicitly in the form of impulse-response functions as in Clarida, Galí and Gertler (1999), or explicitly as in Giannoni and Woodford (2003a, 2003b) and Woodford (2003, chap. 8). Since we eventually obtain a time-invariant equilibrium, we thus show that the time-invariant linear equilibrium which the existing literature finds is optimal among all time-invariant linear equilibrium candidates, is also optimal among all (possibly not time-invariant) linear equilibrium candidates.

Third, we specify the variables prior to optimization as functions of the complete history of the exogenous disturbances without imposing any time-consistency requirement. By contrast, the existing literature either optimizes over the class of equilibrium candidates which depend only on the shocks occurring from date t_0 onwards, as implicitly done by Clarida, Galí and Gertler (1999) who look for the equilibrium in the form of an impulse-response function for a shock occurring at date t_0 or after, or imposes a time-consistency requirement as in the "timeless perspective" of Giannoni and Woodford (2003a, 2003b) and Woodford (2003, chap. 8) which amounts in effect to assume that the economy was already at the equilibrium considered before date t_0. The primary reason why the existing literature does not lwdoicw
time-consistencyalr19(requ)1(ireme)-9an3(ms)-1(a)21(y)bdo t4t, equilibri0y ob(alr)1(d-24t,)-419(i)1(s)-r16(alr)1(1

$$\Delta p_t = \frac{\delta z}{\beta}\varepsilon_t^{pc} - \frac{\gamma^2\delta}{\beta\lambda\left(1-\beta z\right)}\sum_{k=1}^{t-t_0} z^{k+1}\varepsilon_{t-k}^{pc},$$

$$y_t = \frac{-\gamma\delta}{\beta\lambda}\sum_{k=0}^{t-t_0} z^{k+1}\varepsilon_{t-k}^{pc},$$

$$r_t = \frac{1}{\eta}\varepsilon_t^{is} + \frac{\gamma\delta\left[\beta z^2-\left(1+\beta+\gamma\eta\right)z+1\right]}{\beta\eta\lambda\left(1-\beta z\right)}\sum_{k=0}^{t-t_0} z^{k+1}\varepsilon_{t-k}^{pc},$$

where z is a positive constant, expressed in **appendix C** as a function of the parameters. These results hold under the assumption that δ is sufficiently close to β for the following inequality to be satisfied: $\gamma^2\delta+\beta^2\lambda+\delta\lambda>\beta\delta\lambda+\beta\lambda$, which ensures that $z<1$.

As under discretion, the output gap and the inflation rate are insulated from the effects of demand shocks ε^{is}, but not from those of cost-push shocks ε^{pc}. The main difference between the optimal equilibria under discretion and under commitment is that the effect of ε^{pc} is more prolonged here: the shock ε^{pc} is one-shot, but the responses of Δp, y and r are inertial. This is because following a positive cost-push shock, the social planner can now trade off not only between a higher inflation rate and a lower output gap at a given date, but also between the present and the future situations. In other words, the commitment technology enables her to spread the burden of the adjustment to the shock over several periods. This equilibrium is time-inconsistent of course, as the social planner faces no incentive to go on reacting to bygone shocks in this purely forward-looking framework.

2.3 Implementation of the optimal equilibrium

In this subsection, we replace the social planner by a central bank which sets r according to an interest rate rule (3). We look for the interest rate rules implementing the optimal equilibrium determined in the previous subsection and ensuring its strong local determinacy. Note however that as shown by **appendix D**, the economy is "controllable" in the sense that not only the optimal equilibrium, but also any local equilibrium can be implemented by a well-chosen interest rate rule ensuring its strong local determinacy. More precisely, any bounded trajectory $\{\Delta p_t, y_t, r_t\}_{t_0}^{+\infty}$ satisfying (4) and (5) at all dates $t\geq t_0$ and written in the VAR form

$$\begin{bmatrix}\Delta p_t\\ y_t\\ r_t\end{bmatrix} = \sum_{i=1}^{n} S_i\begin{bmatrix}\Delta p_{t-i}\\ y_{t-i}\\ r_{t-i}\end{bmatrix} + T\begin{bmatrix}\varepsilon_t^{pc}\\ \varepsilon_t^{is}\end{bmatrix}$$

where $n\in\mathbb{N}^*$, S_i for $i\in\{1,...,n\}$ are 3×3 matrices and T is a 3×2 matrix, can be implemented by a well-chosen interest rate rule ensuring its strong local determinacy.

We first look for the interest rate rules ensuring strong local equilibrium determinacy. **Appendix D** shows that these interest rate rules are necessarily forward-looking. Loosely speaking, the reason for this "forward-lookingness" is the following. Since y_t and $E_t\{y_{t+k}\}$ for $k\geq1$ can be residually derived from Δp_t and $E_t\{\Delta p_{t+k}\}$ for $k\geq1$ with the Phillips curve, and r_t and $E_t\{r_{t+k}\}$ for $k\geq1$ from y_t, $E_t\{y_{t+k}\}$ and $E_t\{\Delta p_{t+k}\}$ for $k\geq1$ with the IS equation, the interest rate rule will ensure strong local equilibrium determinacy if and only if it pins down Δp_t and $E_t\{\Delta p_{t+k}\}$ for $k\geq1$ uniquely. Now in order to pin down Δp_t uniquely, the interest rate rule should disconnect Δp_t from the non-predetermined variables at date t

The interest rate rule should therefore cancel the effect of y_t and $E_t\{y_{t+1}\}$ on Δp_t, that is to say that its forward-looking part should amount to the term $\frac{1}{\eta}E_t\{y_{t+1}\}$ if $b_0 = -\frac{\beta+\gamma\eta}{\beta\eta}$. More generally, this forward-looking part will be determined conditionally on b_0 and *modulo* the Phillips curve and the IS equation, as made clear by **appendix D**.

We then look more precisely for the interest rate rules which make the strong-locally unique equilibrium selected coincide with the optimal equilibrium determined in the previous subsection. Because y_t, r_t, $E_t\{y_{t+k}\}$ and $E_t\{r_{t+k}\}$ for $k \geq 1$ can be residually derived from Δp_t and $E_t\{\Delta p_{t+k}\}$ for $k \geq 1$ with the Phillips curve and the IS equation, the interest rate rule will select the optimal equilibrium if and only if it implements the optimal values of Δp_t and $E_t\{\Delta p_{t+k}\}$ for $k \geq 1$. Now, as shown in **appendix D**, the Phillips curve, the IS equation and the interest rate rule can be combined to get some initial conditions and a time-invariant linear recurrence equation on Δp_t and $E_t\{\Delta p_{t+k}\}$ for $k \geq 1$. The choice of the interest rate rule then affects the number and the values of the roots of the characteristic polynomial of this recurrence equation, as well as the number and the identity of the initial conditions. Moreover, we can to some extent independently control (the number and the value of) the roots of the characteristic polynomial on the one hand and (the number and the identity of) the initial conditions on the other hand. For instance, adding a term $\omega\varepsilon_{t-k}^{pc}$ where ω /

which involve only the target variables Δp and y, in the terminology of Woodford (2003, chap. 8) and Giannoni and Woodford (2003a, 2003b).

