Disclosure of research results *

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Abstract

In a model where a sender provides verifiable information to a receiver, in order to influence her, we study how strategic reporting affects the incentives to search for information. We show that in equilibrium the sender reveals all the information he obtains and conducts more research when his research effort is not observed by the receiver. In this setting government subsidization of research can be welfare reducing, in particular if the bias of the sender is large or if research quickly provides conclusive results. We also find that when two senders with opposing biases compete, they conduct less research than if they were alone reporting. Finally we study situations where the receiver faces uncertainty about the preferences of the sender or the research technology he uses.

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1 Introduction

Interest groups provide information to decision makers in a variety of ways: they testify in congressional hearings, prepare reports and proposals, have informal contacts with policy makers. All of these activities require gathering precise information: although most organizations delegate this effort, some perform their own research. In both settings, an explicit decision has to be made as to the quantity of research to perform. This raises a number of questions. How does the strategic reporting of results affect the research incentives? Do more biased organizations conduct more or less research? How does competition between groups with conflicting objectives impact the research effort? We will answer all these questions in a general theoretical framework, applicable to other settings.

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Situations where a sender communicates information to a receiver, who will take a decision affecting both their welfare, have received a lot of attention. The cheap talk literature, starting with the seminal paper of Crawford and Sobel (1982), covers cases where the reports by the sender are non verifiable whereas what is referred to as the disclosure literature studies cases where reports are verifiable (Milgrom (1981), Milgrom and Roberts (1986)).

However, in most situations the information transmitted by the sender is not readily available. We will study in this article the interactions between research incentives and disclosure. We view research as having two essential functions: it of course allows better knowledge of the underlying state but also provides evidence that can be used to influence the decision maker. To illustrate the second function, suppose a policy maker expects a lobby group to run 4 experiments on a phenomenon of interest. It might be optimal for the interest group to incur the cost of 6 experiments in order to report only the 4 most favourable results.

In a model where the sender first decides on the quantity of research, then selects what verifiable reports to send to a receiver who decides on the policy, we obtain a series of results. We first show that in the reporting phase, on the equilibrium path, the sender reveals all the information he obtained. The intuition is the same as for Milgrom's result (1981): if some information is withheld, the receiver considers the worst case scenario. However, the total quantity of research performed is not known ex ante as in Milgrom's paper, but is known in equilibrium. For the research phase, we show that the sender always conducts more research when the receiver cannot observe the amount of research performed. The intuition is, in the unobservable case, that if the quantity of research the receiver expects is too low, the sender will have an incentive to search further, obtain more favourable evidence and hide the unfavourable one and thus mislead the receiver into believing the state is higher. The receiver understands these incentives and knows that extra research will be conducted to mislead her, up to the point where the marginal benefits equal the costs. This accounts for the larger amounts spent on research in the unobservable case. In these types of situations where research results are used to convince, we show that subsidies for research are not always optimal: in cases where the sender is strongly biased or uses a research technology that is quickly informative, socially wasteful resources are spent on research. In such situations, taxing research expenditures, would be socially optimal. We also show that when two senders with opposing biases compete to provide information, they conduct less research than if they were each alone reporting. The intuition is that, because in equilibrium the other sender is revealing his information, the marginal benefits from searching more and getting more positive signals to mislead the receiver are smaller.

There are numerous applications of this setup. A similar situation to that of the interest groups is the provision of information by congressional committees to the floor. Another application is the case of a lawyer searching for evidence of her client's innocence. In this case, the search for information is often delegated to private firms. We will also discuss the case of pharmaceutical companies reporting results of tests on their products. Finally, in the literature on media bias, economists have often considered journalists as having a desire to influence opinions and policies (Baron (2004)). Under this assumption, our setup also applies to journalists gathering several pieces of evidence on a particular question.

As suggested previously, the disclosure literature has examined, in situations where reports are verifiable, how much of the information held by the sender will be revealed in his report. Milgrom (1981) shows that, when the decision maker knows the quantity of information held by the sender, in every sequential equilibrium of the game, we observe full disclosure. The idea is that if some information is withheld, the decision maker considers the worst case scenario. Starting from this seminal paper, several authors have tried to relax several assumptions of the model to see if this surprising result still holds (Verrecchia (1983), Dye (1985)). All these authors consider the information as given and one of our contribution is to make it endogenous. We will discuss throughout this article how studying the interaction with the research phase affects disclosure.

We also study in our paper how competition affects research incentives. Dewatripont and Tirole (1999) study competition in a different setting: they examine whether a system with competing advocates dominates one with a single non partisan decision maker. In their model, contracts specify payments as a function of information obtained. They show that when evidence is not concealable, the advocacy system is strictly optimal: it gives incentives for information gathering without abandoning rents. The focus of their paper is different and they therefore do not model research in the same way. We believe the model of research activity we propose, is one of the contributions of our paper.

In most of the literature, research is modelled as a binary decision: either the sender does not search or he searches and incurs a cost c to obtain a signal (that can be continuous). However, none of these models allow us to capture the main idea of this article: more research could be done by the sender not only to obtain more precise information but also to be able to show more results in his favour. We therefore propose a model, where the outcome of research is a series of infinitesimal positive or negative signals. These can be thought of as a series of small experiments. A positive signal corresponds to a small experiment that gave positive results: for example it could be that a drug does work on a certain type of patient. A negative signal in that context would mean that the drug does not work. The aggregate amount of positive signals gives information on the state and allows the agents to update their priors. In this context, increasing Q, the measurable quantity of research, augments the precision but also produces more positive signals that can be used to mislead the decision maker.

It is worth mentioning a few other articles. Shavell (1994) looks at whether buyers or sellers of a good have incentives to search for information about the value of that good.

Sinclair-Desgagne and Gozlan (2003) examine the interaction between a potential polluter and a stakeholder in a situation where disclosure is not mandatory. The firm in their model has full information and can make a continuous costly decision on the precision of the report he submits to the stakeholder. One of the striking conclusion is that, because precision is costly, even a 'clean' firm may decide to provide a partially informative report that could not even differentiate it from a dirty one. Finally Persico (2000) searches for the optimal voting rule in a panel of decision makers. A good voting rule must aggregate information efficiently but also induce the decision makers to search for information. He finds that a voting rule that requires large plurality can only be optimal if the information obtained by each decision maker after a search is sufficiently precise. Indeed, they need a big incentive to search as the probability of being pivotal is low.

Our paper is organized as follows: in section 2 we present the model. In section 3 we study a limit case where research is immediately informative. In this situation, even an infinitesimal amount of research will provide full knowledge of the state. This extreme case provides us with most of the intuitions of the model. In section 4 we consider the general case where the outcome of research is subject to uncertainty. In section 5 we study situations where two senders with opposing views compete to provide information. Finally in section 6 we suppose the receiver faces different types of uncertainty on the bias of the sender or the exact research technology he uses.

2 The Model

We describe in this section the details of the model that will be used throughout this article.

2.1 Preferences

The state of interest to both the sender and the receiver is noted θ.
The receiver sets the policy p.

As in the cheap talk literature, the instantaneous utilities, given a state θ and a policy p are:

For the receiver $u_r(p, \theta) = -(p-\theta)^2$.
For the sender $u_s(p, \theta) = -(p-\theta-\delta)^2$.

The parameter δ describes the extent of the bias. Indeed, if the state is known, the receiver's preferred policy p equals θ, whereas the sender would ideally want $p = \theta + \delta$.

The receiver will choose the policy p that maximizes her expected utility given the reports of the sender. Given the utility specification, the policy set is $p = E[\theta|r]$.

2.2 Research Technology

The sender will be useful to the extent he provides information on the state θ through research he conducts. The sender and the receiver share a common prior on the state: $f(\theta)$ with support on $[0, 1]$.

