Endogenous Persistent Inequality when Markets are Perfect[1].

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This Draft

October 27, 2004

Abstract

This paper exhibits a mechanism of endogenous formation and persistence of inequality through investment in human capital in the overlapping generations framework, with perfect capital market. The results are based on the effects that exact comparative advantages agents have on their occupational choice.

It is shown that entrepreneurial decisions and human capital investment depend on the inherited level of human capital, when agents who inherit a relatively low level of human capital are better off being workers. Those who inherit a relatively high level of human capital are better off being entrepreneurs and they earn higher revenue.

The human capital threshold that separates agents in the two categories is endogenous and varies with the equilibrium wage. Therefore the economy may experience upward or downward mobility depending on the variation of the equilibrium wage. In the long run, inequality persists and the workers never catch up with the entrepreneurs. However the model exhibits long run multiple equilibria (meaning a continuum of steady state with inequality).

Keywords: Inequality, Human Capital, Entrepreneurship, Mobility, Occupational Choice, Education

JEL Classification:D63, J24, J62, J31, M13, I2

[1] I am grateful to H. d'Albis, A. d'Autume, E. H. Fall, J.-O. Hairault and B. Wigniolle for helpful suggestions. The usual disclaimer applies.

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1 Introduction

Which market mechanisms generate formation and persistence of inequality? In market economies, what accumulable variable is determinant in explaining inequality formation and persistence? What is the role of endogenous market prices in generating and perpetuating inequality?

In his seminal contribution Loury [1981] formulates these questions about inequality formation and persistence in an overlapping generations framework with imperfect capital market and education choices. In his paper, the assumptions of imperfect capital market and of random assignment of abilities to agents, yield the stochastic process of the dynamic of the earning distribution among successive generations of workers. However, this process converges to a unique, and globally stable, stationary distribution. Lourys' pioneering work in 1981 has since received many contributions. Among others we can cite Galor and Zeira [1993], Banerjee and Newman [1993], Ljungqvist [1993], Durlauf [1996], Aghion and Bolton [1997], and Piketty [1997].

In all these papers, there are two major types of assumptions that can drive the main results of inequality persistence and long-run effects of initial distribution, imperfect capital market[3] and non convex technology of investment. The role of these assumptions can be summarized as follows. Imperfect credit markets prevent less endowed agents from borrowing and thus prevents them from having access to a high level of education. It limits their capacity for entrepreneurial projects, as well as their occupational choice. At the aggregate level, this limits the aggregate investment in human or physical capital in the economy. Non convexity in the human (or physical) capital production function, namely indivisibility in the investment, yields the long run macroeconomic effects of wealth distribution. This is seen firstly with market prices. It prevents poor agents from accumulating enough to overcome the barrier in investing high return human capital and projects. Secondly, it creates the condition for multiple long run equilibrium.

The aim of this paper, is to exhibit a mechanism of endogenous and persistent inequality through investment in human capital in the overlapping generations framework, with a perfect capital market. Our main interest is how, in a perfect market, agents make different occupational choices, and why less endowed agents never catch up with more endowed agents. It is shown that even in a perfect market, agents may have different comparative advantages in their occupational choice, given their inherited level of human capital. Those agents less endowed in human capital choose to be workers, while those better endowed choose to be entrepreneurs. As a consequence, their investment in human capital differs, and this generates a long run persistence of human capital inequality. It is also shown that economies with different initial distributions of human capital will converge to a different level of steady state inequality, both in terms of the number of agents in each type of occupation and equilibrium wage. In short, the model exhibits multiple long run equilibrium.

Finally, under this model mobility is possible during the transition to steady state. This depends on the variation of market prices.

To simplify the exposition of the model we abstract from capital variable. This is possible because we

[3] Chatterjee [1994] showed that in a neoclassical framework, with the assumption of a perfect market, inequality across individuals can persist while the aggregate capital stock is uniquely determined in steady state.

assume that the capital market is perfect and there are no market imperfections in the economy. One can be sure that the absence of capital variable does not change the qualitative results of the model. Indeed, if we add capital variable as a factor, both in the good production function and in the human capital production function, it does not change agents' decisions because they have free and costless access to the capital market.

The model is an overlapping generations economy in which individuals are identical in their preferences and their production technology of human capital. However, they may differ in the inherited level of human capital and thus in the efficacy of their own investment in education. The individuals' level of human capital increases with the time invested in its education and with the inherited level of human capital. Agents take two decisions as follows. 1) How much time to devote to education in their youth ? 2) Which occupations to choose. Will they be entrepreneurs or workers in their old age? Workers earn the equilibrium wage while entrepreneurs earnings is increasing with the increase in the entrepreneurs human capital. Consequently, the inherited level of human capital of each individual determines his occupational choice and his investment in education. At equilibrium, there is an endogenous human capital threshold depending on the equilibrium wage. This separates agents between workers and entrepreneurs. Agents who inherit less than the human capital threshold are better of being workers while those who inherit more are better of being entrepreneurs. Moreover, entrepreneurs invest more time in education than workers.

The distribution of human capital determines the size of the two groups (workers and entrepreneurs), and the equilibrium wage level. As their investment in education differs, workers and entrepreneurs will converge to a different level of human capital at steady state. Entrepreneurs' human capital is higher than workers human capital at steady state, and therefore, stable steady state is characterized by a two-point distribution of human capital, stable equilibrium wage and the number of agents in each occupation. The stable steady state depends on the initial distribution of human capital through the effects of the latter on equilibrium wage dynamics.

The model exhibits long run multiple equilibrium. Different economies with different initial human capital distribution will converge to different unique stable steady state equilibria, with inequality. These steady states are different in terms of equilibrium wage and the numbers of poor workers and rich entrepreneurs. These countries may differ in the number of poorly educated workers and highly educated entrepreneurs they have, but not in their levels of steady state human capital. Although at steady state, the numbers of workers and entrepreneurs remain constant, mobility can occur during the transition process to steady state. If the dynamics of the equilibrium wage experiences a high variation, there may be upward mobility (workers becoming entrepreneurs), or, downward mobility (entrepreneurs becoming workers).

Among those studies that offer a theory of endogenous formation and persistence of inequality, Matsuyama [2000] is the most closely related. In Matsuyama's paper there is a wealth threshold that separates agents between poor lenders and rich borrowers. The wealth threshold depends on the equilibrium interest rate and in the long run there is a two-point distribution of wealth. His model also exhibits a continuum of steady states with inequality. However there are some important differences.

The result in Matsuyama's paper relate on imperfect credit market and non-convexity in the investment technology. Moreover, in his paper, the wealth distribution determines the equilibrium interest rate, which in turn affects the wealth accumulation. In this paper the equilibrium wage does not affect the accumulation of human capital. Another closely related paper that exhibits similar results is that of Galor and Tsiddon [1997]. However, in their model, multiple long run equilibrium and dynamics are driven by the competition between the home environment externality and a global technological externality in the production of human capital. In this paper, as shown[4] by Mookherjee and Ray [2003] in an imperfect capital market context, multiplicity (in the sense of continuum of steady states) of long run equilibrium is endemic whenever the number of professions is small.

Two critical hypotheses play a fundamental role in the results in this paper. Firstly, the assumption that the production function of the entrepreneurial project is an increasing returns to scale technology, and that it depends on entrepreneurs specific level of human capital. Secondly, the production of human capital is an increasing function of the inherited human capital (parental human capital). It is the combination of these two hypotheses that make the occupational choice depends on the inherited level of human capital, given the equilibrium wage.

Since Romer [1986], it is frequent in the field of human capital and growth (Lucas [1988] and Azariadis and Drazen [1990] among others) to introduce technology with increasing returns to scale. However the way increasing returns is introduced and its role in this paper is different to that in the papers mentioned above. In these papers it is a spillover, or externalities effects, that generate increasing returns to scale in the accumulation of human capital. Therefore these externalities imply multiple long run equilibrium whenever there exists any type of threshold in access to efficient technology. In this paper, increasing returns make entrepreneurial profit dependent on entrepreneurs' specific human capital. It also allows the coexistence at equilibrium, of different sizes of businesses.