3 Application to a small open economy model

This section applies our point to the canonical New Keynesian model of a small open economy. We first present the reduced form of this model. We then directly apply our results to the case of a flexible exchange rate regime under discretion and under commitment, as the reduced form of the model is then isomorphic to that of the closed economy model previously considered. We finally consider the case of a fixed exchange rate regime, that is to say we determine analytically the unique equilibrium in this case and characterize the interest rate rules implementing this equilibrium and ensuring its strong local determinacy.

3.1 Reduced form of the model

What we call the canonical New Keynesian model of a small open economy is an intertemporal general equilibrium model laid out by Clarida, Galí and Gertler (2001), very close to the model

Galí and Gertler (2001), Galí and Monacelli (2002) and Monacelli (2003), who consider serially correlated shocks. Finally, because the general form of the interest rate is the same as in the closed economy framework, the results obtained in subsection 2.3 can also be readily applied to our small open economy framework under a flexible exchange rate regime.

3.3 Fixed exchange rate regime

We look for the fixed exchange rate regime equilibrium among all equilibrium candidates which express the variables as (possibly not time-invariant) linear combinations of the entire history of shocks, that is to say for variables Δp and y in particular:

$$\begin{aligned} \Delta p_t &= \sum_{k=0}^{+\infty} \left(g_t^k \varepsilon_{t-k}^{pc} + h_t^k \varepsilon_{t-k}^{is} \right) \\ \text{and } y_t &= \sum_{k=0}^{+\infty} \left(s_t^k \varepsilon_{t-k}^{pc} + w_t^k \varepsilon_{t-k}^{is} \right). \end{aligned}$$

As shown in **appendix E**, the fixed exchange rate regime equilibrium turns out to be unique and to express the variables as time-invariant linear combinations of the entire history of shocks:

$$\Delta p_t = \gamma x \varepsilon_t^{is} - \gamma (1-x) \sum_{k=1}^{+\infty} x^k \varepsilon_{t-}^{is}$$

backward-looking ($N_1 \geq 1$), and that the set of these rules of size N_1 is a $3N_1 + 1$-dimension vectorial space. For instance, the unique adequate interest rate rule of size $N_1 = 1$ satisfying the quadruple constraint $(b_0, c_{-1}, d_0, d_{-1}) = \left(-\frac{\beta+\gamma\eta}{\beta\eta}, 0, 0, 0\right)$ is the following one:

$$
\begin{aligned}
r_t \quad = \quad & \frac{1}{\eta} E_t \{y_{t+1}\} - \frac{\beta + \gamma\eta}{\beta\eta} y_t - \frac{1-x}{\beta x} \Delta p_t + \frac{\gamma}{\beta(1-\beta x)} y_{t-1} \\
& + \frac{\beta + \gamma\eta}{\beta\eta} \varepsilon_t^{is} - \frac{\gamma}{\beta(1-\beta x)} \varepsilon_{t-1}^{is}.
\end{aligned}
$$

Since the nominal interest rate keeps constantly equal to its steady state value ($r = 0$) under such an interest rate rule, the central bank can be said to be active *ex ante* and passive *ex post*.

Conclusion

This paper aims at giving a new insight into the design of interest rate rules precluding multiple equilibria. Our main contribution consists in examining the interest rate rules which ensure strong local equilibrium determinacy, that is to say which make sure that there is a unique equilibrium starting out in the neighbourhood of the steady state and that this equilibrium remains constantly in that neighbourhood. We first show in a general framework that such interest rate rules do exist. We then show, in the more specific but very popular canonical New Keynesian framework, that they are forward-looking so as to insulate the current situation from the private agents' expectations about the future situation, and this result provides what is to our knowledge a new theoretical justification for the adoption of forward-looking interest rate rules by central banks. We characterize moreover, in this specific framework, the set of interest rate rules which make the strong-locally unique equilibrium selected coincide with the desired equilibrium.

The issue of strong local equilibrium indeterminacy is admittedly more restrictive than that of global equilibrium indeterminacy. In particular, our interest rate rules may well not be effective outside the neighbourhood of the steady state, as they are only meant to prevent the economy from leaving this neighbourhood. But ensuring strong local equilibrium determinacy is still enough for an interest rate rule to preclude all kinds of equilibria described in the existing literature, including endogenous fluctuations around the steady state and equilibria converging to the liquidity trap, a deterministic cycle or a chaotic cycle, provided they originate near the steady state. At last but not least, strong local equilibrium determinacy rules out the "boom and bust" equilibria identified in this paper, which may be of practical importance as most post-war American recessions have been due, according to a widespread point of view, to a monetary policy tightening putting an end to a period of increasing inflation rate and could therefore be interpreted as such "boom and bust" equilibria.

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Appendix

A Existence of interest rate rules ensuring strong local determinacy

As argued in subsection 1.2, strong local equilibrium determinacy is ensured if and only if there is a unique set of sequences $\{\Delta p_t, y_t, r_t\}_{t_0}^{+\infty}$ satisfying (1), (2) and (3) at all dates $t \geq t_0$ and the corresponding sequences $(\Delta p_t)_{t_0}^{+\infty}$ and $(y_t)_{t_0}^{+\infty}$ are bounded. We show that interest rate rules ensuring strong local equilibrium determinacy do exist by providing examples of such interest rate rules. We consider two alternative cases in turn, according to whether the Phillips curve is forward-looking or not, that is to say whether $(a_i^{pc}, b_i^{pc}) = (0,0)$ for $i \in \{1, ..., N_2^{pc}\}$ or not.