The outcome of research is modelled as a series of infinitesimal positive and negative signals. This can be thought of as a series of small experiments, of small pieces of evidence. The positive signals indicate a higher value for the state whereas the negative signals indicate a lower value.

The sender decides on the quantity Q of research he performs. This research is costly, the unit cost is noted C.

If the sender conducts an amount Q of research, he will obtain x positive signals given by the distribution $f(x|\theta, Q)$. This aggregate amount of positive signals x thus provides information on the value of θ: the sender can derive a posterior distribution on the state.

Note that if the receiver knows the quantity Q of research performed and the amount x of positive signals obtained, the negative signals have no informational value. However, we will compare in the article cases where Q is observable to others where it is not. In a situation where Q is not observed, the quantity of negative signals does provide information. Note also that we could express everything in terms of quantity of negative signals, this would be exactly symmetrical.

The model chosen to represent the research process allows us to measure explicitly the amount of research performed and to conduct comparative statics on this variable. Viewing research as a series of positive or negative signals also enables us to study the exact opportunities senders have to mislead receivers: by conducting more research, a sender can accumulate more positive signals and can replace some of the negative signals he would have reported, by this more favourable evidence.

2.3 Reporting

Once the research is performed, the sender reports a subset of the evidence to the receiver. We suppose throughout the article that the evidence is verifiable.

We denote r the quantity of positive signals the sender reports. The verifiability of the information imposes the constraint: $r < x$ (the sender cannot fabricate information and cannot therefore report more positive signals than he obtained).

All these infinitesimal signals are independent pieces of information. So, the sender can choose to report any selection of those. If he obtained 1 positive, then 2 negative and finally another positive, he can choose to report only the 2 positive.

2.4 Timing of the model

To summarize, the timing of the model is the following:

1) The sender decides on Q, quantity of research to be performed
2) The sender conducts Q research and obtains signals
3) The sender reports a subset or all of these signals
4) The receiver sets the policy p

3 A useful benchmark

Different fields require different amounts of research to reach the same degree of certainty. For example research on the impacts of climate change, in spite of the amounts already performed, remains very imprecise. On the other hand testing a new drug for specific effects can lead to quick conclusions. In this benchmark, we consider the limit case where research is immediately informative.

We therefore suppose that the distribution of signals takes a particular form $f(x|\theta, Q) = 1_{x=\theta Q}$: if the sender conducts an amount Q of research, he knows he will obtain θQ positive signals and will therefore be able to infer the exact value of the state. We described in the introduction two functions of research: achieve better knowledge of the state and influence the receiver. This benchmark case abstracts from the first function to concentrate on the second one. It will provide useful intuitions to understand better in the general setup of section 4, the strategic motive of research.

As in the rest of the article, we will compare the situation where the amount of research performed by the sender is not observed by the receiver, to the observable case and to the social optimum. In this benchmark case, if the receiver observes the amount of research performed by the sender, only an infinitesimal amount will be conducted, enough to reveal the exact state. This is also the social optimum as more research does not bring better information and is therefore a social waste. We describe in the following proposition the unobservable case.

There are a lot of pure strategy Bayesian Nash equilibria depending on the beliefs of the receiver off the equilibrium path (when the sender reveals he has conducted more research than the equilibrium amount Q^* by reporting $x > Q^*$ positive signals). However they all share common properties given by the following proposition.

Proposition 1 *All Pure Strategy Bayesian Nash Equilibria of the game where the quantity of research is not observable are characterized by:*

a) No information is withheld by the sender on the equilibrium path.

b) The amount of research performed is $Q^* = \frac{2\delta E(\theta)}{C}$.

Proof:

For clarity concerns, I make a simplifying assumption: the bias of the sender in favor of a higher policy is large ($\delta > 1$ for instance). Such an agent would never withhold information that could lead the receiver to set a higher policy. His optimal reporting strategy is therefore to always report all the positive signals he obtained. The reports takes the very simple form r=x. I show in the appendix how the result generalizes for all values of the bias.

Let the beliefs off the equilibrium path (when $x > Q^*$ positive signals are reported) such that the policy set by the receiver is h(x), where h is assumed differentiable and the derivative is bounded.

The strategies of the receiver are therefore:

- If the sender reports $r \leq Q^*$ (on the equilibrium path), the receiver believes the report and therefore sets the policy $E[\theta|r] = \frac{r}{Q*}$.
- If the sender reports $r > Q^*$ (off the equilibrium path), the receiver sets policy h(r).

If the sender performs an amount Q' of research, he obtains $\theta Q'$ positive signals, his reporting strategy is:

- $r = \theta Q'$ if $\theta Q' < Q^*$
- $\tilde{r}(\theta Q')$ if $\theta Q' > Q^*$ (where $\tilde{r}$ is an optimal reporting strategy given the policy h set by the receiver off the equilibrium path).

Note that the optimal reporting off the equilibrium path $\tilde{r}$ could imply hiding some positive signals.

The reporting strategy described here is clearly a best response to the receiver's strategy. We now have to determine the optimal research strategy: the sender will choose Q' to maximize

$$max_{Q'} - \int_0^{min(Q^*/Q',1)} [\frac{\theta Q'}{Q^*} - \theta - \delta]^2 f(\theta) d\theta$$
$$- \int_{min(Q^*/Q',1)}^1 [h(\tilde{r}(\theta Q')) - \theta - \delta]^2 f(\theta) d\theta - CQ'$$

For $Q' > Q^*$, The FOC is:

$$C = - \int_0^{Q^*/Q'} \frac{2\theta}{Q^*} [\frac{\theta Q'}{Q^*} - \theta - \delta] f(\theta) d\theta$$
$$- \int_{Q^*/Q'}^1 2\ \theta\ \tilde{r}'(\theta Q')\ h'(\tilde{r}(\theta Q'))\ [h(\tilde{r}(\theta Q')) - \theta - \delta] f(\theta) d\theta$$
$$+ (Q^*/Q')^2 (1 - \theta - \delta)^2 - (Q^*/Q')^2 (h(\tilde{r}(Q^*) - \theta - \delta)^2$$

For Q^* to be an equilibrium, we need this first order condition to be verified at Q^*, we therefore take the limit as $Q' \to Q^*$.

The first term is continuous in Q' and therefore the limit is:
$- \int_0^1 \frac{2\theta}{Q^*} [\theta - \theta - \delta] f(\theta) d\theta = \frac{2\delta E(\theta)}{Q^*}$

The integrand of the second term is bounded, given the assumption on h, and therefore when Q' converges to Q^*, this term converges to 0.

Finally, the third term equals 0, as $h(\tilde{r}(Q^*) = 1$. Indeed this is on the equilibrium path, therefore the truth Q^* is reported, leading to a policy of 1.

The same type of argument can be made for $Q' < Q^*$, leading to the same result. Therefore we find that Q^* is characterized by:

$$Q^* = \frac{2\delta E(\theta)}{C} \tag{1}$$

Furthermore, the FOC are sufficient as the second order derivative is negative at the optimum $V''(Q^*) = -\frac{1}{(Q^*)^2}$.

The arguments presented here were general and are therefore valid for any belief function h, off the equilibrium path. We have proved result b).
We see that on the equilibrium path, the optimal reporting strategy is to reveal all the information obtained. We have therefore shown result a). ■

Result a) of that proposition can be linked to the results of Milgrom (1981) and Milgrom and Roberts (1986). In equilibrium, we see that the receiver knows, though she can't observe, the quantity of information obtained by the sender and therefore Milgrom's idea applies. This even suggests that if our infinitesimal signals took a more complicated form (not just binary signals), we would still get full revelation because, as in Milgrom's article, the only possible equilibrium belief of the receiver would be one of extreme skepticism. Since Milgrom's seminal paper, several authors, in particular in the accounting literature (Verrecchia (1983,2001), Dye (1985,2001)), have shown how relaxing certain assumptions can weaken this result of full disclosure. We will examine this point in more details in section 6 where we modify some of the conditions of the model.