The importance of the parental education input in the formation of the human capital of the child, has been explored theoretically as well as empirically. The empirical significance of the parental effect was documented by Becker and Tomes [1986] among others. In this model, this feature of human capital transmission plays an important role in the persistence of inequality. Even if we add the availability of public education, some persistency continues to exist due to this characteristic of the production of human capital.

The fact that the inherited level of human capital is the determinant factor in the occupational choice is also crucial in the model. Indeed, if the financial capital variable was the critical factor then in perfect markets all agents could choose to be entrepreneurs and there would be no inequality at equilibrium.

Numerous contributions in the literature have emphasized the importance of human capital, individual talent or ability in entrepreneurship. Bates [1990], Lentz and Laband [1990], Iyigun and Owen [1997], Irigoyen [2002] are some of those to have done this. Dunn and Holtz-Eakin [2000] state that human capital transmission is the strongest channel by which parents influence entrepreneurship decisions.

In section 2 we set up the model and derive the endogenous human capital occupational threshold. In the section 3 we discuss the existence and uniqueness of short run equilibrium. In section 4, we will analyze the steady state equilibrium in terms of persistent inequality and multiplicity of equilibrium. The final section, 5, studies the dynamics of this model and the concern about mobility. Section 6 concludes.

[4]See Mookherjee and Ray [2003] proposition 3.

2 The model

The framework is a two-period OLG model with human capital intergenerational transmission within families. Each agent is a member of an infinitely-lived family. The only source of heterogeneity within a group of same aged-individuals is their human capital inheritance.

2.1 Entrepreneurship.

A would be entrepreneur faces the following technology

$$Y_{it+1} = h_{it+1}^{\psi} L_{it+1}^{\alpha} \tag{1}$$

where $0 < \alpha, \psi < 1$ and $\psi + \alpha > 1$, where h_{it+1} is the human capital of the entrepreneurs (the owner of the business) and L_{it+1} is the labor factor hired by the entrepreneur. L_{it+1} is an amount of efficient human capital[5]. As we abstract form capital variable, there is one explicit market[6] in this economy: the labor market which determines the equilibrium wage per unit of human capital and therefore workers earnings. The assumption of increasing returns to scale plays an important role. It means that entrepreneurs' earnings is not the wage but it is rather a part of the output. After paying production factors at market equilibrium prices, there is something left that is for the entrepreneurs. The entrepreneurs' human capital express his managerial ability and therefore the size of business he can run. The profit maximizing program of an entrepreneur is therefore

$$\max_{L_{it+1}} \left[h_{it+1}^{\psi} L_{it+1}^{\alpha} - w_{t+1} L_{it+1} \right]$$

Solving this problem yields the optimal labor demand of the entrepreneur depending on his proper level of human capital:

$$L_{it+1} = \left(\frac{\alpha h_{it+1}^{\psi}}{w_{t+1}} \right)^{\frac{1}{1-\alpha}} \tag{2}$$

Therefore entrepreneurs profit or earnings is written:

$$\Pi_i \left(h_{it+1} \right) = h_{it+1}^{\frac{\psi}{1-\alpha}} \left(\frac{\alpha}{w_{t+1}} \right)^{\frac{\alpha}{1-\alpha}} (1 - \alpha) \tag{3}$$

The profit is increasing and convex with entrepreneurs' human capital. This implies that there is not one optimal level of production and profit in the economy but for any given level of entrepreneurial human capital there is one optimal level of production and profit. Thus there is no constraint on setting up an entrepreneurial project. But entrepreneurs will differ on the size of the business they can run and therefore on their earnings. This means that there is a continuum of enterprise that can exist in this model depending on entrepreneurs different level of human capital.

[5] Agents are endowed with one unit of time each period and they supply all their unit of time in the second period, one can write $H_{it+1} = 1 * h_{it+1}$ as labor supply of efficient human capital of an agent. Thus labor demand by one entrepreneneur is $L_{it+1} = \Sigma H_{it+1}$

[6] Note that the good market exists but is implicit by the Walras law.

2.2 Agents occupational choice.

In their youth, every agent receives a human capital h_{it} from his parents and an endowment of one unit of time. During his youth, an agent allocates his time between training in education (u_{it}), and leisure $(l_{it} = 1 - u_{it})$. The time invested in education determines his second period human capital h_{it+1}. During his old age an agents chooses his occupation between being a worker or an entrepreneur. If he chooses to be an entrepreneur he will earn at the end of the period the profit from running his business $\Pi(h_{it+1})$. The profit depends on his amount of human capital in a manner specified above in the entrepreneurs' program. If he chooses to be a worker, he will earn $w_{t+1}h_{it+1}$ where w_{t+1} is the equilibrium wage per unit of efficient human capital. The agent consumes all his revenue before leaving the economy. The agents utility function depends on his first period leisure l_{it} and his second period consumption c_{it+1}. For simplicity we abstract from first period consumption[7] and, we assume that agents inherit their parents' human capital. The agents' program is written:

$$\underset{l_{it},c_{it+1}}{Max} \ \left\{U\left(l_{it},c_{it+1}\right)=l_{it}^{1-\sigma}c_{it+1}^{\sigma}\right\} \tag{4}$$

$$s.c. \quad \begin{cases} l_{it}+u_{it}=1 \\ h_{it+1}=\theta\left(u_{it}h_{it}\right)^{\gamma} \\ c_{it+1}=y_{it+1} \\ y_{it+1}=\max\left\{w_{t+1}h_{it+1},\,\Pi\left(h_{it+1}\right)\right\} \end{cases}$$

where $0 < \sigma < 1$ and $\theta > 0$ represent the efficacity of the education system..

From an agent perspective, the occupational choice is a maximizing value function. The agent will choose to be an entrepreneur if his value function of being an entrepreneur, let denote it $V^E(h_{it}, w_{t+1})$, is greater than his value function of being a worker $V^W(h_{it}, w_{t+1})$. Therefore to solve the global program of the agent, one can compute the two distinct value functions of being an entrepreneur or a worker. The program of the entrepreneur writes :

$$\underset{l_{it},c_{it+1}}{Max} \left\{U\left(l_{it},c_{it+1}\right)=l_{it}^{1-\sigma}c_{it+1}^{\sigma}\right\}$$

$$s.t. \quad \begin{cases} l_{it}+u_{it}=1 \\ c_{it+1}=\theta^{\frac{\psi}{1-\alpha}}\left(u_{it}h_{it}\right)^{\frac{\gamma\psi}{1-\alpha}}\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha) \end{cases}$$

Solving this program gives his optimal investment in training:

$$u_{it}^{E}=\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi} \tag{5}$$

and his second period human capital:

$$h_{it+1}^{E}=\theta\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\gamma}h_{it}^{\gamma} \tag{6}$$

[7] Glomm and Ravikumar [1992] make the same simplificating assumption on first period consumption.