If the Phillips curve is not forward-looking, then consider an interest rate rule (3) of the kind

$$\begin{aligned} r_t \quad = \quad & \frac{1}{c_K^{is}} \left[y_{t-K} - \sum_{i=K+1}^{N_2^{is}} a_i^{is} E_t \left\{ \Delta p_{t-K+i} \right\} - \sum_{i=K+1}^{N_2^{is}} b_i^{is} E_t \left\{ y_{t-K+i} \right\} - \right. \\ & \sum_{i=-N_1^{is}}^{K} a_i^{is} \Delta p_{t-K+i} - \sum_{i=-N_1^{is}}^{K-1} b_i^{is} y_{t-K+i} - \sum_{i=-N_1^{is}}^{K-1} c_i^{is} r_{t-K+i} \; + \\ & \left(\Delta p_{t-K} - \sum_{i=-N}^{0} A_i \Delta p_{t-K+i} - \sum_{i=-N}^{0} B_i y_{t-K+i} \right) + \\ & \left. \sum_{i=-N}^{0} d_i \varepsilon_{t-K+i}^{pc} + \sum_{i=-N}^{0} f_i \varepsilon_{t-K+i}^{is} \right] \end{aligned}$$

where $K \equiv \max \left\{ i \in \left\{ 0, ..., N_2^{is} \right\} \text{ such that } c_i^{is} \neq 0 \right\}$, $N \in \mathbb{N}$ and $(A_i, B_i) \in \mathbb{R}^2$ for $i \in \{-N, ..., 0\}$ with A_0 and B_0 such that $(1 - A_0)\, b_0^{pc} \neq B_0$.

We first show by recurrence that Δp_t, y_t, r_t, $E_t\{\Delta p_{t+n}\}$, $E_t\{y_{t+n}\}$ and $E_t\{r_{t+n}\}$ for $n \geq 1$ are uniquely determined with such an interest rate rule. Indeed, the introduction of this interest rate rule (taken in expectations $E_t\{.\}$ at date $t + K$) into the IS equation taken at date t leads to

$$\Delta p_t = \sum_{i=-N}^{0} A_i \Delta p_{t+i} + \sum_{i=-N}^{0} B_i y_{t+i} - \sum_{i=-N}^{0} d_i \varepsilon_{t+i}^{pc} - \sum_{i=-N}^{0} f_i \varepsilon_{t+i}^{is} - \varepsilon_t^{is}. \qquad (7)$$

The two equations (1) and (7) taken at date t pin down the two unknowns Δp_t and y_t (which are the only non-predetermined endogenous variables in these equations) uniquely as $(1 - A_0)\, b_0^{pc} \neq B_0$. For $n \in \mathbb{N}^*$ similarly, if Δp_t and y_t as well as (if $n \geq 2$) $E_t\{\Delta p_{t+k}\}$ and $E_t\{y_{t+k}\}$ for $k \in \{1, ..., n-1\}$ are already determined, then the two equations (1) and (7) taken in expectations $E_t\{.\}$ at date $t + n$ pin down the two unknowns $E_t\{\Delta p_{t+n}\}$ and $E_t\{y_{t+n}\}$ uniquely as $(1 - A_0)\, b_0^{pc} \neq B_0$. Thus Δp_t, y_t, $E_t\{\Delta p_{t+n}\}$ and $E_t\{y_{t+n}\}$ for $n \geq 1$ are uniquely determined, and r_t and $E_t\{r_{t+n}\}$ for $n \geq 1$ are then residually uniquely determined with the interest rate rule considered.

We then show that N and the coefficients A_i, B_i, d_i and f_i for $i \in \{-N, ..., 0\}$ can be chosen so that the uniquely determined sequences $(\Delta p_t)_{t_0}^{+\infty}$ and $(y_t)_{t_0}^{+\infty}$ are bounded. Take for simplicity $(d_0, f_0) = (0, -1)$ and (if $N \geq 1$) $(d_i, f_i) = (0, 0)$ for $i \in \{-N, ..., -1\}$. Equations (1) and (7) then correspond to the following linear cross-recurrence equations for $t \geq t_0$:

$$\begin{aligned} \Delta p_t - b_0^{pc} y_t &= \sum_{i=-N_1^{pc}}^{-1} a_i^{pc} \Delta p_{t+i} + \sum_{i=-N_1^{pc}}^{-1} b_i^{pc} y_{t+i} + \varepsilon_t^{pc}, \\ (1 - A_0)\, \Delta p_t - B_0 y_t &= \sum_{i=-N}^{-1} A_i \Delta p_{t+i} + \sum_{i=-N}^{-1} B_i y_{t+i}. \end{aligned}$$

The choice of $N = N^*$, $(A_0, B_0) = (a_0^* + 1, b_0^*)$ and (if $N \geq 1$) $(A_i, B_i) = (a_i^*, b_i^*)$ for $i \in \{-N, ..., -1\}$ then ensures that $(\Delta p_t)_{t_0}^{+\infty}$ and $(y_t)_{t_0}^{+\infty}$ are bounded.

If the Phillips curve is forward-looking, then consider an interest rate rule (3) of the kind

$$
\begin{aligned}
r_t \;=\; & \frac{1}{c_K^{is}} \left[y_{t-K} - \sum_{i=K+1}^{N_2^{is}} a_i^{is} E_t \left\{ \Delta p_{t-K+i} \right\} - \sum_{i=K+1}^{N_2^{is}} b_i^{is} E_t \left\{ y_{t-K+i} \right\} - \right. \\
& \left. \sum_{i=-N_1^{is}}^{K} a_i^{is} \Delta p_{t-K+i} - \sum_{i=-N_1^{is}}^{K-1} b_i^{is} y_{t-K+i} - \sum_{i=-N_1^{is}}^{K-1} c_i^{is} r_{t-K+i} \right] + \\
& \left[\Delta p_{t-K-H} - \sum_{i=1-H}^{0} a_{H+i}^{pc} \Delta p_{t-K+i} - \sum_{i=1-H}^{0} b_{H+i}^{pc} y_{t-K+i} - \right] \\
& \sum_{i=-N}^{-H} A_i \Delta p_{t-K+i} - \sum_{i=-N}^{-H} B_i y_{t-K+i} + \\
& \sum_{i=-N}^{0} d_i \varepsilon_{t-K+i}^{pc} + \sum_{i=-N}^{0} f_i \varepsilon_{t-K+i}^{is}
\end{aligned}
$$

where $K \equiv \max \left\{ i \in \left\{ 0, ..., N_2^{is} \right\} \text{ such that } c_i^{is} \neq 0 \right\}$, $H \equiv \max \left[i \in \{1, ..., N_2^{pc}\} \text{ such that } \left(a_i^{pc}, b_i^{pc} \right) \neq (0,0) \right]$, $N \in \mathbb{N}$ such that $N \geq H$, and $(A_i, B_i) \in \mathbb{R}^2$ for $i \in \{-N, ..., -H\}$ with A_{-H} and B_{-H} such that $A_{-H} b_H^{pc} \neq \left(B_{-H} - b_0^{pc} \right) a_H^{pc}$.