If the decision maker could observe the quantity of research conducted by the sender, the latter would only perform an infinitesimal amount, which would be sufficient to reveal the exact value of the state. Result b) therefore implies that $Q^* = \frac{\delta E(\theta)}{C}$ is the extra amount of research conducted due to the fact that research is unobservable. The intuition is that, if the quantity of research the receiver expects is too low, the sender will have an incentive to search further and obtain more positive signals. He will then still report the quantity of signals the receiver expects, but replace some negative signals by positive ones and thus mislead the receiver into believing the state is higher. The receiver understands these incentives and knows that extra research will be conducted to mislead her, up to the point where the marginal benefits equal the costs.

The results in Proposition 1 are also linked to results obtained by Shavell (1994). The focus of the two papers and the mechanism studied differ significantly. Shavell examines whether it is optimal for sellers to make expenditures to acquire information on the value of a good. The results depend on the assumption that there are different types of sellers having different costs to acquire information. Furthermore research is modelled as a binary decision. In this article, we want to study explicitly the continuous decision of how much research to perform and how this leads to different reporting strategies.

We observe that the difference between the two research efforts, $Q^* - 0 = \frac{\delta E(\theta)}{C}$, is decreasing in the unit cost of research, as we would expect. It is also increasing in the bias: therefore a sender with extreme views will have to spend more on research activity than one with more moderate positions, for an identical result. This type of sender will therefore tend to favour other modes of action. This is a phenomenon we often observe with lobby groups such as environmental NGOs: the National Research Defence Council will tend to lobby directly decision maker whereas a group such as Greenpeace, with more extreme views, will favour public actions. The intuition is not that more extreme views tend to bias more their reports but that they have bigger incentives to conduct more research in order to mislead the receiver.

3.1 Correcting the inefficiencies

In the previous section we observed socially wasteful research was conducted due to the fact that the research effort was not observed by the receiver. In this section, we want to examine solutions to correct this inefficiency.

The first obvious solution is a policy that would render the research effort observable. We address this in section 3.3.1.

The other solution would be a tax. We want to be very explicit here: we are placing ourselves throughout this article in a very specific context where research is used not only to attain better knowledge of the phenomenon, but also to influence a decision maker. We do not make any statements on research in general.

We examine in this section two types of taxes:

a) A tax on the quantity of research.

b) A tax on the reports.

We want to emphasize that the tax in case a) does not make the research effort observable. The tax declaration submitted to the IRS is not available to the public, and even if it were it would be hard to allocate an aggregate amount to different issues. Furthermore, an alternative solution could be to tax the research providers.

3.1.1 Mandatory disclosure

The New York Times on June 15 2004, reported that editors of some of the top journal in medical science were considering a proposal 'that would require drug makers to register clinical trials at their start in a public database in order for results, whether successful or not, to be later considered for publication'. This proposal was put forward after concerns that drug companies concentrate on successful trials and thus withhold information from the public. There is however another effect that they do not consider: this measure makes the amount of research performed observable and thus eliminates wasteful research in cases where research is quickly informative (benchmark case).

Mandatory disclosure seems an appropriate solution when the relation between the sender and the receiver is well established and formalized. This is the case for pharmaceutical companies and medical journals. It is also the case for lawyers and juries. You could imagine a similar system where lawyers would need to declare at the start of the case funds they spend on searching for evidence to be able to use the results during the trial.

However it seems hard to implement for interest groups and decision makers where the relation is much more informal. This is why we examine in the next section the use of taxes.

3.1.2 Tax on the quantity of research

Let us consider a tax τ on the quantity of research. The unit cost of research now becomes $(C + \tau)$.

Proposition 2 *In all Pure Strategy Bayesian Nash Equilibria:*
a) The sender performs an amount of research $Q^*(\tau) = \frac{\delta E(\theta)}{C+\tau}$.
b) The government can raise revenue with a tax without changing the welfare of any of the parties.

Proof:
a) The unit cost of research C just becomes $(C + \tau)$ and we can apply Prop1.
b) We have:

- The final policy implemented by the receiver is the same: the sender reports truthfully and research reveals the exact state θ.
- The cost of research for the sender remains the same (equal to $\delta E(\theta)$), so his utility is unchanged.
- The government raises revenue $\tau Q^*(\tau)$. ■

This tax is the "ideal tax": the government raises revenue leaving all agents utilities unchanged. This is true however in the benchmark case, i.e in a limit case. We will determine in section 4 if the results are still valid in intermediate cases. One of the limitations of this tax is that it would have to specify very clearly what type of research it applies to: the idea is to limit excessive research conducted to influence a decision maker, not to hinder research in general. Alternatively, the desirability of this type of tax could be judged on the empirical mix between research used to convince and research purely used to attain a better knowledge of the state.

3.1.3 Tax on the reports

We study here the properties of a unit tax τ on the reports.

Proposition 3 *In all Pure Strategy Bayesian Nash Equilibria:*
a) The sender performs an amount of research $Q^*(\tau) = \frac{\delta E(\theta)}{C+\tau E(\theta)}$.

b) The government can raise revenue with this tax without changing the welfare of any of the parties.

Proof: see appendix.

We are again in this "ideal fiscal environment". This tax would however be harder to implement than the previous one and probably less politically acceptable as the tax rate τ would need to be bigger to raise the same revenue. On the other hand it has the virtue of targeting specifically the type of research we are concerned about: only the research used to convince is taxed in this situation.

4 General Setup

We wish to determine in this section if the results obtained in the benchmark case are still valid when we make the research technology more realistic. We now suppose that the outcome of research is subject to uncertainty. If the sender conducts an amount of research Q, the quantity of positive signals he obtains is given by the distribution $f(x|\theta, Q)$. We suppose that this density is differentiable with respect to Q.

We use the following notations:

- $g(x|Q)$ is the unconditional distribution of signals.
- $f(\theta|x, Q)$ is the posterior distribution of the state given the signals and the amount of research performed.
- $V[\theta|x, Q]$ is the variance of the posterior distribution of θ given the signals and the amount of research performed.

We make the following assumption:

$$\int_0^\infty V[\theta|x, Q]\ g(x|Q)dx \text{ is decreasing and convex in Q} \qquad \text{(A)}$$

Assumption (A) means that the more research is performed, the more precise its outcome becomes on average: on average the variance of the posterior on θ decreases with Q. It also implies that the marginal gains in precision are decreasing with Q (ie convexity). This is a reasonable property for the results of experiments: the more research we conduct, the better informed we become, but the marginal gains decrease as our understanding improves.

It turns out this assumption guarantees the uniqueness of the solution in the case where the research effort is observable:

Proposition 4 *Under assumption (A), in the unique pure strategy equilibrium when Q is* *observed by the receiver, the amount of research performed Q_0 is solution to:*

$$C = -\frac{\partial}{\partial Q'}[\int_0^\infty \; V[\theta|x, Q'] \; g(x|Q')dx] \;|_{Q'=Q_0}$$

Proof: see appendix.

If the interest group's research activity was observed by the policy maker, it would search up to the point where the marginal costs equals the benefits from getting better information (benefits from decreasing the variance of the posterior).

We now turn to the case where the quantity of research performed by the sender is unobservable. In this case, he has to make 2 strategic decisions:

1) The quantity of research to perform.
2) What to report if x positive signals are obtained.

Let's examine the second consideration.
If he does an amount of research Q' and obtains x signals, his desired policy is $E[\theta|x, Q'] + \delta$.
Therefore, if the receiver's belief is that an amount Q^* of research was conducted, the sender's ideal report would be r such that $E[\theta|r, Q^*] = E[\theta|x, Q'] + \delta$.
The only constraint on the report is that $r \leq x$.