The optimal corresponding value function of an entrepreneur is :

$$V^E\left(h_{it}, w_{t+1}\right) = \left(1-\alpha\right)^{\sigma} \theta^{\frac{\sigma\psi}{1-\alpha}} \left(\frac{\left(1-\sigma\right)\left(1-\alpha\right)}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{1-\sigma} \left(\frac{\sigma\gamma\psi}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{\frac{\sigma\gamma\psi}{1-\alpha}} \left(\frac{\alpha}{w_{t+1}}\right)^{\frac{\sigma\alpha}{1-\alpha}} h_{it}^{\frac{\sigma\gamma\psi}{1-\alpha}} \tag{7}$$

The program of a worker is written:

$$\underset{l_{it}, c_{it+1}}{Max} \left\{U\left(l_{it}, c_{it+1}\right) = l_{it}^{1-\sigma} c_{it+1}^{\sigma}\right\}$$

$$s.t. \quad \begin{cases} l_{it} + u_{it} = 1 \\ c_{it+1} = w_{t+1}\theta\left(u_{it}h_{it}\right)^{\gamma} \end{cases}$$

His optimal time investment in training is:

$$u_{it}^W = \frac{\sigma\gamma}{1-\sigma+\sigma\gamma} \tag{8}$$

and his second period human capital is:

$$h_{it+1}^W = \theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma} h_{it}^{\gamma} \tag{9}$$

The optimal corresponding value function of a worker is:

$$V^W\left(h_{it}, w_{t+1}\right) = \theta^{\sigma}\left(\frac{1-\sigma}{1-\sigma+\sigma\gamma}\right)^{1-\sigma}\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\sigma\gamma} w_{t+1}^{\sigma} h_{it}^{\sigma\gamma} \tag{10}$$

Due to concavity assumptions on the utility function and on the production function of the human capital, the time investment in training is independent of agents' level of human capital and of next period expected wage as given by equations (5) and (8). That is all would be entrepreneurs will devote the same time to training even if they do not have the same level of human capital. It is the same for all would be worker. Moreover, would be entrepreneurs will invest more time in training than would be worker. As a consequence of the optimal investment in training, the second period human capital of agents is independent of the next period expected wage. This feature of human capital transition equation has important implication for long run dynamics of human capital.

Agents value function is an increasing function of their inherited human capital. But, while it is increasing with the equilibrium wage for the workers, it is decreasing with the equilibrium wage for entrepreneurs. The higher is the equilibrium wage, the least is the profit for entrepreneurs.

2.3 Human capital occupational threshold.

Given the distribution of human capital, what will be the frontier between being workers and entrepreneurs ? Let us consider $V(h_i, w_{t+1}) \equiv V^W(h_i, w_{t+1}) - V^E(h_i, w_{t+1})$. For each agent when $V(h_i, w_{t+1})$ is positive, he chooses to be workers, if rather it is negative, he chooses to be entrepreneurs. At the macroeconomic level we show that there exists a unique human capital level that separates the group of workers and that of entrepreneurs. As illustrated by figure 1 below, this function $V(h_i, w_{t+1})$ is positive for all $h_i < \underline{h_{it}}$ and negative for all $h_i > \underline{h_{it}}$ and, there is a unique positive level of human capital for which $V(h_i, w_{t+1}) = 0$. That is, agents that are endowed with that level of human capital are indifferent[8] between being workers or entrepreneurs. This level of human capital is the human capital occupational threshold. It separates workers from entrepreneurs in the economy.

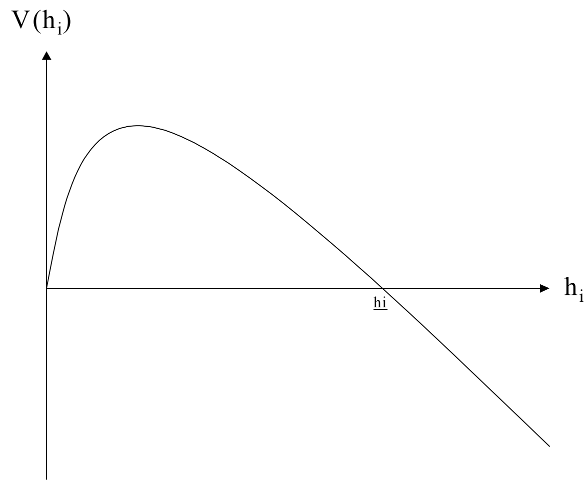

Figure 1: Human capital occupational threshold.

Proposition 1 *: There is a unique threshold depending on the equilibrium wage that separates workers from entrepreneurs.*

Proof. Since[9] $V^{''}(h_i, w_{t+1}) < 0$, then $V(h_i, w_{t+1})$ is strictly concave and, as $V(0) = 0$ and $\lim_{h_i \to \infty} V(h_i) = -\infty$ there exists a unique and positive human capital level $\underline{h_i}$ for which $V(\underline{h_i}, w_{t+1}) = 0$. Thus for all $h_i < \underline{h_{it}}$, $V(h_i, w_{t+1}) > 0$ and for all $h_i > \underline{h_{it}}$, $V(h_i, w_{t+1}) < 0$. ■

Equalizing $V^w(h_i, w_{t+1}) = V^E(h_i, w_{t+1})$ gives the occupational threshold $\underline{h_{it}}$. It is written:

$$\begin{aligned} \underline{h_{it}} \quad = \quad & (w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}} \left(\frac{[(1-\sigma)(1-\alpha) + \sigma\gamma\psi]}{(1-\sigma+\sigma\gamma)(1-\alpha)} \right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma\gamma(\psi+\alpha-1)}} \left(\frac{(1-\sigma)(1-\alpha) + \sigma\gamma\psi}{\sigma\gamma\psi} \right)^{\frac{\psi}{(\psi+\alpha-1)}} {\frac{1}{\theta}}^{\frac{1}{\gamma}} \\ & \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma} \right)^{\frac{1-\alpha}{(\psi+\alpha-1)}} \left(\frac{1}{\alpha} \right)^{\frac{\alpha}{\gamma(\psi+\alpha-1)}} \left(\frac{1}{1-\alpha} \right)^{\frac{1-\alpha}{\gamma(\psi+\alpha-1)}} \end{aligned} \qquad (11)$$

This occupational threshold $\underline{h_{it}}(w_{t+1})$ is increasing with the equilibrium wage. Indeed, when the equilibrium wage is high, more agents are better of being workers than being entrepreneurs. A higher equilibrium wage per unit of human capital implies more salary for workers and less benefits for entrepreneurs.

In this model the distribution of human capital affects the short run equilibrium characterized by the equilibrium wage and the number of workers and entrepreneurs. The effect of the initial human capital distribution is twofold. On the one hand, if there are many agents in the initial distribution of human capital with low level of human capital there may be many agents who choose to be workers implying a high level of occupational threshold. On the other hand many agents below the threshold yields a high level of

[8] But for intergenerational interests they will choose to be entrepreneur.

[9] See appendix for calculous details.

labor supply what implies a low level of equilibrium wage. However, a low level of wage lessens the labor cost what makes a lower level of human capital "entrepreneurial profitable". This implies in turn a low level of occupational threshold

3 Equilibrium with Endogenous Inequality.

Now turn to the general equilibrium of this model. First, since agents are working only during one period, their second period of life, contracts in markets are stroked up by same aged agents. This implies that the next period expected wage equilibrium is really known today because it is completely determined by the choices made by today's generation of agents. Therefore we write equilibrium of the model by solving the next period market. That is the short-run equilibrium is given by agents today choices given the next period equilibrium wage. Second, consequently, the dynamics of human capital in this economy is derived by the dynamics of human capital transmission.

3.1 Short-Run Equilibrium.

Let $\Lambda_0(h_i)$ denote the initial distribution function of human capital across agents, defined in $\mathbb{R}_+$ between $h_i^{\min}$ and $h_i^{\max}$ with values in $(0,1)$.

Labor market equilibrium: Aggregated labor supply is given by the sum of workers human capital:

$$H_{t+1} = \int_{h_{it}^{\min}}^{\underline{h_{it}}(w_{t+1})} h_{it+1}^{W} d\Lambda_t(h_i) = \int_{h_{it}^{\min}}^{\underline{h_{it}}(w_{t+1})} \theta \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma} h_{it}^{\gamma} d\Lambda_t(h_i)$$

While aggregated labor demand is the sum of entrepreneurs labor demand:

$$L_{t+1} = \int_{\underline{h_{it}}(w_{t+1})}^{h_{it}^{\max}} L_{it+1} d\Lambda_t(h_i) = \int_{\underline{h_{it}}(w_{t+1})}^{h_{it}^{\max}} \left(\frac{\alpha}{w_{t+1}}\right)^{\frac{1}{1-\alpha}} \theta^{\frac{\psi}{1-\alpha}} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\alpha}} h_{it}^{\frac{\gamma\psi}{1-\alpha}} d\Lambda_t(h_i)$$

Therefore, the equilibrium condition of the labor market is written:

$$\theta \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma} \int_{h_{it}^{\min}}^{\underline{h_{it}}(w_{t+1})} h_{it}^{\gamma} d\Lambda_t(h_i) = \left(\frac{\alpha}{w_{t+1}}\right)^{\frac{1}{1-\alpha}} \theta^{\frac{\psi}{1-\alpha}} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\alpha}} \int_{\underline{h_{it}}(w_{t+1})}^{h_{it}^{\max}} h_{it}^{\frac{\gamma\psi}{1-\alpha}} d\Lambda_t(h_i) \tag{12}$$

This equilibrium of the labor market gives the equilibrium wage per unit of human capital. But, this equilibrium condition is an implicit function of the wage due to the occupational threshold. Thus, equations (11) and (12) define the temporary equilibrium of the model.