We first show by recurrence that Δp_t, y_t, r_t, $E_t \{\Delta p_{t+n}\}$, $E_t \{y_{t+n}\}$ and $E_t \{r_{t+n}\}$ for $n \geq 1$ are uniquely determined with such an interest rate rule. Indeed, the introduction of this interest rate rule (taken in expectations $E_t \{.\}$ at date $t + K$) into the IS equation taken at date t leads to

$$
\begin{aligned}
\Delta p_{t-H} \;=\; & \sum_{i=1-H}^{0} a_{H+i}^{pc} \Delta p_{t+i} + \sum_{i=1-H}^{0} b_{H+i}^{pc} y_{t+i} + \sum_{i=-N}^{-H} A_i \Delta p_{t+i} + \\
& \sum_{i=-N}^{-H} B_i y_{t+i} - \sum_{i=-N}^{0} d_i \varepsilon_{t+i}^{pc} - \sum_{i=-N}^{0} f_i \varepsilon_{t+i}^{is} - \varepsilon_t^{is}, \qquad (8)
\end{aligned}
$$

while the substraction of (1) taken at date t from (8) taken in expectations $E_t \{.\}$ at date $t + H$ leads to

$$
\begin{aligned}
0 \;=\; & A_{-H} E_{t_0} \{\Delta p_t\} + \sum_{i=-N_1^{pc}}^{-1} \left(A_{i-H} - a_i^{pc} \right) E_{t_0} \{\Delta p_{t+i}\} + \\
& \sum_{i=-N_1^{pc}}^{0} \left(B_{i-H} - b_i^{pc} \right) E_{t_0} \{y_{t+i}\} + \\
& \sum_{i=-N+H}^{-N_1^{pc}-1} A_{i-H} E_{t_0} \{\Delta p_{t+i}\} + \sum_{i=-N+H}^{-N_1^{pc}-1} B_{i-H} E_{t_0} \{y_{t+i}\} . \qquad (9)
\end{aligned}
$$

Equations (8) and (9) taken at date t pin down the two unknowns Δp_t and y_t (which are the only non-predetermined endogenous variables in these equations) uniquely as $A_{-H} b_H^{pc} \neq \left(B_{-H} - b_0^{pc} \right) a_H^{pc}$. For $n \in \mathbb{N}^*$ similarly, if Δp_t and y_t as well as (if $n \geq 2$) $E_t \{\Delta p_{t+k}\}$ and $E_t \{y_{t+k}\}$ for $k \in \{1, ..., n-1\}$ are already determined, then equations (8) and (9) taken in expectations $E_t \{.\}$ at date $t + n$ pin down the two unknowns $E_t \{\Delta p_{t+n}\}$ and $E_t \{y_{t+n}\}$ uniquely as $A_{-H} b_H^{pc} \neq \left(B_{-H} - b_0^{pc} \right) a_H^{pc}$. Thus Δp_t, y_t, $E_t \{\Delta p_{t+n}\}$ and $E_t \{y_{t+n}\}$ for $n \geq 1$ are uniquely determined, and r_t and $E_t \{r_{t+n}\}$ for $n \geq 1$ are then residually uniquely determined with the interest rate rule considered.

We then show that N, the coefficients A_i and B_i for $i \in \{-N, ..., -H\}$ and the coefficients d_i and f_i for $i \in \{-N, ..., 0\}$ can be chosen so that the uniquely determined sequences $(\Delta p_t)_{t_0}^{+\infty}$ and $(y_t)_{t_0}^{+\infty}$ are bounded. Take for simplicity $N \geq N_1^{pc} + H$, $d_{-H} = -1$, $f_0 = -1$, $d_i = 0$ for $i \in \{-N, ..., 0\} \setminus \{-H\}$ and $f_i = 0$ for $i \in \{-N, ..., -1\}$. For $t \geq t_0 + H$, equations (1) and (9) taken in expectations $E_{t_0} \{.\}$ at date t then correspond to the following linear cross-recurrence equations:

$$-a_H^{pc} E_{t_0}\{\Delta p_t\} - b_H^{pc} E_{t_0}\{y_t\} = \sum_{i=1-H}^{-1} a_{i+H}^{pc} E_{t_0}\{\Delta p_{t+i}\} + E_{t_0}\{\Delta p_{t-H}\} + \sum_{i=-N_1^{pc}-H}^{-1-H} a_{i+H}^{pc} E_{t_0}\{\Delta p_{t+i}\} + \sum_{i=-N_1^{pc}-H}^{0} b_{i+H}^{pc} E_{t_0}\{y_{t+i}\} + E_{t_0}\left(\varepsilon_{t-H}^{pc}\right),$$

$$-A_{-H} E_{t_0}\{\Delta p_t\} + \left(b_0^{pc} - B_{-H}\right) E_{t_0}\{y_t\} = \sum_{i=-N_1^{pc}}^{-1} \left(A_{i-H} - a_i^{pc}\right) E_{t_0}\{\Delta p_{t+i}\} + \sum_{i=-N_1^{pc}}^{-1} \left(B_{i-H} - b_i^{pc}\right) E_{t_0}\{y_{t+i}\} + \sum_{i=-N+H}^{-N_1^{pc}-1} A_{i-H} E_{t_0}\{\Delta p_{t+i}\} + \sum_{i=-N+H}^{-N_1^{pc}-1} B_{i-H} E_{t_0}\{y_{t+i}\}.$$

The choice of $N = \max\left(N^* + H, N_1^{pc} + H\right)$, $(A_{-H}, B_{-H}) = (a_0^* + 1, b_0^* + b_0^{pc})$ and (if $N \geq H+1$) $A_i = a_{H+i}^* 1\!\!1_{i \geq -N^*-H} + a_{H+i}^{pc} 1\!\!1_{i \geq -N_1^{pc}-H}$ and $B_i = b_{H+i}^* 1\!\!1_{i \geq -N^*-H} + b_{H+i}^{pc} 1\!\!1_{i \geq -N_1^{pc}-H}$ for $i \in \{-N, ..., -H-1\}$ then ensures that $\left(E_{t_0}\{\Delta p_t\}\right)_{t_0}^{+\infty}$ and $\left(E_{t_0}\{y_t\}\right)_{t_0}^{+\infty}$ converge geometrically towards zero, and therefore that $(\Delta p_t)_{t_0}^{+\infty}$ and $(y_t)_{t_0}^{+\infty}$ are bounded.

B Determination of the optimal equilibrium under discretion

At each date t the social planner chooses Δp_t, y_t and r_t so as to minimize L_t subject to the Phillips curve and the IS equation taken at all dates, or equivalently at each date t the social planner chooses Δp_t and y_t so as to minimize L_t subject to the Phillips curve taken at date t, while r is residually determined with the IS equation. As a consequence, at each date t the social planner considers the current expectations about the future situation ($E_t\{x_{t+k}\}$ for $x \in \{\Delta p, y, r\}$ and $k \geq 1$) as given when minimizing L_t.