Because of this constraint, the sender cannot always obtain his preferred policy. The intervals on which this ideal policy is not attainable are denoted $A_k = [x_{k1}, x_{k2}]$ $k \in 1, .., K$. In these intervals, the constraint is binding and therefore, the sender reports r=x.

At the boundaries of the interval, x_{k1} and x_{k2}, we have $E[\theta|x, Q^*] = E[\theta|x, Q'] + \delta$ (the ideal policy becomes attainable).

To illustrate this, suppose $E[\theta|x, Q] = \frac{x}{Q}$. The ideal report is therefore characterized by $\frac{r}{Q^*} = \frac{x}{Q'} + \delta$. However, we have the constraint $r = \frac{Q^*}{Q'}x + \delta \leq x$. Therefore, on $A_k = [0, \frac{\delta}{1-\frac{Q^*}{Q'}}]$, the ideal policy is not attainable whereas on $(A_k)^c = [\frac{\delta}{1-\frac{Q^*}{Q'}}, +\infty]$ it is.

Not that this is an example where there is only 1 interval of each type, but that there could exist multiple intervals of each type if the research technology was less smooth.

Let us now turn to the research phase. The sender can only obtain his preferred policy in $A_k)^c$. He therefore chooses Q' in order to:

$$max_{Q'} \;\; -\int_0^1 \int_{\bigcup A_k} (E[\theta|x, Q^*] - \theta - \delta)^2 \; f(x|\theta, Q')f(\theta) \; d\theta \; dx$$

$$-\int_0^1 \int_{(\bigcup A_k)^c} (E[\theta|x,Q'] + \delta - \theta - \delta)^2 \ f(x|\theta,Q') \ f(\theta) \ d\theta \ dx \ \ - CQ'$$

The solution to this problem is characterized by the properties in Proposition 5:

Proposition 5 *When the amount of research is unobservable, the unique equilibrium is characterized by an amount of research Q^*, solution to:*

$$C = -\frac{\partial}{\partial Q'}[\int_0^\infty \ V[\theta|x,Q'] \ g(x|Q')dx] \ |_{Q'=Q^*}$$
$$-2\delta \int_0^1 \int_0^\infty \ \frac{\partial}{\partial Q'}[E(\theta|x,Q')]|_{Q'=Q^*} \ f(x|\theta,Q^*) \ f(\theta)dx \ d\theta$$

Proof: see appendix

The first term in this equation represents the marginal benefits from obtaining a better knowledge of the state. This gain did not exist in the benchmark case where research was immediately perfectly informative. The second term represents the potential benefits from conducting more research in order to mislead the receiver: it is the expected gain from the fact that the receiver sets a higher policy because more positive signals are reported. Note that if you consider the values of the benchmark case ($f(x|\theta,Q) = 1_{x=thetaQ}$ and $E(\theta|x,Q) = \frac{x}{Q}$), proposition 5 confirms the result of proposition 1, i.e the second term equals $\frac{2\delta E(\theta)}{Q}$. In equilibrium marginal benefits from more research have to be equal to marginal costs.

Corollary: $Q_O < Q^*$, more research is performed when the research effort is unobservable.

Proof: $\frac{\partial E}{\partial Q'}(\theta|x,Q')$ is negative because the number of positive signals is weakly increasing in the quantity of research performed.
$\int_0^\infty V[\theta|x,Q']g(x|Q')dx$ is decreasing in Q' according to the hypothesis made on the variance. Therefore, we have $Q_0 < Q^*$. ■

As in the benchmark model, we find that the interest group conducts more research when the policy maker cannot observe its research effort.

4.1 Should we tax or subsidize research?

We reevaluate in this section the question whether research should be subsidized or taxed.

Proposition 6 *The optimal tax (or subsidy) in the case of lump sum redistribution is given by:*

$$\tau^* \; = \; -\,2\delta \int_0^1 \int_0^\infty \; \frac{\partial}{\partial Q'}[E(\theta|x,Q')]|_{Q'=Q_W} \; f(x|\theta,Q_W) \; f(\theta)dx \; d\theta \; - \; \frac{C}{2}$$

where Q_W *is the socially optimal amount of research given by:*

$$\frac{C}{2} = -\frac{\partial}{\partial Q'}[\int_0^\infty \; V[\theta|x,Q'] \; g(x|Q')dx] \; |_{Q'=Q_W}$$

Proof: see appendix

The first term represents the positive benefits from potentially misleading the receiver (as in proposition 5). The second term represents the cost of the extra research needed at the social optimum: the sender, when he decides on his research effort, ignores the positive effects of the information he gathers on the receiver's welfare. Therefore in the observable case, he performs a socially suboptimal amount of research ($Q_0 < Q_W$). The fact that the research effort is unobservable introduces a strategic consideration, described in the benchmark case and in proposition 5, that acts towards correcting this ignored externality. Whether it corrects partially, perfectly or excessively determines whether τ^* is a tax or a subsidy. Note that in the benchmark case, research always needed to be taxed as there was no externality to correct (in both the observable case and the social optimum, only an infinitesimal amount was needed).

Corollary: There exits δ^* such that $\forall \delta > \delta^*$, research should be taxed: $\tau^* > 0$.

This corollary states that for any research technology and for any values of the other parameters, if the bias of the sender is big enough, it is socially optimal to tax research. The intuition is that if the bias is too big, the strategic consideration overcorrects the ignored externality. Indeed, this externality is independent of the bias of the sender. In such a case the amount of research performed in the unobservable case is socially excessive. Overall, whether research should be taxed or subsidized becomes an empirical question: it will depend on the distribution of biases, costs and research technologies.

4.1.1 An example

We illustrate, in a specific example, how the tax varies with the different parameters of the model.

We suppose that:

The state follows a BETA distribution $\theta \sim BETA(\alpha, \beta)$.

Given the state and the quantity of research, the signal is a binomial $x \sim BIN(\theta, Q)$.

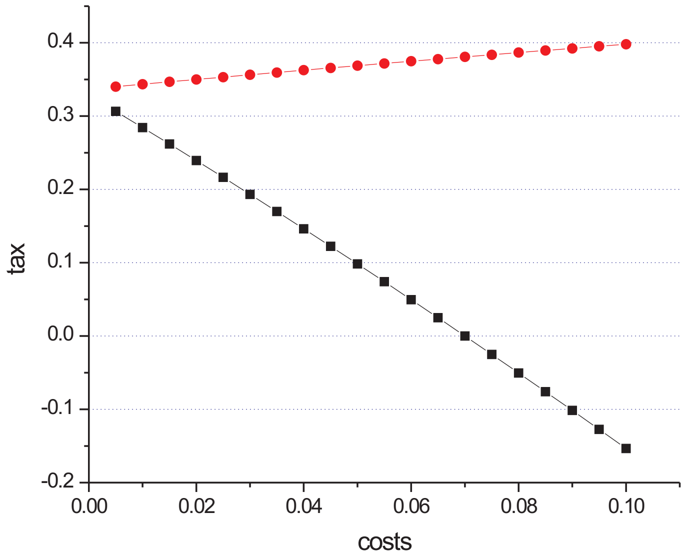

Figure 1: Variation of tax with costs C

Under these assumptions, we obtain:

The posterior distribution is $\theta|x \sim BETA(\alpha + x, \beta + Q - x)$.