3.2 Existence and Uniqueness of the Equilibrium.

Given an initial distribution of human capital, the economic equilibrium is therefore defined by a system of two equations (11) and (12) of two unknown variables the equilibrium wage w_{t+1} and the occupational threshold $\underline{h_{it}}$ that separates workers from entrepreneurs.

Proposition 2 *: Given the date t human capital distribution, there exists a unique equilibrium of the economy, characterized by the occupational threshold* $\underline{h_{it}}$ *and the equilibrium wage* w_{t+1}.

Proof. Let us rewrite equation (11) as $\underline{h_{it}} = A * (w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}$ where A is a positive coefficient. By substituting this expression in the equation (12), we rewrite the equilibrium condition as a function $\Phi(w_{t+1}) = 0$. For a given distribution of human capital, we show[10] that $\Phi'(w_{t+1})$ is positive and $\lim_{w_{t+1} \to w_{t+1}^{\min}} \Phi(w_{t+1}) < 0$ and $\lim_{w_{t+1} \to w_{t+1}^{\max}} \Phi(w_{t+1}) > 0$. Therefore, there exists a unique w_{t+1} that realizes $\Phi(w_{t+1}) = 0$. As $\underline{h_{it}}$ is strictly increasing with w_{t+1}, the equilibrium is uniquely determined by a couple $(w_{t+1}, \underline{h_{it}})$. ■

Thus, at any date there exists a wage equilibrium for which some agents choose endogenously to be worker while some else choose to be entrepreneurs. As entrepreneurs earnings profile is higher than that of workers they will be the rich in the economy while the workers will be the poor.

This equilibrium requires some comments. First, even if the economic equilibrium requires the presence of the two categories of agents in the economy, the inequality that arises in this economy is endogenous[11]. It is endogenous in the sense that agents status depends on their optimal choice given their endowment and on the equilibrium price, which, in turn depends on agents choices. Second, the frontier between rich agents (entrepreneurs) and poor agents (workers) is endogenous and is moving with the equilibrium price (the wage equilibrium). This feature of the inequality in this model looks very like with that in Matsuyama's [2000] model. Note that the Matsuyama's (2000) model is not a framework of maximizing agents, but rather it is the imperfection in the credit market and the non convexity in the investment technology that yield the endogenous inequality threshold. While in this model, even if markets are perfect, endogenous inequality emerges due to the structure of earnings in the economy. Also, there is no non convexity in the investment technology at the individual level. Indeed, every amount of investment in the entrepreneurial project is available in this economy for agents. In other words, there is no technological or institutional barrier that prevents agents to be entrepreneurs. What is really deterministic is their initial endowment of human capital. Third, the class agents that emerge in this economy are heterogenous class agents. Indeed, since agents have different level of human capital there will be different level of workers and of entrepreneurs. Will this endogenous inequality persist in the long run?

4 Endogenous Persistent Inequality.

As seen in the previous section the economic equilibrium implies an equilibrium wage for which agents choose endogenously their occupational status and therefore their investment in education. The short run equilibrium is completely determined since the expected next period wage equilibrium depends entirely on today's human capital distribution. All the dynamics in this economy is driven by the transmission of human capital. The transition equation of human capital distribution is written:

[10] See appendix for calculous

[11] We rule out trivial initial distribution, like totally equal initial distribution of human capital.

$$h_{it+1}=\begin{cases}\theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}h_{it}^{\gamma} & \text{if } h_{it}<\underline{h_{it}}(w_{t+1})\\ \theta\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\gamma}h_{it}^{\gamma} & \text{if } \underline{h_{it}}(w_{t+1})\leq h_{it}\end{cases} \tag{13}$$

The first part of equation (13) is the transition equation of workers while the second part is the transition equation of entrepreneurs. As seen in equation (5) for entrepreneurs and equation (8) for workers, the would be entrepreneur invest more in education than the would be worker. The would be entrepreneur invest more in education because the returns optimal point of the project technology requires a higher level of human capital. It is an ex-post justification and not an ex-ante one. In other words, would be entrepreneurs renounce to more leisure today, not because they have less preference on leisure, but because it is required by the optimal condition of their entrepreneurial investment. All same class agents invest the same amount[12] in education, and therefore they have the same transition path of human capital.

Note also that human capital transition equations are independent of the equilibrium wage, while the occupational threshold between being worker or entrepreneur is evolving with the equilibrium wage.

4.1 Steady State Equilibrium with Inequality.

The steady state is associated with the limit distribution of human capital and the limit equilibrium wage w_{∞}. Define a steady state as a state which replicates itself over time, once the economy is settled in, and where all the households hold a constant level of human capital. That is, all workers and entrepreneurs have reach their respective long run level of human capital and the number of agents in each group is constant. In such a steady state, the human capital of all workers must converge to the fixed point of the map,

$$h_{i\infty}^{W}=\theta^{\frac{1}{1-\gamma}}\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\frac{\gamma}{1-\gamma}} \tag{14}$$

and next, the human capital of all entrepreneurs must converge to the fixed point of the map,

$$h_{i\infty}^{E}=\theta^{\frac{1}{1-\gamma}}\left(\frac{\sigma\gamma\psi}{(1-\sigma)\,(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma}{1-\gamma}} \tag{15}$$

The following Figure 2 illustrates the evolution of workers and entrepreneurs human capital.

[12] This is a very common feature of human capital model, see d'Autume and Michel [1994] for detailed analysis.

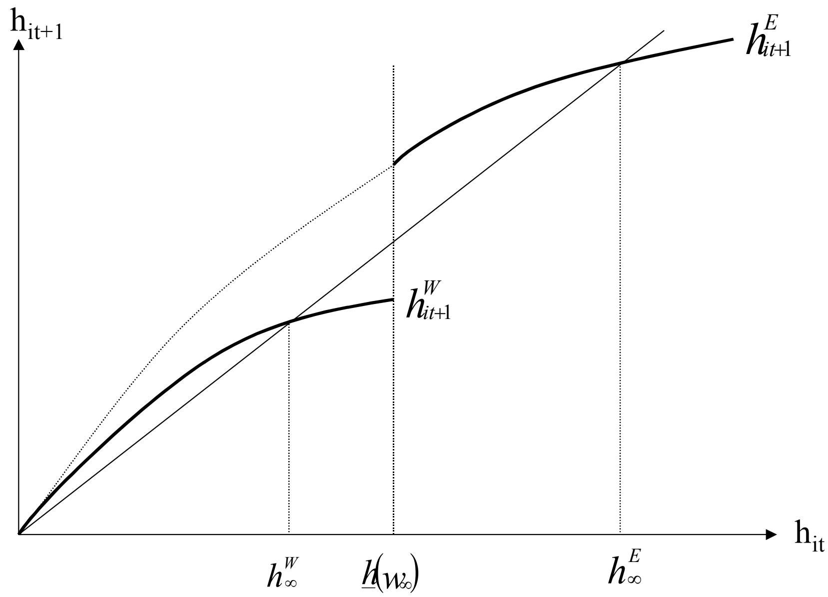

Figure 2: Inequality at steady state.