The first-order condition of the minimisation of L_t is written $\lambda y_t + \gamma \Delta p_t = 0$. Similarly, the first-order condition of the minimisation of L_{t+k}, taken in expectations $E_t\{.\}$, is written $\lambda E_t\{y_{t+k}\} + \gamma E_t\{\Delta p_{t+k}\} = 0$ for $k \geq 1$. Using the Phillips curve taken in expectations $E_t\{.. \quad k$

In the first case, the social planner has not observed shocks $\varepsilon_{t_0}^{pc}$ and $\varepsilon_{t_0}^{is}$ when she minimizes L_{t_0} at date t_0. She has nonetheless observed past shocks $\varepsilon_{t_0+k-j}^{pc}$ and $\varepsilon_{t_0+k-j}^{is}$ for $j \geq k+1$, so that the variables can be rewritten in the following way prior to the minimization of L_{t_0}:

$$\begin{aligned}
\Delta p_{t_0+k} &\equiv \left[\sum_{j=0}^{k}\left(a_k^{k-j}\varepsilon_{t_0+k-j}^{pc}+b_k^{k-j}\varepsilon_{t_0+k-j}^{is}\right)\right]+g_k,\\
y_{t_0+k} &\equiv \left[\sum_{j=0}^{k}\left(c_k^{k-j}\varepsilon_{t_0+k-j}^{pc}+d_k^{k-j}\varepsilon_{t_0+k-j}^{is}\right)\right]+h_k\\
\text{and } r_{t_0+k} &\equiv \left[\sum_{j=0}^{k}\left(e_k^{k-j}\varepsilon_{t_0+k-j}^{pc}+f_k^{k-j}\varepsilon_{t_0+k-j}^{is}\right)\right]+i_k
\end{aligned}$$

for $k \geq 0$. We look for the coefficients a_k^{k-j}, b_k^{k-j}, c_k^{k-j}, d_k^{k-j}, g_k and h_k for $k \geq 0$ and $0 \leq j \leq k$ which minimize L_{t_0} subject to the Phillips curve considered at all dates, *i.e.* which minimize the following Lagrangian:

$$\begin{aligned}
&E_{t_0}\left\{\left[\sum_{k=0}^{+\infty}\delta^k\left((\Delta p_{t_0+k})^2+\lambda\,(y_{t_0+k})^2\right)\right]\right\}\\
&-\left[\sum_{k=0}^{+\infty}\mu_k\left(\Delta p_{t_0+k}-\beta E_{t_0+k}\left\{\Delta p_{t_0+k+1}\right\}-\gamma y_{t_0+k}-\varepsilon_{t_0+k}^{pc}\right)\right].
\end{aligned}$$

The coefficients e_k^{k-j}, f_k^{k-j} and i_k for $k \geq 0$ and $0 \leq j \leq k$ are then residually determined with the help of the IS equation. In a straighforward manner, we find that $\forall k \geq 0$ and $\forall j \in \{0,...,k\}$, $b_k^{k-j} = d_k^{k-j} = 0$. Now, the first-order conditions of the Lagrangian's minimization with respect to a_k^k for $k \geq 0$, a_k^{k-j} for $k \geq 1$ and $j \in \{1,...,k\}$, c_k^{k-j} for $k \geq 0$ and $j \in \{0,...,k\}$, g_0, g_k for $k \geq 1$, h_k for $k \geq 0$ can be respectively written in the following way:

$$\begin{aligned}
2\delta^k V(\varepsilon^{pc})\,a_k^k-\mu_k\varepsilon_{t_0+k}^{pc} &= 0 \text{ for } k \geq 0,\\
2\delta^k V(\varepsilon^{pc})\,a_k^{k-j}-\mu_k\varepsilon_{t_0+k-j}^{pc}+\beta\mu_{k-1}\varepsilon_{t_0+k-j}^{pc} &= 0 \text{ for } k \geq 1 \text{ and } j \in \{1,...,k\},\\
2\delta^k\lambda V(\varepsilon^{pc})\,c_k^{k-j}+\gamma\mu_k\varepsilon_{t_0+k-j}^{pc} &= 0 \text{ for } k \geq 0 \text{ and } j \in \{0,...,k\},\\
2g_0-\mu_0 &= 0,\\
2\delta^k g_k-\mu_k+\beta\mu_{k-1} &= 0 \text{ for } k \geq 1,\\
2\delta^k\lambda h_k+\gamma\mu_k &= 0 \text{ for } k \geq 0,
\end{aligned}$$

where $V(\varepsilon^{pc})$ represents the variance of the shock ε^{pc}. Moreover, the Phillips curve considered at all dates leads to the following three additional equations:

$$\begin{aligned}
\beta a_{k+1}^k-a_k^k+\gamma c_k^k+1 &= 0 \text{ for } k \geq 0,\\
\beta a_{k+1}^{k-j}-a_k^{k-j}+\gamma c_k^{k-j} &= 0 \text{ for } k \geq 1 \text{ and } j \in \{1,...,k\},\\
\beta g_{k+1}-g_k+\gamma h_k &= 0 \text{ for } k \geq 0.
\end{aligned}$$

Let us note $u \equiv k-j$, $v \equiv j$, $A_{u,v} \equiv a_k^{k-j}$ and $C_{u,v} \equiv c_k^{k-j}$, so that $A_{u,v}$ and $C_{u,v}$ characterize respectively the reactions of Δp_{t_0+u+v} and y_{t_0+u+v} to $\varepsilon_{t_0+u}^{pc}$. Our nine equations are then equivalent to the following systems of equations:

$$\left\{\begin{array}{ll}
\gamma g_0+\lambda h_0=0 & \\
g_k-\beta g_{k+1}-\gamma h_k=0 & \text{for } k \geq 0\\
\gamma\delta g_{k+1}+\delta\lambda h_{k+1}-\beta\lambda h_k=0 & \text{for } k \geq 0
\end{array}\right. \quad (10)$$

$$\text{and} \quad \begin{cases} \gamma A_{u,0} + \lambda C_{u,0} = 0 & \text{for } u \geq 0 \\ \gamma\delta A_{u,v+1} + \delta\lambda C_{u,v+1} - \beta\lambda C_{u,v} = 0 & \text{for } u \geq 0 \text{ and } v \geq 0 \\ \beta A_{u,v+1} - A_{u,v} + \gamma C_{u,v} = 0 & \text{for } u \geq 0 \text{ and } v \geq 1 \\ \beta A_{u,1} - A_{u,0} + \gamma C_{u,0} + 1 = 0 & \text{for } u \geq 0 \end{cases}. \tag{11}$$