The conditional expectation is: $E[\theta|x] = \frac{\alpha+x}{\alpha+\beta+Q}$

The conditional variance is: $V[\theta|x] = \frac{(\alpha+x)(\beta+Q-x)}{(\alpha+\beta+Q)^2(\alpha+\beta+Q+1}$

In this context, following Proposition 6, the tax or subsidy is $\tau = 2\delta\frac{\alpha(\alpha+\beta)+\alpha Q_W}{(\alpha+\beta)(\alpha+\beta+Q_W)^2} - \frac{C}{2}$

Given these preliminary results we examine how different aspects of the environment affect the choice between tax and subsidy in this particular example. We do not make claims on how these results generalize.

a) τ is increasing with the bias

We confirm the result obtained in the previous section.

b) τ is decreasing with β

β is a parameter linked to the variance of the prior. As the variance of the prior increases, Q_W, the socially optimal amount of research grows, as more information needs to be obtained. In this particular example the tax is decreasing in Q_W and in β. Therefore overall, as β increases, subsidies should look more and more attractive to a social planner.

c) For high values of the bias, τ is increasing in the unit cost of research C and for low values it decreases with C.

The first part of τ $2\delta\frac{\alpha(\alpha+\beta)+\alpha Q_W}{(\alpha+\beta)(\alpha+\beta+Q_W)^2}$ (ie the benefits from the strategic reporting) is decreasing with Q_W and therefore increasing with C. So, if the bias is big enough, this term dominates and the overall effect is that as the unit cost of research increases, subsidies should look less and less attractive to a social planner. We observe this in figure 1: the upper curve is the tax for a high bias ($\delta = 2$) and the lower one corresponds to a low bias ($\delta = 0.1$). This example shows that there is no monotonic link between the cost of research and the desirability of a subsidy scheme.

5 Competition between parties with opposing views

In this section, we study the case of two senders with opposing desires, gathering information and transmitting it strategically to the decision maker. We suppose that the first sender prefers a higher policy than the decision maker and the second favours a lower policy. In all the potential applications of this model that we described in the introduction, this seems to be an essential case to consider. If we want to study some aspect of lobbying for instance, it is important to consider the competition between two lobbies with opposing views. The natural conjecture is that competition tends to increase the incentives for research. We will show that this is not the case here, as competition decreases the marginal returns to attempting to mislead the receiver.

The utility function of sender 1 is given by $u_1 = -(p - \theta - \delta_1)^2$ and that of sender 2 is $u_2 = -(p - \theta - \delta_2)^2$ (with $\delta_1 > 0$ and $\delta_2 < 0$).

We present the results in the setup of section 3 and in the case where both senders have large biases. The results are the same in the case of small biases.

We will see in proposition 7 that, in equilibrium, both parties reveal all their information and therefore report the same state (research is perfectly informative). However, a lot of equilibria will exist depending on the beliefs of the receiver off the equilibrium path (when conflicting reports are given by the two senders). However, when we impose a reasonable restriction on these beliefs, all equilibria share a surprising property: less research is conducted by both senders than if they were left alone to communicate.

In this section we choose to describe the reports made by the senders as reports on the state. The actual reports are the aggregate amounts of positive and negative signals, but given a set of beliefs of the receiver, they are equivalent to reporting the state. Let $p(\theta_1, \theta_2)$ be the policy set by the decision maker when she receives message θ_1 from sender 1 and θ_2 from the second sender. We have on the equilibrium $p(\theta, \theta) = \theta$. They both have opposite incentives and perfect information therefore if they make the same reports the optimal response is to believe the report.

Restriction A: $\quad p(\theta_1, \theta_2) \in [\theta_2, \theta_1]$

This restriction is very reasonable. Sender 1 observes perfectly the state and has a bias towards a higher policy, so he would not voluntarily hide positive signals: the state cannot be bigger than θ_1. In the same way, based on the report of sender 2, the state cannot be smaller than θ_2. Imposing this restriction leads to the result in proposition 7.

Proposition 7 *Under restriction A, for all PBNE where the quantity of research is not observed by the receiver, the equilibrium amounts of research Q_1, Q_2, conducted by each sender are such that:*

$$Q_i < \frac{2\delta_i E(\theta)}{C}$$

Proof:

In equilibrium, sender 2 reports truthfully, so the problem of sender 1 is:

$$max_{Q'} - \int_0^{Q^*/Q'} [p(\frac{\theta Q'}{Q^*}, \theta) - \theta - \delta]^2 f(\theta) d\theta - \int_{Q^*/Q'}^1 [p(1, \theta) - \theta - \delta]^2 f(\theta) d\theta - CQ'$$

The FOC are:

$$C = -2 \int_0^{Q^*/Q'} \frac{\theta}{Q^*} \frac{\partial p}{\partial \theta_1} [\frac{\theta Q'}{Q^*}, \theta] \; [p(\frac{\theta Q'}{Q^*}, \theta) - \theta - \delta] f(\theta) d\theta$$

The FOC at the equilibrium become:

$$Q^* = \frac{-2}{C} \int_0^1 \theta \; \frac{\partial p}{\partial \theta_1} [\theta, \theta] \; [p(\theta, \theta) - \theta - \delta] f(\theta) d\theta = \frac{2\delta}{C} \int_0^1 \theta \; \frac{\partial p}{\partial \theta_1} (\theta, \theta) \; f(\theta) d\theta$$

We have: $\frac{\partial p}{\partial \theta_1}(\theta, \theta) = lim_{h \to 0} \frac{p(\theta + h, \theta) - p(\theta, \theta)}{h}$

Hypothesis A implies $\theta < p(\theta + h, \theta) < \theta + h$. Therefore, we have $0 < \frac{\partial p}{\partial \theta_1}(\theta, \theta) < 1$.
So, we find $Q^* < \frac{2\delta}{C} \int_0^1 \theta \; f(\theta) d\theta$ ■

The natural conjecture would be that the two parties, when forced to compete, would conduct more research. Proposition 7 leads to the opposite conclusion: both senders conduct less research than if they were alone to provide information to the decision maker. The intuition is the following: because the other sender is revealing all his information in equilibrium, the marginal benefit from searching for more signals to mislead the receiver is not as big as if he were alone. Indeed the marginal effect of more information is smaller as the information provided by both parties are perfect substitutes.

The difference between the general setup of section 4 and the benchmark case is that the decision on research is determined by two considerations: the strategic reporting and the information gathering. The results on competition obtained in this section generalizes on both accounts. Firstly, the intuition for the strategic function is the same as in Proposition 7: because the other sender is revealing all his information in equilibrium, the marginal benefits from obtaining more signals to mislead are smaller. Secondly, the fact that the other sender is obtaining information to get a better knowledge of the state, also reduces the benefits from research as an information tool for the first sender. Combining these two effects, in the general setup, we also obtain that less research will be conducted by each sender than if they were alone reporting.

6 Uncertainty and partial disclosure

In the model of the previous sections we supposed that the receiver knew perfectly both the preferences of the sender and the research technology he used. We examine in this section how critical these assumptions were for the results previously obtained. We will in particular concentrate on the result that, in the reporting phase, all the information obtained by the sender is revealed to the receiver. The idea is that, in equilibrium, the receiver can determine the quantity of research performed by the sender and therefore the result of Milgrom (1981) applies. We show that, as other authors have suggested in different contexts (in particular in the accounting literature Dye(1985), Verrecchia (1983)), when the receiver faces uncertainty it creates incentives for the sender to withhold some information.

6.1 Research is not always informative

We take in this section an approach similar to that of Dye (1985): we suppose that each infinitesimal signal has a probability q of being non informative. Each little experiment therefore has 3 outcomes (positive, negative or non informative). We find that the results are very similar to the baseline case.

If q is fixed and common knowledge, although we have partial revelation, the same quantity of research is performed. The intuition is that the receiver can anticipate perfectly the number of uninformative signals which will be obtained.

Proposition 8 *In all Pure Strategy Bayesian Nash Equilibria of the game where the research effort is not observable:*
1) The sender reports only the positive signals and reports all the rest as uninformative
2) the quantity of research performed is $Q = \frac{2\delta E(\theta)}{C}$

Proof: When the receiver obtains a report r, she sets the policy at the expected value $\frac{r}{qQ^*}$. In turn, when the sender decides on the quantity of research to perform, if he does an amount Q', he expects to get $q\theta Q'$ positive signals and the policy will therefore be $\frac{\theta Q'}{Q^*}$. So, the problem of the sender is the same as in the baseline model.