As seen in Figure 2, in the long run, all entrepreneurs converge to the same level of human capital $h^E_{i\infty}$ and therefore the same level of earnings just as all workers reach the same level of human capital $h^W_{i\infty}$ and earn the same revenue. Let denote $0 < \mu_\infty < 1$ the steady state fraction of workers (therefore the fraction of entrepreneurs is $1 - \mu_\infty$). Substituting equations (14) and (15) in equation (12) yields the steady state equilibrium condition of the economy :

$$\theta^{\frac{1}{1-\gamma}} \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma} \right)^{\frac{\gamma}{1-\gamma}} \mu_\infty = \left(\frac{\alpha}{w_\infty} \right)^{\frac{1}{1-\alpha}} \theta^{\frac{1}{1-\gamma}\frac{\psi}{1-\alpha}} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi} \right)^{\frac{\gamma}{1-\gamma}\frac{\psi}{1-\alpha}} (1-\mu_\infty) \tag{16}$$

Rewriting this equilibrium condition to express the relationship between the fraction of workers and the steady state equilibrium wage yield:

$$\mu_\infty = \frac{\theta^{\frac{1}{1-\gamma}\frac{\psi}{1-\alpha}} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi} \right)^{\frac{\gamma}{1-\gamma}\frac{\psi}{1-\alpha}}}{\left(\frac{w_\infty}{\alpha} \right)^{\frac{1}{1-\alpha}} \theta^{\frac{1}{1-\gamma}} \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma} \right)^{\frac{\gamma}{1-\gamma}} + \theta^{\frac{1}{1-\gamma}\frac{\psi}{1-\alpha}} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi} \right)^{\frac{\gamma}{1-\gamma}\frac{\psi}{1-\alpha}}} \tag{17}$$

As given by equation(17) the relation between the number of poor workers and the equilibrium wage is strictly monotonic. This implies the uniqueness of the economic steady state equilibrium. That is for any steady state equilibrium a unique couple (w_∞, μ_∞). For the rest of the paper we will characterize a steady state equilibrium by a couple (w_∞, μ_∞). Note that, the stability of steady state equilibrium follows from the stability[13] of steady state entrepreneurs and workers human capital.

4.2 Multiple Steady States with Inequality.

To demonstrate the existence of a two-point steady-state distribution of human capital, it suffices to show that at steady state equilibrium some agents are better of being workers while some else are better of being

[13] See the appendix for details.

entrepreneurs. From the steady state equilibrium condition, define the equilibrium wage as a function of the number of poor workers:

$$w_\infty(\mu_\infty) = \alpha \left(\frac{1-\mu_\infty}{\mu_\infty}\right)^{(1-\alpha)} \theta^{\frac{\alpha+\psi-1}{(1-\gamma)}} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\gamma}} \left(\frac{1-\sigma+\sigma\gamma}{\sigma\gamma}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}} \tag{18}$$

For any given steady state, the class of workers exists if and only if $V^W(h^W_\infty, w_\infty(\mu)) > V^E(h^W_\infty, w_\infty(\mu))$, that is given the equilibrium wage, they are better[14] of being workers. This implies a restriction on the steady state fraction of workers:

$$\mu_\infty < \left[1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}{\psi(1-\sigma+\sigma\gamma)}\right)^{\frac{\gamma\gamma\psi}{(1-\gamma)(1-\alpha)}} \left(\frac{(1-\alpha)(1-\sigma+\sigma\gamma)}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{1-\sigma}{\sigma}}\right]^{-1} \equiv \mu^+_\infty$$

Likewise, there are steady state entrepreneurs if they are better of being entrepreneurs. That is, if $V^E(h^E_\infty, w_\infty(\mu)) > V^W(h^E_\infty, w_\infty(\mu))$. This condition implies the following restriction on the steady state fraction of entrepreneurs:

$$\mu_\infty > \left[1 + \frac{(1-\alpha)}{\alpha}\left(\frac{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}{\psi(1-\sigma+\sigma\gamma)}\right)^{\frac{\gamma\gamma}{1-\gamma}} \left(\frac{(1-\sigma+\sigma\gamma)(1-\alpha)}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{1-\sigma}{\sigma}}\right]^{-1} \equiv \mu^-_\infty$$

These two restrictions taken together give the following range for the existence of steady state with two-point distribution of human capital:

$$\mu^-_\infty < \mu_\infty(w_\infty) < \mu^+_\infty$$

As there is a monotonic relationship between μ_∞ and w_∞ , we also have a corresponding range of equilibrium wage that support the existence of inequality at steady state. That is $w^-_\infty < w_\infty < w^+_\infty$ where[15] $w^-_\infty \equiv w_\infty(\mu^+_\infty)$ and $w^+_\infty \equiv w_\infty(\mu^-_\infty)$. To summarize,

Proposition 3 *There exists a continuum of steady states with a two-point distribution of human capital,* $h^W_{i\infty} = \theta^{\frac{1}{1-\gamma}}\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\frac{\gamma}{1-\gamma}}$ *and* $h^E_{i\infty} = \theta^{\frac{1}{1-\gamma}}\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma}{1-\gamma}}$ *with* $\mu_\infty \in]\mu^-_\infty, \mu^+_\infty[$ *and* $w_\infty \in]w^-_\infty, w^+_\infty[$.

All these steady states are characterized by a two-point distribution of wealth and human capital, and the degree of inequality differ across these steady states. A low steady state equilibrium wage is associated with greater inequality. That is more poor workers that earn less salary. Indeed, at steady state the presence of a large number of workers keeps the equilibrium wage low. A lower equilibrium wage favours the few rich entrepreneurs at the expense of the poor workers, which increase the wealth gap. In the other extreme, a high equilibrium wage is associated with lesser inequality. The presence of a relatively fewer workers pins down the equilibrium wage at a high level, what assure a higher level of revenue to workers and reduce the wealth gap.

Notice that, $w_\infty \notin]w^-_\infty, w^+_\infty[$ are possible steady state without inequality. But, in reality, all these steady states are not feasible stable steady states. For example, if $w_\infty > w^+_\infty$ no agent has interest in becoming entrepreneurs and therefore there could not be any wage contract in the labor market. The reason is that the wage is enough high to imply that it is strictly preferable to be worker, as worker's value function is

[14] See the appendix for details.

[15] See the appendix the expressions of w^-_∞ and w^+_∞.

increasing with the wage, than to be entrepreneur, as entrepreneur's value function is decreasing with the equilibrium wage. If, in the other side, $w_\infty < w_\infty^-$ then all agents prefer to be entrepreneurs and therefore there is no labor supply in the economy. However, if we allow for self employment, the unique stable steady state with equality would be the one where all agents are entrepreneurs and they employ themselves. That is the economy does not need salaried activity.

The exact equilibrium steady state (w_∞, μ_∞) which occurs depends on the initial human capital distribution $\Lambda_0(h_i)$. This corresponds to the so called multiple[16] equilibrium steady state. That is the same economic parameters are consistent with numerous steady state outcomes, with varying degrees of inequality, output, and productive efficiency. The multiplicity of long-run outcomes may simply reflect the possible multiplicity of initial conditions. Such multiplicity of equilibrium steady states is very frequent in the economic literature on inequality or development. See for example among many others, Banerjee and Newman [1993], Galor and Zeira [1993], Lundqvist [1993], Durlauf [1996], Piketty [1997], and Matsuyama [2000]. This paper differs from these in several respects. These papers are all build with the hypothesis of imperfect market capital that prevent less endowed agents to borrow and invest in the high return project or occupation in the economy. This with the assumption of indivisible level of investment generates the long run dependency.

Another kind of model in the growth literature, originating with Romer [1986] and Lucas [1988] exhibit such multiple equilibria. These papers, in general, consider technological increasing returns stemming from the production technology itself or some kinds of productivity spillover. In these models, distribution does not play a determinant role.

Unlike these models, in this paper there is no credit market imperfections and all level of investment in the entrepreneurial project are admitted. What is crucial to generate different class of agents is the exact comparative advantage agents have in choosing such occupations given their endowment. It is the hypothesis implicitly included in the human capital production function and in the technology of the entrepreneurial project that are determinant for agents comparative advantage. By choosing their occupational status agents choose the long run path of their dynasty as seen in Figure 2. Therefore, this model exhibits long run dependence on occupational choice and on initial human capital distribution. This persistency is transmitted to the equilibrium wage since the equilibrium wage depends strongly on the number of agents in each occupation. This joint dynamics of the equilibrium wage and of the human capital distribution therefore yield a long run dependency on initial human capital distribution.