System (10) implies that the coefficients g_k satisfy the following equations:

$$\begin{aligned} \beta\lambda g_1 - \left(\gamma^2 + \lambda\right) g_0 &= 0, \\ \beta\delta\lambda g_{k+2} - \left(\gamma^2\delta + \beta^2\lambda + \delta\lambda\right) g_{k+1} + \beta\lambda g_k &= 0 \text{ for } k \geq 0. \end{aligned}$$

The latter equation corresponds to a recurrence equation on the g_k for $k \geq 0$. The corresponding (second-order) characteristic polynomial has two positive real roots, one noted z potentially lower than one, the other noted z' strictly higher than one:

$$\begin{aligned} z &= \frac{\left(\beta^2\lambda + \gamma^2\delta + \delta\lambda\right) - \sqrt{\left(\beta^2\lambda + \gamma^2\delta + \delta\lambda\right)^2 - 4\beta^2\delta\lambda^2}}{2\beta\delta\lambda}, \\ z' &= \frac{\left(\beta^2\lambda + \gamma^2\delta + \delta\lambda\right) + \sqrt{\left(\beta^2\lambda + \gamma^2\delta + \delta\lambda\right)^2 - 4\beta^2\delta\lambda^2}}{2\beta\delta\lambda}, \end{aligned}$$

where $z < 1$ under our assumption that δ is sufficiently close to β for the following inequality to hold: $\gamma^2\delta + \beta^2\lambda + \delta\lambda > \beta\delta\lambda + \beta\lambda$. The general form of the solution to the recurrence equation is therefore $g_k = qz^k + q'z'^k$ for $k \geq 0$, where q and q' are two real numbers. Two equations are then needed to determine q and q'. The first one is provided by the initial condition $\beta\lambda g_1 - \left(\gamma^2 + \lambda\right) g_0 = 0$. The second one is simply $q' = 0$ and comes from the fact that $\delta z'^2 \geq 1$, as can be readily checked, so that no solution with $q' \neq 0$ would fit the bill as L_t would then be infinite. We thus eventually obtain $g_k = 0$ for $k \geq 0$ and therefore $h_k = 0$ for $k \geq 0$.

The solution of system (11) will be time-invariant since the coefficients of this system do not depend on u. For the sake of simplicity, we consider therefore a given $u \geq 0$ in the following. Let us first determine the coefficients $A_{u,v}$ for $v \geq 0$, and then residually obtain the coefficients $C_{u,v}$ for $v \geq 0$. System (11) implies that the coefficients $A_{u,v}$ satisfy the following equations:

$$\begin{aligned} \beta\lambda A_{u,1} - \left(\gamma^2 + \lambda\right) A_{u,0} + \lambda &= 0, \\ \beta\delta\lambda A_{u,v+2} - \left(\gamma^2\delta + \beta^2\lambda + \delta\lambda\right) A_{u,v+1} + \beta\lambda A_{u,v} &= 0 \text{ for } v \geq 0. \end{aligned}$$

The latter equation corresponds to a recurrence equation on the $A_{u,v}$ for $v \geq 0$ which is identical to the recurrence equation on the g_k for $k \geq 0$ obtained above. The general form of the solution to this recurrence equation is therefore $A_{u,v} = wz^v + w'z'^v$ for $v \geq 0$, where w and w' are two real numbers. Two equations are then needed to determine w and w'. In a similar way as above, the first one is provided by the initial condition $\beta\lambda A_{u,1} - \left(\gamma^2 + \lambda\right) A_{u,0} + \lambda = 0$ and the second one is simply $w' = 0$. At the end of the day, we thus obtain $A_{u,v}$ for $v \geq 0$ and therefore $C_{u,v}$ for $v \geq 0$, from which we derive the results for Δp and y displayed in subsection 2.2. Finally, the results for r are residually determined with the help of the IS equation.

In the second case, the social planner has observed shocks $\varepsilon_{t_0}^{pc}$ and $\varepsilon_{t_0}^{is}$ when she minimizes L_{t_0} at date t_0. The variables can then be rewritten in the following way prior to the minimization of L_{t_0}:

$$\begin{aligned}
\Delta p_{t_0+k} &\equiv \left(\sum_{j=0}^{k-1} \left(a_k^{k-j}\varepsilon_{t_0+k-j}^{pc} + b_k^{k-j}\varepsilon_{t_0+k-j}^{is}\right)\right) + g_k, \\
y_{t_0+k} &\equiv \left(\sum_{j=0}^{k-1} \left(c_k^{k-j}\varepsilon_{t_0+k-j}^{pc} + d_k^{k-j}\varepsilon_{t_0+k-j}^{is}\right)\right) + h_k \\
\text{and } r_{t_0+k} &\equiv \left(\sum_{j=0}^{k-1} \left(e_k^{k-j}\varepsilon_{t_0+k-j}^{pc} + f_k^{k-j}\varepsilon_{t_0+k-j}^{is}\right)\right) + i_k
\end{aligned}$$

for $k \geq 0$. We now look for the coefficients a_k^{k-j}, b_k^{k-j}, c_k^{k-j}, d_k^{k-j}, g_k and h_k for $k \geq 0$ and $0 \leq j \leq k-1$ which minimize the same Lagrangian as above, before residually determining the coefficients e_k^{k-j}, f_k^{k-j} and i_k for $k \geq 0$ and $0 \leq j \leq k-1$ with the help of the IS equation. In a similar way as above, we find in a straighforward manner that $\forall k \geq 0$ and $\forall j \in \{0, ..., k-1\}$, $b_k^{k-j} = d_k^{k-j} = 0$. Now, the first-order conditions of the Lagrangian's minimization with respect to a_k^k for $k \geq 1$, a_k^{k-j} for $k \geq 2$ and $j \in \{1, ..., k-1\}$, c_k^{k-j} for $k \geq 1$ and $j \in \{0, ..., k-1\}$, g_0, g_k for $k \geq 1$, h_k for $k \geq 0$ can be respectively written in the following way:

$$\begin{aligned}
2\delta^k V(\varepsilon^{pc}) a_k^k - \mu_k \varepsilon_{t_0+k}^{pc} &= 0 \text{ for } k \geq 1, \\
2\delta^k V(\varepsilon^{pc}) a_k^{k-j} - \mu_k \varepsilon_{t_0+k-j}^{pc} + \beta\mu_{k-1}\varepsilon_{t_0+k-j}^{pc} &= 0 \text{ for } k \geq 2 \text{ and } j \in \{1, ..., k-1\}, \\
2\delta^k \lambda V(\varepsilon^{pc}) c_k^{k-j} + \mu_k \gamma \varepsilon_{t_0+k-j}^{pc} &= 0 \text{ for } k \geq 1 \text{ and } j \in \{0, ..., k-1\}, \\
2g_0 - \mu_0 &= 0, \\
2\delta^k g_k - \mu_k + \beta\mu_{k-1} &= 0 \text{ for } k \geq 1, \\
2\delta^k \lambda h_k + \gamma\mu_k &= 0 \text{ for } k \geq 0,
\end{aligned}$$

and the Phillips curve considered at all dates provides the following four additional equations:

$$\begin{aligned}
\beta a_{k+1}^k - a_k^k + \gamma c_k^k + 1 &= 0 \text{ for } k \geq 1, \\
\beta a_{k+1}^{k-j} - a_k^{k-j} + \gamma c_k^{k-j} &= 0 \text{ for } k \geq 2 \text{ and } j \in \{1, ..., k-1\}, \\
\beta g_1 - g_0 + \gamma h_0 + \varepsilon_{t_0}^{pc} &= 0, \\
\beta g_{k+1} - g_k + \gamma h_k &= 0 \text{ for } k \geq 1.
\end{aligned}$$

These ten equations are then easily shown, through similar computations, to lead to the same results for Δp, y and r as in the first case.

D Characterization of the adequate interest rate rules

An adequate interest rate rule is defined as an interest rate rule which ensures strong local equilibrium determinacy and which makes the strong-locally unique equilibrium selected coincide with the desired equilibrium. We first look for the interest rate rules which ensure strong local equilibrium determinacy, and show that they are necessarily forward-looking in a well-defined manner. We then look more precisely for the interest rate rules which make the strong-locally unique equilibrium selected coincide with the desired equilibrium, and show that they form a vectorial space whose dimension depends on the desired equilibrium considered.

Equations (3), (4) and (5), taken at date t and in expectations $E_t\{.\}$ at dates $t+k$ for $k \geq 1$, are equivalent to the following five equations:

$$y_t = \frac{1}{\gamma}\left[-\varepsilon_t^{pc} + \Delta p_t - \beta E_t\{\Delta p_{t+1}\}\right], \tag{12}$$

$$E_t\left\{y_{t+k}\right\} = \frac{1}{\gamma}\left[E_t\left\{\Delta p_{t+k}\right\} - \beta E_t\left\{\Delta p_{t+k+1}\right\}\right] \text{ for } k \geq 1, \tag{13}$$

$$r_t = \frac{1}{\gamma\eta}\left[\gamma\varepsilon_t^{is} + \varepsilon_t^{pc} - \Delta p_t + (1+\beta+\gamma\eta) E_t\left\{\Delta p_{t+1}\right\} - \beta E_t\left\{\Delta p_{t+2}\right\}\right], \tag{14}$$

$$\begin{aligned} E_t\left\{r_{t+k}\right\} &= \frac{1}{\gamma\eta}\left[-E_t\left\{\Delta p_{t+k}\right\} + (1+\beta+\gamma\eta) E_t\left\{\Delta p_{t+k+1}\right\}\right. \\ &\quad \left. -\beta E_t\left\{\Delta p_{t+k+2}\right\}\right] \text{ for } k \geq 1 , \end{aligned} \tag{15}$$

and the equation noted (16) corresponding to the interest rate rule (3) taken at date t and in expectations $E_t\{.\}$ at dates $t+k$ for $k \geq 1$, where the terms y_t, $E_t\{y_{t+j}\}$ for $j \geq 1$, r_t and $E_t\{r_{t+j}\}$ for $j \geq 1$ are expressed as in (12), (13), (14) and (15).

Given the block-recursivity of this system of equations, Δp_t, $E_t\{\Delta p_{t+k}\}$ for $k \geq 1$, y_t, $E_t\{y_{t+k}\}$ for $k \geq 1$, r_t and $E_t\{r_{t+k}\}$ for $k \geq 1$ will be uniquely determined, that is to say strong local equilibrium determinacy will be ensured, if and only if Δp_t and $E_t\{\Delta p_{t+k}\}$ for $k \geq 1$ are themselves uniquely determined by (16) taken at date t and in expectations $E_t\{.\}$ at dates $t+k$ for $k \geq 1$. Now, this equation considered in expectations $E_t\{.\}$ at dates $t+k$ for $k \geq N_1+1$ corresponds to a time-invariant linear recurrence equation on the expected future inflation rates:

$$\forall k \geq N_1 + 1, \quad \sum_{i=-N_1}^{N_2+2} m_i E_t\left\{\Delta p_{t+k+i}\right\} = 0,$$

where $m_i \in \mathbb{R}$ for $i \in \{-N_1, ..., N_2+2\}$. For the interest rate rule to ensure strong local determinacy, there must naturally exist an $i \in \{-N_1, ..., N_2+2\}$ such that $m_i \neq 0$, so that we can define $M \equiv \max\left(i \in \{-N_1, ..., N_2+2\},\ m_i \neq 0\right)$. Given this recurrence equation, strong local equilibrium determinacy will thus be ensured if and only if Δp_t and (if $N_1 + M \geq 1$) $E_t\{\Delta p_{t+k}\}$ for $1 \leq k \leq N_1 + M$ are uniquely determined by (16) taken at date t and (if $N_1 \geq 1$) in expectations $E_t\{.\}$ at dates $t+k$ for $1 \leq k \leq N_1$. For these $N_1 + M + 1$ unknowns to be uniquely determined by these $N_1 + 1$ linear equations, we must have $M \leq 0$.