If we make this probability q a random variable with distribution f(q), we show that the amount of research conducted in equilibrium will be different. Depending on the distribution of q and θ, it will be either more or less than in the baseline model. However, the qualitative result that more research is performed than if the research effort was observable remains valid.

6.2 The bias of the sender is unknown

We introduce in this section another type of uncertainty: the receiver does not know the exact bias of the sender. In the standard model without research this would not modify the full disclosure result obtained by Milgrom and Roberts. Indeed in the disclosure phase, both types reveal all their information. On the contrary, in our model, where we consider the interaction with research, some equilibria are characterized by partial disclosure. Furthermore the concealed information is favorable to the sender's objectives.

We suppose that the bias can take two values: δ_L with probability p and δ_H with probability 1-p, and that both senders are biased in the same direction ($\delta_L > 0$ and $\delta_H > 0$). We also make in the simplifying assumption that the state is uniformly distributed on $[0, 1]$.

Proposition 9 *Under the assumption that* $\theta \sim U[0, 1]$*, when research is unobservable, we find a PBNE characterized by three amounts* Q_L*,* Q_H *and* Q_B *such that:*
1) The high type δ_H *conducts an amount of research* Q_H*, greater than if his type was known.*
2) The low type δ_L *conducts an amount* Q_L ($Q_L < Q_H$)*, smaller than if his type was known.*
3) Q_B *is such that, in the reporting phase, if the number of positive signals obtained by the high type is in* $[Q_L, Q_B]$*, he doesn't disclose all his positive information.*

Proof: see appendix.

This section is meant as an illustration: we do not list all the possible equilibria. However this proposition offers some interesting perspectives.

Result 3) characterizes the partial disclosure: although the agent always prefers a high policy, in certain situations he withholds some of his positive signals. The intuition is the following: because the low type in equilibrium conducts an amount of research Q_L, when the high type obtains a quantity of positive signals close but greater than Q_L, presenting all this favorable evidence would reveal his type and lead to a lower policy. The high type, in such a situation prefers withholding some of the information he obtained. This effect, which should be present in all equilibria, is a result of the interaction between research and disclosure. Because different types choose different amounts, the quantity of research performed characterizes the type in equilibrium. As pointed out earlier, in a simple disclosure model, this effect would not exist and both types would reveal all their information.

Results 1) and 2) describe how the unknown biases affect the amount of research performed. For the low type, the impact is quite straightforward: the marginal benefit of showing more positive signals is lower than if his type was known, because the receiver takes into account the possibility that he could be a high type and thus sets a lower policy. The intuition for the high type is the reverse: he always benefits from the fact that the policy is set at a higher level than if his type was known. These intuitions seem general, but we have only shown the results for a particular equilibrium. We did highlight however the importance of studying research and disclosure jointly.

7 Conclusion

We have studied in this article the interaction between research and the strategic disclosure of its results. Let us summarize our findings in terms of the interest group example. An interest group attempting to influence a policy maker will have to conduct more research in equilibrium if the research effort is unobservable. This result implies that when research is quickly informative, all parties involved prefer research to be taxed to avoid wasting resources. Furthermore the amount of research is increasing in the bias of the interest group, which implies that informational lobbying is more costly for more extreme groups and could explain why they often select other types of activities such as direct action. When research is not immediately informative, the choice between tax and subsidy on research is determined by the parameters of the model: such as the bias, research technology or cost. In particular, the more biased the sender, the less research should be subsidized. In a situation where two interest groups with conflicting views, compete to influence the decision maker, we show that they will spend less on research than if they were alone reporting.

The last section suggests further directions for research. In sections 2 to 5, we obtain the result that all the information obtained is disclosed in equilibrium. In section 6, we consider situations where the receiver faces more uncertainty and we observe partial disclosure. In particular, when the bias of the sender is unknown, we show that the interaction between research and disclosure can lead to partial revelation. In equilibrium, different types conduct different amounts of research and some information can be withheld so as not to reveal the type. This effect is not present if the disclosure phase is studied in isolation. Although the analysis of the last section was partial, it suggests that a lot more work remains to understand more clearly the determinants of disclosure. The set of results also show that research and disclosure often need to be studied jointly.

Appendix

Proposition 1

When the bias is small, another strategic consideration appears: the sender could wish to hide some of his positive signals.

If he does an amount of research Q', he gets $\theta Q'$ positive signals.
His desired policy is $\theta + \delta$.
So ideally he would want to report r to induce the policy $\frac{r}{Q^*} = \theta + \delta$.

We call the policy set off the equilibrium path when $r > Q^*$ is reported, $h(r, Q^*)$. We do not make explicit the shape of these beliefs: we even allow for the improbable scenario where the sender could obtain his preferred policy off the equilibrium path. We will see in this proof that these beliefs do not matter.

The strategic reporting would take this form:
$\frac{r}{Q^*} = \theta + \delta$ if $r < Q^*$
and $h(r, Q^*) = \theta + \delta$ if $r > Q^*$ (if the beliefs off the equilibrium path allow it)

There is however one constraint on the value of the report: it needs to verify $r < \theta Q'$. The sender needs to show hard evidence of his claims and cannot therefore report more positive signals than he obtained.

If $r < Q^*$, this constraint can be written $\theta > \frac{\delta Q^*}{Q' - Q^*}$.

Case I: $Q' < (1+\delta)Q^*$

$Q' < (1+\delta)Q^*$ is equivalent to $\frac{\delta Q^*}{Q' - Q^*} > 1$. Therefore, in this case, the sender will always report all his positive signals, because the report that would lead to his preferred policy is always such that $r > \theta Q'$. Therefore when $Q' < (1+\delta)Q^*$, we are brought back to the calculations of Prop 1 and we know there is no optimal deviation from $Q^* = \frac{2\delta E(\theta)}{C}$.

Case II: $Q' \geq (1+\delta)Q^*$

We need to show that there is no Q' verifying this constraint that would be an optimal deviation from Q^*.

For these values of Q', there can be strategic reporting:
- if $\theta < \frac{\delta Q^*}{Q' - Q^*}$, the sender reports $\theta Q'$.
- if $\frac{\delta Q^*}{Q' - Q^*} < \theta < 1 - \delta$, the sender reports r such that $\frac{r}{Q^*} = \theta + \delta$.

- if $\theta > 1-\delta$, his optimal report r is such that $r > Q^*$. We do not explicitly define the report and the policy set in place for these values of θ as it would require us to give a specific form to the beliefs off the equilibrium path. We call the policy implemented $h(\theta, Q')$.

The problem of the sender is then

$$max_{Q'} - \int_0^{\frac{\delta Q^*}{Q'-Q^*}} [\frac{\theta Q'}{Q^*} - \theta - \delta]^2 f(\theta) d\theta - \int_{\frac{\delta Q^*}{Q'-Q^*}}^{1-\delta} [\theta + \delta - \theta - \delta]^2 f(\theta) d\theta$$
$$- \int_{1-\delta}^{1} [h(\theta, Q') - \theta - \delta]^2 f(\theta) d\theta - CQ'$$

We consider the best of cases for the beliefs off the equilibrium path that would set the third term to 0 and concentrate on the function corresponding to the other terms.
Let f(Q) be:

$$f(Q') = - \int_0^{\frac{\delta Q^*}{Q'-Q^*}} [\frac{\theta Q'}{Q^*} - \theta - \delta]^2 f(\theta) d\theta - CQ'$$

We have

$$f'(Q') = - \int_0^{\frac{\delta Q^*}{Q'-Q^*}} \frac{2\theta}{Q^*} [\frac{\theta Q'}{Q^*} - \theta - \delta] f(\theta) d\theta - C$$

and

$$f''(Q') = -2 \int_0^{\frac{\delta Q^*}{Q'-Q^*}} [\frac{\theta}{Q^*}]^2 f(\theta) d\theta < 0$$

So, the function f is concave and single peaked. Furthermore, we know that
$f'((1+\delta)Q^*) = C\frac{\int_0^1 \theta(1-\theta)f(\theta)d\theta}{E(\theta)} - C < 0$. Therefore on $[(1+\delta)Q^*, +\infty]$, f is decreasing. So, the best deviation would be for $Q' = (1+\delta)Q^*$.