5 Dynamics and Mobility

The precise map between initial conditions and the steady state reached, and the transitory process is of interest. It permits to ask wether inequality is increasing or decreasing over time. That is wether there is some kind of mobility in this economy. Note that, as seen in Figure 2, there is no possible mobility at steady state. That is, once the economy has reached the steady state all workers stay worker forever as well as all entrepreneurs. But, during the transitory process as the occupational threshold moves with the equilibrium wage, some mobility may be possible. Though that, a complete mathematical analysis of the joint dynamics

[16] See Mookherjee and Ray [2003] for detailed analysis of such multiple equilibrium steady states.

of the human capital distribution and of the wage is beyond the scope of this paper, some insight into this dynamics may be given. Recall that the path of human capital accumulation by workers and entrepreneurs as given by the equations (6) and (9) is independent of the wage, while the occupational threshold is moving with equilibrium wage.

The following figure illustrates the different possibility of the joint dynamics of the human capital distribution and of the wage.

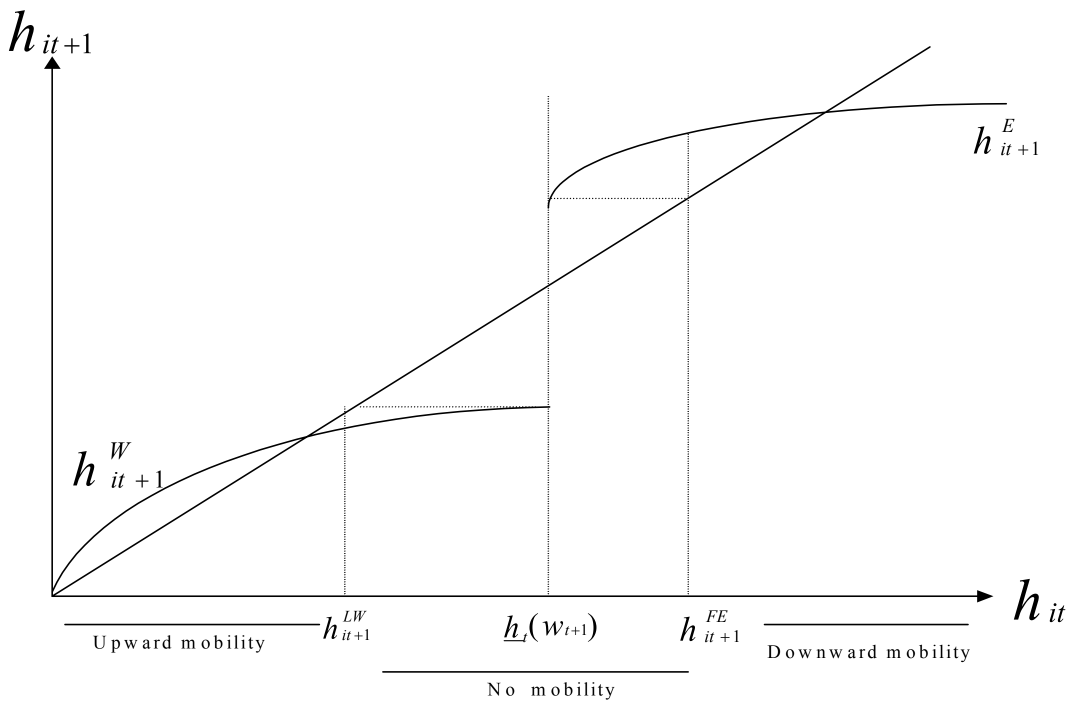

Figure 3: How mobility occur.

Figure 3 illustrates the configuration possible from one period to the following period. Define mobility as some dynasty changing of occupation. That is some offspring of workers becoming entrepreneurs or some offspring of entrepreneurs becoming workers. Since the difference equations that define the path of the two categories of human capital are order preserving, to see wether there are mobility or not it suffices to check the situation of the offspring of the last worker and of the offspring of the first entrepreneurs. The last worker (h_{it}^{LW} in the Figure 3) is the worker who has the highest level of human capital across workers, he is the worker whose human capital is just below the occupational threshold. The first entrepreneurs (h_{it}^{FE} in the Figure 3) is the entrepreneur who has the lowest level of human capital across entrepreneurs, his human capital is equal to or just above the occupational threshold. Thus at the equilibrium of a given period t we have $h_{it}^{LW} < \underline{h_{it}}(w_{t+1}) < h_{it}^{FE}$. Focusing on the variation of the equilibrium wage, one can derive some conditions for mobility:

Upward mobility: that is from first period to the second period some workers-dynasty become entrepreneurs. At least one worker family changes of category if $h_{it}^{LW} < \underline{h_{it}}(w_{t+1})$ and $h_{it+1}^{LW} > \underline{h_{it+1}}(w_{t+2})$. This is the case if $\theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}\left(h_{it}^{LW}\right)^{\gamma} > A\left(w_{t+2}\right)^{\frac{1}{\gamma(\psi+\alpha-1)}}$. This condition depends on the specific human capital of the last worker. But a bound of the last worker human capital is the first period occupational threshold. Thus, the second period human capital of the last worker whose human capital is equal to the first period occupational threshold[17] is $h_{it+1}^{LW} = \theta \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma} A^{\gamma} \left(w_{t+1}\right)^{\frac{1}{\psi+\alpha-1}}$. Therefore if this is greater than the second period human capital occupational threshold then there is upward mobility. This condition is checked iff:

$$\frac{\left(w_{t+2}\right)^{\frac{1}{\gamma}}}{\left(w_{t+1}\right)} < \theta^{(\psi+\alpha-1)} \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma(\psi+\alpha-1)} A^{(\gamma-1)(\psi+\alpha-1)} \tag{19}$$

Equation (19) gives a condition on the evolution of the equilibrium wage to guarantee some upward mobility as illustrated in Figure 3. If this condition is verified during some successive periods then there is upward mobility during some periods before the economy reaches its steady state.

Downward mobility: that is from first period to the second period some entrepreneurs dynasty become workers. There is downward mobility if $h_{it+1}^{FE} < \underline{h_{it+1}}(w_{t+2})$. The human capital of the first entrepreneurs is bounded at the bottom by the human capital occupational threshold. Computing this condition with the human capital threshold gives the following condition for downward mobility:

$$\theta^{(\psi+\alpha-1)} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\gamma(\psi+\alpha-1)} A^{(\gamma-1)(\psi+\alpha-1)} < \frac{\left(w_{t+2}\right)^{\frac{1}{\gamma}}}{w_{t+1}} \tag{20}$$

No mobility: that is all dynasty of workers stay workers as well as all dynasty of entrepreneurs stay entrepreneurs. The condition of no mobility follow directly from the reverse of previous conditions:

$$\theta^{(\psi+\alpha-1)} \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma(\psi+\alpha-1)} A^{(\gamma-1)(\psi+\alpha-1)} < \frac{\left(w_{t+2}\right)^{\frac{1}{\gamma}}}{w_{t+1}} < \theta^{(\psi+\alpha-1)} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\gamma(\psi+\alpha-1)} A^{(\gamma-1)(\psi+\alpha-1)} \tag{21}$$

As illustrated in the Figure 3, if from the first period to the following period, the second period occupational threshold $\underline{h_{it+1}}(w_{t+2})$ is between h_{it+1}^{LW} and h_{it+1}^{FE} then there is no mobility between the two periods. That is $h_{it+1}^{LW} < \underline{h_{it+1}}(w_{t+2}) < h_{it+1}^{FE}$. If this is the case for all successive periods until the steady state, then there is no mobility during the transitory process. That is dynasty occupation is determined once for all at the beginning of the economy.