Given (13) and (15), $M \leq 0$ can only happen if the forward-looking part of the interest rate rule (3), namely $\sum_{i=1}^{N_2} a_i E_t\{\Delta p_{t+i}\} + \sum_{i=1}^{N_2} b_i E_t\{y_{t+i}\} + \sum_{i=1}^{N_2} c_i E_t\{r_{t+i}\}$, amounts to $\frac{1+\beta+\gamma\eta+\beta\eta b_0}{\gamma\eta} E_t\{\Delta p_{t+1}\} - \frac{\beta}{\gamma\eta} E_t\{\Delta p_{t+2}\}$. This forward-looking part is characterized *modulo* the IS equation and the Phillips curve, by which we mean that there are an infinity of distinct though equivalent expressions for this forward-looking part which are linked to each other through the IS equation or the Phillips curve. If $b_0 = -\frac{\beta+\gamma\eta}{\beta\eta}$ for instance, this forward-looking part can be written $\frac{1}{\eta} E_t\{y_{t+1}\}$, or equivalently $\frac{1}{\gamma\eta} E_t\{\Delta p_{t+1}\} - \frac{\beta}{\gamma\eta} E_t\{\Delta p_{t+2}\}$, or still equivalently $\frac{1}{\eta} E_t\{y_{t+2}\} - E_t\{r_{t+1}\} + E_t\{\Delta p_{t+2}\}$, *etc.*

Having characterized the forward-looking part of the interest rate rule, we now turn to its coefficients a_i, b_i, d_i, f_i for $i \in \{-N_1, ..., 0\}$ and (if $N_1 \geq 1$) c_i for $i \in \{-N_1, ..., -1\}$. The interest rate rule will make the strong-locally unique equilibrium selected coincide with the desired equilibrium if and only if these $5N_1 + 4$ coefficients satisfy a certain number of linear constraints. Because of the linearity of these constraints, the set $\{r - r_0\}$ of adequate interest rate rules relatively to a given benchmark adequate interest rate rule r_0 (rather than the set $\{r\}$ of adequate interest rate rules *per se*) will be a vectorial space.

Whatever the desired equilibrium considered, a number $2(N_1+1)$ of constraints come from (16) taken at date t and (if $N_1 \geq 1$) in expectations $E_t\{.\}$ at dates $t+k$ for $1 \leq k \leq N_1$, as the coefficients should indeed be such that Δp_t and (if $N_1 \geq 1$) $E_t\{\Delta p_{t+1}\}$, ..., $E_t\{\Delta p_{t+N_1}\}$ depend on the two shocks ε_t^{pc} and ε_t^{is} in the desired way. For instance, whether under discretion or under commitment we must have $f_0 = \frac{-1}{\eta}$ and (if $N_1 \geq 1$) $c_i = \frac{f_i}{\eta}$ for $i \in \{-N_1, ..., -1\}$ to get the optimal impulse-response function of Δp with respect to ε^{is}.

If the desired equilibrium considered is the optimal equilibrium under commitment (whether for a closed economy or a small open economy with a flexible exchange rate) or the fixed exchange rate equilibrium (for a small open economy), then one additional constraint comes from the fact that z or x must be a root of the characteristic polynomial of the recurrence equation on the expected future inflation rates, given the results obtained in subsections 2.2 and 3.3. Besides, this requirement implies $N_1 \geq 1$, for the degree of the characteristic polynomial to be at least one, while N_1 can be nil when the desired equilibrium considered is the optimal equilibrium under discretion (whether for a closed economy or a small open economy with a flexible exchange rate).

As a consequence, the set of adequate interest rate rules is a $3N_1+2$-dimension vectorial space with $N_1 \geq 0$ when the desired equilibrium considered is the optimal equilibrium under discretion (whether for a closed economy or a small open economy with a flexible exchange rate), and a $3N_1 + 1$-dimension vectorial space with $N_1 \geq 1$ when the desired equilibrium considered is the optimal equilibrium under commitment (whether for a closed economy or a small open economy with a flexible exchange rate) or the fixed exchange rate equilibrium (for a small open economy).

Note finally that any given impulse-response function of Δp with respect to ε^{pc} and ε^{is} (provided that this impulse-response function is that of an ARMA process with ε^{pc} and ε^{is} as white noises) can be implemented by a suitable choice of N_1 and the coefficients a_i, b_i, d_i, f_i for $i \in \{-N_1, ..., 0\}$ and (if $N_1 \geq 1$) c_i for $i \in \{-N_1, ..., -1\}$, since by this choice we can impose any set of initial conditions for the recurrence equation on the expected future inflation rates and any set of roots for the characteristic polynomial of this recurrence equation. As a consequence, the economy is "controllable" in the sense given to this term in subsection 2.3.

E Determination of the fixed exchange rate equilibrium

The fixity of the nominal exchange rate ($\Delta e = 0$) implies, *via* the non-covered interest rate parity, that the nominal interest rate should keep constantly equal to its steady state value ($r = 0$). Now consider a given date t. The IS equation and the Phillips curve taken in expectations $E_t\{.\}$ at dates $t+n$ for $n \geq 1$, together with the condition $r = 0$, imply that the expected future inflation rates $E_t\{\Delta p_{t+n}\}$ for $n \geq 1$ follow a recurrence equation whose second-order characteristic polynomial has x and x' for roots, where

$$0 < x = \frac{1+\beta+\gamma\eta - \sqrt{(1+\beta+\gamma\eta)^2 - 4\beta}}{2\beta} < 1$$

$$\text{and } x' = \frac{1+\beta+\gamma\eta + \sqrt{(1+\beta+\gamma\eta)^2 - 4\beta}}{2\beta} > 1.$$

Now the long-run PPP condition and the fixity of the nominal exchange rate imply that $E_t\{\Delta p_{t+n}\}$ cannot diverge as $n \to +\infty$, so that we get: $E_t\{\Delta p_{t+n+1}\} = xE_t\{\Delta p_{t+n}\}$ for $n \geq 1$. The long-run PPP condition then becomes $p_{t-1} + \Delta p_t + \frac{1}{1-x}E_t\{\Delta p_{t+1}\} = 0$. Finally, the IS equation and the Phillips curve taken at date t, together with the condition $r_t = 0$, imply $\Delta p_t - \frac{1}{x}E_t\{\Delta p_{t+1}\} = \gamma\varepsilon_t^{is} + \varepsilon_t^{pc}$. Because p_{t-1} does not depend on ε_t^{is} and ε_t^{pc} and because they hold whatever the shocks ε_t^{is} and ε_t^{pc}, these three equations imply that:

$$\begin{cases} g_{t+n+1}^{n+1} = x g_{t+n}^{n} \text{ for } n \geq 1 \\ g_t^0 + \frac{1}{1-x} g_{t+1}^1 = 0 \\ g_t^0 - \frac{1}{x} g_{t+1}^1 = 1 \end{cases} \quad \text{and} \quad \begin{cases} h_{t+n+1}^{n+1} = x h_{t+n}^{n} \text{ for } n \geq 1 \\ h_t^0 + \frac{1}{1-x} h_{t+1}^1 = 0 \\ h_t^0 - \frac{1}{x} h_{t+1}^1 = \gamma \end{cases}.$$

These systems enable us to get g_{t+n}^n and h_{t+n}^n for $n \geq 0$. Because the results obtained do not depend on t, we have $g_t^n = g_{t+n}^n$ and $h_t^n = h_{t+n}^n$ for $n \geq 0$, so that we finally get Δp_t and (residually with the Phillips curve) y_t as displayed in subsection 3.3.