In conclusion we just need to compare $f((1+\delta)Q^*)$ to the value function at Q^*.

The value function at Q^* equals $-\delta^2 - CQ^*$ and $f((1+\delta)Q^*) = \int_0^1 [\delta(1-\theta)]^2 f(\theta) d\theta - C(1+\delta)Q^*$. The difference between this two terms is $\delta^2 E[\theta^2]$ and therefore this is not an optimal deviation from Q^* and there is no such deviation.

This proof is valid for any belief off the equilibrium path and therefore we have shown that Q^* is still an equilibrium. ■

Proposition 3

The sender, if he performs an amount of research Q' and the receiver believes he did a quantity Q^*, will obtain $\theta Q'$ positive signals and will report r to maximize:
$(\frac{r}{Q^*} - \theta - \delta)^2 - \tau r$ with $r \leq \theta Q'$.
In the case of a big bias, the sender will report $\theta Q'$.
Therefore in the first phase, the sender choose research quantity Q' so as to maximize:

$$max_{Q'} - \int_0^{Q^*/Q'} ([\frac{\theta Q'}{Q^*} - \theta - \delta]^2 - \tau\theta Q')f(\theta)d\theta - \int_{Q^*/Q'}^1 ([1-\theta-\delta]^2 - \tau Q^*)f(\theta)d\theta - CQ'$$

The FOC are:

$$C = -\int_0^{Q^*/Q'} (\frac{\theta}{Q^*}[\frac{\theta Q'}{Q^*} - \theta - \delta] - \tau\theta)f(\theta)d\theta$$

Therefore the equilibrium amount of research will be:

$$Q^* = \frac{\delta E(\theta)}{C + \tau E(\theta)} \quad \blacksquare$$

Proposition 4

When the research effort is observable, the sender solves:

$$max_{Q'} \ -\int_0^\infty \int_0^\infty (E[\theta|x,Q'] - \theta - \delta)^2 \ f(x|\theta,Q') \ f(\theta) \ dx \ d\theta \ - CQ'$$

We have $f(x|\theta,Q') \ f(\theta) = f(\theta|x,Q') \ g(x|Q')$, so the problem can be rewritten:

$$\begin{aligned} max_{Q'} \ & -\int_0^\infty \int_0^\infty (E[\theta|x,Q'] - \theta)^2 \ f(\theta|x,Q') \ g(x|Q') \ dx \ d\theta \\ & +2\int_0^\infty \int_0^\infty (E[\theta|x,Q'] - \theta)\delta \ f(\theta|x,Q') \ g(x|Q') \ dxd\theta \\ & -\delta^2 \int_0^\infty \int_0^\infty f(\theta|x,Q') \ g(x|Q') \ dxd\theta \ - CQ' \end{aligned}$$

By definition $\int_0^\infty (E[\theta|x,Q'] - \theta)f(\theta|x,Q')d\theta = 0$ and $\int_0^\infty (E[\theta|x,Q'] - \theta)^2 \ f(\theta|x,Q')d\theta = V[\theta|x,Q']$ so the problem is:

$$max_{Q'} \ -\int_0^\infty V[\theta|x,Q'] \ g(x|Q') \ dx \ - \delta^2 \ - CQ'$$

The hypothesis we made on the variance therefore guarantees that this problem has a unique solution given by the FOC of proposition 5. $\blacksquare$

Proposition 5

The problem of the sender can be rewritten:

$$max_{Q'} \; - \int_0^1 \int_{\bigcup A_k} [(E[\theta|x,Q^*] - \theta - \delta)^2 - (E[\theta|x,Q'] - \theta)^2] \; f(x|\theta,Q') \; f(\theta) \; dx \; d\theta$$
$$- \int_0^1 \int_0^\infty (E[\theta|x,Q'] - \theta)^2 \; f(x|\theta,Q') \; f(\theta) \; dx \; d\theta \; - CQ'$$

As in the proof of proposition 2a, we can rewrite the third term as a function of the variance. The objective becomes

$$max_{Q'} \; - \int_0^\infty \; V[\theta|x,Q'] \; g(x|Q')dx \; - CQ'$$
$$- \int_0^\infty \int_{\bigcup A_k} [\; (E[\theta|x,Q^*] - \theta - \delta)^2 - (E[\theta|x,Q'] - \theta)^2] \; f(x|\theta,Q') \; f(\theta) \; dx \; d\theta$$

Let's examine the first order conditions.

$$C = -\frac{\partial}{\partial Q'}[\int_0^\infty \; V[\theta|x,Q'] \; g(x|Q')dx]$$
$$+ \sum_k \int_0^1 \frac{\partial x_{k1}}{\partial Q'}[\; (E[\theta|x_{k1},Q^*] - \theta - \delta)^2 - (E[\theta|x_{k1},Q'] - \theta)^2] \;\; f(\theta) \; d\theta$$
$$- \sum_k \int_0^1 \frac{\partial x_{k2}}{\partial Q'}[\; (E[\theta|x_{k2},Q^*] - \theta - \delta)^2 - (E[\theta|x_{k2},Q'] - \theta)^2] \;\; f(\theta)d\theta$$
$$- \int_0^1 \int_{\bigcup A_k} [\; (E[\theta|x,Q^*] - \theta - \delta)^2 - (E[\theta|x,Q'] - \theta)^2] \; \frac{\partial f}{\partial Q'}(x|\theta,Q') \; f(\theta)dx \; d\theta$$
$$+2 \int_0^1 \int_{\bigcup A_k} \frac{\partial E}{\partial Q'}(\theta|x,Q') \; (E[\theta|x,Q'] - \theta) \; f(x|\theta,Q') \; f(\theta) \; dx \; d\theta$$

For x_{k1} and x_{k2}, we have $E[\theta|x,Q^*] = E[\theta|x,Q'] + \delta$, so

$$C = -\frac{\partial}{\partial Q'}[\int_0^\infty \; V[\theta|x,Q'] \; g(x|Q')dx]$$
$$- \int_0^1 \int_{\bigcup A_k} [\; (E[\theta|x,Q^*] - \theta - \delta)^2 - (E[\theta|x,Q'] - \theta)^2] \; \frac{\partial f}{\partial Q'}(x|\theta,Q') \; f(\theta)dx \; d\theta$$
$$+2 \int_0^1 \int_{\bigcup A_k} \frac{\partial E}{\partial Q'}(\theta|x,Q') \; (E[\theta|x,Q'] - \theta) \; f(x|\theta,Q') \; f(\theta) \; dx \; d\theta$$

For Q^* to be an equilibrium these FOC need to be verified at Q^*. As we did previously, we therefore take the FOC when Q' converges to Q^*. The first thing to observe is that $\bigcup A_k \rightarrow [0, +\infty]$ (indeed at the limit, when the receiver has the right beliefs about the quantity of research, there is no more opportunities to hide information). All the functions are continuous, so taking the limit results in:

$$C = -\frac{\partial}{\partial Q'}[\int_0^\infty \ V[\theta|x, Q^*] \ g(x|Q^*)dx]$$
$$+\int_0^1 \int_0^\infty \delta \ (2E[\theta|x, Q^*] - 2\theta - \delta) \ \frac{\partial f}{\partial Q'}(x|\theta, Q^*) \ f(\theta)dx \ d\theta$$
$$+2\int_0^1 \int_0^\infty \frac{\partial E}{\partial Q'}(\theta|x, Q^*) \ (E[\theta|x, Q^*] - \theta) \ f(x|\theta, Q^*) \ f(\theta) \ dx \ d\theta$$