The intuition behind these results is easy to grasp. If the second period equilibrium wage is very high then for the entrepreneurs with low level of human capital, it is preferable to become workers. This is because a high level of equilibrium wage decreases their entrepreneurs profit as it increases the labor cost. Therefore their value function of being workers is greater than their's one of being entrepreneurs implying downward mobility. The workers will become entrepreneurs if the variation in the equilibrium wage is not important or if the equilibrium wage decreases. Thus the labor cost decreases, what makes better, for their level of human capital, to become entrepreneurs implying upward mobility. If rather the second period equilibrium wage is quite close to the first period equilibrium wage then no agent has strict benefits in moving from one occupation to another. That is the movement in the equilibrium wage does not imply enough variation in the value function of any agent as he would be better in changing of occupation. This may be the case if the initial equilibrium wage is very close to the steady state equilibrium wage implying a quick convergence to the stationary equilibrium.

[17] The first period occupational threshold given by equation (11) is rewritten $\underline{h_{it}}(w_{t+1}) = A\left(w_{t+1}\right)^{\frac{1}{\gamma(\psi+\alpha-1)}}$ where A is the coefficient given by all the terms in this equation, it is positive.

The exact dynamics of equilibrium wage that an economy experiences depends strongly on the shape of its initial distribution of human capital and therefore what kind of mobility will occur during the transitory process. However, this model provides some insight on how the rise and fall of families may occur.

6 Concluding Remarks

Since Loury [1981] seminal paper, many contributions have emphasized the role of imperfect capital market in the formation of inequality and its persistence. Among many others, Galor and Zeira [1993], Banerjee and Newman [1993], Ljungqvist [1993], Durlauf [1996], Aghion and Bolton [1997] and Mookherjee and Ray [2003] have shown that both assumptions of imperfect capital market and non convex investment technology are important to generate the formation and persistence of inequality. In this line, Matsuyama [2000] modeled endogenous formation of inequality. In his model there is an endogenous threshold depending on the equilibrium interest rate which separates rich agents from poor agents. In some cases, this inequality persists in the long run.

In contrast to these analyses, in this paper there is no credit market imperfections and all level of investment in the entrepreneurial project are admitted. However, the model generates endogenous persistent inequality. That is there exists a unique human capital threshold which separates workers from entrepreneurs. In this model, less endowed agents in human capital have comparative advantage in choosing to become workers, and therefore they invest less in education. Agents with relatively higher endowment of human capital choose to be entrepreneurs and they invest more in education. These two class of agents persist in the long run. However there is a place for upward or downward mobility during the transition process. Indeed, as the human capital threshold depends on the equilibrium wage, high variation of equilibrium wage during the transition process implies agents mobility. If from one period to the next, the equilibrium wage increases enough, then some entrepreneurs dynasty are better off becoming workers, implying downward mobility. At the reverse, if the equilibrium wage decreases enough then some workers dynasty could become entrepreneurs implying upward mobility.

The model also exhibits a continuum of steady states, each of which is characterized by a two-point distribution and an equilibrium wage. This reflect the fact that different economies with different initial distribution of human capital will converge to different level of steady states. That is at steady states, they will differ by the number of agents they have in each occupations and by the equilibrium wage.

The model presented above have some strong implications on theoretical and policy analysis. Indeed, if inequality emerges even if markets are perfect then there is a debate wether inequality are due mainly to economic structure or to market imperfections. If the mechanisms designed here are a stylized version of which produces inequality then market imperfections only increase the degree of inequality in the economy. This model can be extended to take into account market imperfections. There are two ways to add a role for financial capital variable. One is in financing education or training of children by parents. Another is that the entrepreneurial project requires a financial investment. Therefore in presence of borrowing constraints, the financial variable will also play a role on the occupational choice. Indeed, agents who inherit less wealth or who save less would be subject to borrowing constraint and therefore would choose to be workers.

The model also implicitly implies that intergenerational human capital transmission is the key variable which explain inequality persistency. In that case, the policy debate is still wether it is preferable to focus on ex-ante redistribution in schooling or education or on ex-post redistribution on revenue. The model can be extended to take into account this debate.

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Appendix.

Existence and uniqueness of the occupational threshold.

Let us define $V(h_i) \equiv V^w - V^E$

$$V(h_i) = \left(\frac{1-\sigma}{1-\sigma+\sigma\gamma}\right)^{1-\sigma} w_{t+1}^{\sigma}\theta^{\sigma}\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\sigma\gamma} h_{it}^{\sigma\gamma} - h_{it}^{\frac{\sigma\gamma\psi}{1-\alpha}}\left(\frac{(1-\sigma)(1-\alpha)}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{1-\sigma}$$
$$(1-\alpha)^{\sigma}\theta^{\frac{\sigma\psi}{1-\alpha}}\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\sigma\gamma\psi}{1-\alpha}}\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{\sigma\alpha}{1-\alpha}}$$

The derivative of this function $V(h_i)$ is written:

$$V'(h_i) = \left(\frac{1-\sigma}{1-\sigma+\sigma\gamma}\right)^{1-\sigma} w_{t+1}^{\sigma}\theta^{\sigma}\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\sigma\gamma}\sigma\gamma h_{it}^{\sigma\gamma-1} - \frac{\sigma\gamma\psi}{1-\alpha}h_{it}^{\frac{\sigma\gamma\psi+\alpha-1}{1-\alpha}}\left(\frac{(1-\sigma)(1-\alpha)}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{1-\sigma}$$
$$(1-\alpha)^{\sigma}\theta^{\frac{\sigma\psi}{1-\alpha}}\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\sigma\gamma\psi}{1-\alpha}}\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{\sigma\alpha}{1-\alpha}}$$

The second derivative of this function is:

$$V''(h_i) = (\sigma\gamma-1)\sigma\gamma\left(\frac{1-\sigma}{1-\sigma+\sigma\gamma}\right)^{1-\sigma} w_{t+1}^{\sigma}\theta^{\sigma}\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\sigma\gamma} h_{it}^{\sigma\gamma-2} - \frac{\sigma\gamma\psi}{1-\alpha}\frac{\sigma\gamma\psi+\alpha-1}{1-\alpha}h_{it}^{\frac{\sigma\gamma\psi+2\alpha-2}{1-\alpha}}$$
$$*\left(\frac{(1-\sigma)(1-\alpha)}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{1-\sigma}(1-\alpha)^{\sigma}\theta^{\frac{\sigma\psi}{1-\alpha}}\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\sigma\gamma\psi}{1-\alpha}}\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{\sigma\alpha}{1-\alpha}}$$

$V''(h_i) < 0$ iff $\sigma\gamma\psi + \alpha > 1$ what is assumed. Therefore $V(h_i)$ is strictly concave and, as $V(0) = 0$ and $\lim_{h_i\to\infty} V(h_i) = -\infty$ there exists a unique human capital level $\underline{h_i}$ for which $V(\underline{h_i}) = 0$.

Existence and uniqueness of the short run equilibrium.

Recall that, substituting the occupational threshold equation in the equilibrium condition of the labor market yield the following short-run economic equilibrium:

$$\theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}\int_{h_{it}^{\min}}^{A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}} h_{it}^{\gamma}\,d\Lambda(h_{it}) =$$
$$\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{1}{1-\alpha}}\theta^{\frac{\psi}{1-\alpha}}\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\alpha}}\int_{A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}}^{h_{it}^{\max}} h_{it}^{\frac{\gamma\psi}{1-\alpha}}\,d\Lambda(h_{it})$$

We can rewrite this equation as $\Phi(w_{t+1}) = 0$. To show that there is a unique $w_{t+1} > 0$ that solves this equation to zero, we will show that this function is strictly increasing and it goes from $-\infty$ to a positive value. Therefore, there exist a unique w_{t+1} for which this function is equal to zero.