Let's examine the 3rd term:

$$2\int_0^1 \int_0^\infty \frac{\partial E}{\partial Q'}(\theta|x, Q^*) \ (E[\theta|x, Q^*] - \theta) \ f(x|\theta, Q^*) \ f(\theta) \ dx \ d\theta$$
$$= 2\int_0^\infty \frac{\partial E}{\partial Q'}(\theta|x, Q^*) \ g(x|Q') \ [\int_0^1 \ (E[\theta|x, Q^*] - \theta) \ f(\theta|x, Q^*) \ d\theta] \ dx$$
$$= 0$$

Let's now look at the second term:

$$\int_0^1 \int_0^\infty \delta \ (2E[\theta|x, Q^*] - 2\theta - \delta) \ \frac{\partial f}{\partial Q'}(x|\theta, Q^*) \ f(\theta)dx \ d\theta$$
$$= -\int_0^1 \int_0^\infty \delta^2 \ \frac{\partial f}{\partial Q'}(x|\theta, Q^*) \ f(\theta)dx \ d\theta$$
$$+\int_0^1 \int_0^\infty 2\delta \ (E[\theta|x, Q^*] - \theta) \ \frac{\partial f}{\partial Q'}(x|\theta, Q^*) \ f(\theta)dx \ d\theta$$

We have $\int_0^1 \int_0^\infty \ f(x|\theta, Q^*) \ f(\theta)dx \ d\theta = 1$.
So, by taking the derivative with respect to Q'
$\int_0^1 \int_0^\infty \delta^2 \ \frac{\partial f}{\partial Q'}(x|\theta, Q^*) \ f(\theta)dx \ d\theta = 0$.

We have also $\int_0^1 \int_0^\infty \ (E[\theta|x, Q^*] - \theta) \ f(x|\theta, Q^*) \ f(\theta)dx \ d\theta = 0$.

So, take the derivative,

$$\int_0^1 \int_0^\infty 2\delta \ (E[\theta|x, Q^*] - \theta) \ \frac{\partial f}{\partial Q'}(x|\theta, Q^*) \ f(\theta)dx \ d\theta \tag{2}$$
$$= -2\delta \int_0^1 \int_0^\infty \frac{\partial E}{\partial Q'}(\theta|x, Q^*) \ f(x|\theta, Q^*) \ f(\theta)dx \ d\theta \tag{3}$$

Therefore, the first order conditions can be rewritten:

$$C = -\frac{\partial}{\partial Q'}[\int_0^\infty \; V[\theta|x,Q'] \; g(x|Q')dx] \; |_{Q'=Q^*}$$
$$-2\delta \int_0^1 \int_0^\infty \; \frac{\partial}{\partial Q'}[E(\theta|x,Q')]|_{Q'=Q^*} \; f(x|\theta,Q^*) \; f(\theta)dx \; d\theta \quad \blacksquare$$

Proposition 6

The optimal tax social with redistribution is solution to the problem:

$$max_\tau \; -\int_0^\infty \int_0^1 (E[\theta|x,Q_\tau] - \theta - \delta)^2 \; f(x|\theta,Q_\tau) \; f(\theta)dxd\theta$$
$$-\int_0^\infty \int_0^1 (E[\theta|x,Q_\tau] - \theta)^2 \; f(x|\theta,Q_\tau) \; f(\theta)dxd\theta \; -(C+\tau)Q_\tau + \tau Q_\tau$$

where Q_τ is chosen optimally by the sender given a tax τ.

Let Q_W be the solution to:

$$max_{Q'} \; -\int_0^\infty \int_0^1 (E[\theta|x,Q'] - \theta - \delta)^2 \; f(x|\theta,Q') \; f(\theta)dxd\theta$$
$$-\int_0^\infty \int_0^1 (E[\theta|x,Q'] - \theta)^2 \; f(x|\theta,Q') \; f(\theta)dxd\theta \; - CQ'$$

The FOC corresponding to this problem are:

$$\frac{C}{2} = -\frac{\partial}{\partial Q'}[\int_0^\infty \; V[\theta|x,Q'] \; g(x|Q')dx] \; |_{Q'=Q_W}$$

If there exists a tax such that $Q_\tau = Q_W$ then this is the socially optimal tax. We see from proposition 5 that if the tax is:

$$\tau \;=\; -2\delta \int_0^1 \int_0^\infty \; \frac{\partial}{\partial Q'}[E(\theta|x,Q')]|_{Q'=Q_W} \; f(x|\theta,Q_W) \; f(\theta)dx \; d\theta - \frac{C}{2}$$

the FOC of the two problems are equivalent and therefore $Q_\tau = Q_W$. $\blacksquare$

Proposition 9

We study an equilibrium where the low type conducts an amount of research Q_L and the high type an amount Q_H, with $Q_L < Q_H$. In all equilibria, if the high type obtains an

amount of positive signals close and greater than Q_L, he will not report all of them in order to hide his type. However, he cannot in this case always report Q_L positive signals because then the low type would never report Q_L and things would break down. Therefore, there exists 2 values Q_0 and Q_B such that:
- If research yields a quantity of positive signals in $[0, Q_0]$, all the signals are reported.
- If research yields a quantity of positive signals in $[Q_0, Q_B]$, both types mix their reports so that the decision maker sets a policy q.
- If research yields a quantity of positive signals in $[Q_B, Q_H]$, the high type discloses all his information.

The following properties have to be true in equilibrium:
- Q_B is such that $q = \frac{Q_B}{Q_H}$ (at Q_B, the high type is indifferent between reporting truthfully and revealing his type and obtaining the policy q).
- $q = p\frac{Q_L - Q_0}{Q_L} + (1-p)\frac{Q_B - Q_0}{Q_H}$. The policy is set at the expected value of the state given the reports.

We study in this example a particular equilibrium where at Q_0, the sender is exactly indifferent between obtaining q and reporting Q_0. This means we impose the condition $q = p\frac{Q_0}{Q_L} + (1-p)\frac{Q_0}{Q_H}$. However, other equilibria exist where $q > p\frac{Q_0}{Q_L} + (1-p)\frac{Q_0}{Q_H}$.
Under this condition we obtain $q = \frac{p}{1+p}$.

The problem faced by the senders is:

$$max_{Q'} - \int_0^{\frac{Q_0}{Q'}} [p\frac{\theta Q'}{Q_L} + (1-p)\frac{\theta Q'}{Q_H} - \theta - \delta_i]^2 f(\theta) d\theta - \int_{\frac{Q_0}{Q'}}^{\frac{q}{Q'}} [q - \theta - \delta_i]^2 f(\theta) d\theta$$
$$- \int_{\frac{q}{Q'}}^{\frac{Q_H}{Q'}} [\frac{\theta Q'}{Q_H} - \theta - \delta_i]^2 f(\theta) d\theta - CQ'$$

The FOC for the low type can be written

$$-2\int_0^{\frac{Q_0}{Q_L}} [\theta\frac{q}{Q_0}][\frac{Q_L}{Q_0}\theta q - \theta - \delta_L] f(\theta) d\theta = C$$

The FOC for the high type are:

$$-2\int_0^{\frac{Q_0}{Q_H}} [\theta\frac{q}{Q_0}][p\theta\frac{Q_H}{Q_L} - p\theta - \delta_H] f(\theta) d\theta + 2\int_q^1 \frac{\theta}{Q_H}\delta_H f(\theta) d\theta = C$$

Results 2) and 3) are then obtained, using the fact that the state is uniformly distributed.

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