Using the Liebniz formulae we show that $\Phi'(w_{t+1}) > 0$:

$$\Phi'(w_{t+1}) = \theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}\left[A^{\gamma+1}\frac{(w_{t+1})^{\frac{1-\gamma(\psi+\alpha-1)}{\gamma(\psi+\alpha-1)}+\frac{1}{(\psi+\alpha-1)}}}{\gamma(\psi+\alpha-1)}\Lambda\left(A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}\right)\right]$$
$$+\theta^{\frac{\psi}{1-\alpha}}\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\alpha}}\left[\begin{array}{c}\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{1}{1-\alpha}}A^{\frac{\gamma\psi}{1-\alpha}+1}\frac{(w_{t+1})^{\frac{1-\gamma(\psi+\alpha-1)}{\gamma(\psi+\alpha-1)}+\frac{\psi}{(\psi+\alpha-1)(1-\alpha)}}}{\gamma(\psi+\alpha-1)}\Lambda\left(A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}\right)\\ +\int_{A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}}^{h_{it}^{\max}}\frac{1}{1-\alpha}\left(\frac{\alpha}{w_{t+1}^2}\right)\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{\alpha}{1-\alpha}}h_{it}^{\frac{\gamma\psi}{1-\alpha}}\Lambda(h_{it})\,dh_{it}\end{array}\right]$$

As it is a sum of positive terms $\Phi^{'}(w_{t+1}) > 0$. Thus $\Phi(w_{t+1})$ is increasing and monotonic with w_{t+1}.

The limits of $\Phi(w_{t+1})$

Define $w_{t+1}^{\min} = \left(\frac{h_{it}^{\min}}{A}\right)^{\gamma(\psi+\alpha-1)}$ as the wage level for which the agents with the lowest level of human capital would be entrepreneurs and $w_{t+1}^{\max} = \left(\frac{h_{it}^{\max}}{A}\right)^{\gamma(\psi+\alpha-1)}$ as the wage level for which the agents with the highest level of human capital would be workers.

We proceed by showing that $\lim_{w_{t+1}\to 0}\Phi(w_{t+1}) = -\infty$ and thereafter for a sufficient low $w_{t+1}^{\min}$, $\lim_{w_{t+1}\to w_{t+1}^{\min}}\Phi(w_{t+1}) < 0$:

$$\lim_{w_{t+1}\to 0}\Phi(w_{t+1}) = \lim_{w_{t+1}\to 0}\theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}\left[E\left(h_{it}^{\gamma}\right) - \int_{A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}}^{h_{it}^{\max}} h_{it}^{\gamma}d\Lambda(h_{it})\right]$$

$$-\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{1}{1-\alpha}}\theta^{\frac{\psi}{1-\alpha}}\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\alpha}}\int_{A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}}^{h_{it}^{\max}} h_{it}^{\frac{\gamma\psi}{1-\alpha}}d\Lambda(h_{it})$$

$$\lim_{w_{t+1}\to 0}\Phi(w_{t+1}) = \theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}\left[E\left(h_{it}^{\gamma}\right) - E\left(h_{it}^{\gamma}\right)\right]$$

$$-\left(\frac{\alpha}{w_{t+1}}\right)^{\frac{1}{1-\alpha}}\theta^{\frac{\psi}{1-\alpha}}\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\alpha}} E(h_{it}^{\frac{\gamma\psi}{1-\alpha}}) = -\infty$$

As $\lim_{w_{t+1}\to 0} A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}} = 0$ and $\int_{A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}}^{h_{it}^{\max}} h_{it}^{\gamma}d\Lambda(h_{it}) = \int_{0}^{h_{it}^{\max}} h_{it}^{\gamma}d\Lambda(h_{it}) = \int_{h_{it}^{\min}}^{h_{it}^{\max}} h_{it}^{\gamma}d\Lambda(h_{it}) = E\left(h_{it}^{\gamma}\right)$

Therefore it suffices to assume that $h_{it}^{\min}$ is sufficiently low such that for the corresponding $w_{t+1}^{\min}$

$$\lim_{w_{t+1}\to w_{t+1}^{\min}}\Phi(w_{t+1}) = \theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}\int_{h_{it}^{\min}}^{A*\left(w_{t+1}^{\min}\right)^{\frac{1}{\gamma(\psi+\alpha-1)}}} h_{it}^{\gamma}d\Lambda(h_{it}) - \left(\frac{\alpha}{w_{t+1}^{\min}}\right)^{\frac{1}{1-\alpha}}\theta^{\frac{\psi}{1-\alpha}}$$

$$\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\alpha}}\int_{A*\left(w_{t+1}^{\min}\right)^{\frac{1}{\gamma(\psi+\alpha-1)}}}^{h_{it}^{\max}} h_{it}^{\frac{\gamma\psi}{1-\alpha}}d\Lambda(h_{it}) < 0$$

Likewise, we show that $\lim_{w_{t+1}\to+\infty}\Phi(w_{t+1}) > 0$ and thereafter for a sufficient high $w_{t+1}^{\max}$, $\lim_{w_{t+1}\to w_{t+1}^{\max}}\Phi(w_{t+1}) > 0$:

$$\lim_{w_{t+1}\to\infty}\Phi(w_{t+1}) = \theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}\int_{h_{it}^{\min}}^{A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}} h_{it}^{\gamma}d\Lambda(h_{it}) - \left(\frac{\alpha}{w_{t+1}}\right)^{\frac{1}{1-\alpha}}\theta^{\frac{\psi}{1-\alpha}}$$

$$\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\alpha}}\int_{A*(w_{t+1})^{\frac{1}{\gamma(\psi+\alpha-1)}}}^{h_{it}^{\max}} h_{it}^{\frac{\gamma\psi}{1-\alpha}}d\Lambda(h_{it})$$

$$\lim_{w_{t+1}\to\infty}\Phi(w_{t+1}) = \theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}\int_{h_{it}^{\min}}^{h^{\max}} h_{it}^{\gamma}d\Lambda(h_{it})$$

$$\lim_{w_{t+1}\to\infty}\Phi(w_{t+1}) = \theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}E\left(h_{it}^{\gamma}\right) > 0$$

Therefore it suffices to assume that $h_{it}^{\max}$ is sufficiently high such that for the corresponding $w_{t+1}^{\max}$

$$\lim_{w_{t+1}\to w_{t+1}^{\max}}\Phi(w_{t+1}) = \theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma}\int_{h_{it}^{\min}}^{A*\left(w_{t+1}^{\max}\right)^{\frac{1}{\gamma(\psi+\alpha-1)}}} h_{it}^{\gamma}d\Lambda(h_{it}) - \left(\frac{\alpha}{w_{t+1}^{\max}}\right)^{\frac{1}{1-\alpha}}\theta^{\frac{\psi}{1-\alpha}}$$

$$\left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\alpha}}\int_{A*\left(w_{t+1}^{\max}\right)^{\frac{1}{\gamma(\psi+\alpha-1)}}}^{h_{it}^{\max}} h_{it}^{\frac{\gamma\psi}{1-\alpha}}d\Lambda(h_{it}) > 0$$

Existence and uniqueness

Since $\lim\limits_{w_{t+1}\to w_{t+1}^{\min}} \Phi\left(w_{t+1}\right) < 0$ and $\lim\limits_{w_{t+1}\to w_{t+1}^{\max}} \Phi\left(w_{t+1}\right) > 0$ and $\Phi\left(w_{t+1}\right)$ is increasing, there exists a unique w_{t+1} that realizes $\Phi\left(w_{t+1}\right) = 0$. Since $\underline{h_{it}}(w_{t+1})$ is strictly increasing, the equilibrium is uniquely well defined.

Existence and Stability of steady state:

To show that steady states are stable, it suffices to show that steady states human capital levels are stable. Thus when all agents have reach their steady states human capital,

stability is checked:

$$\begin{aligned}
h_{it+1}^{W} &= \theta\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma} h_{it}^{\gamma} \\
h_{it+1}^{\prime W} &= \theta\gamma\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\gamma} h_{it}^{\gamma-1} \\
h_{it+1}^{\prime W}\left(h_{i\infty}^{W}\right) &= \gamma<1
\end{aligned}$$

$$\begin{aligned}
h_{it+1}^{E} &= \theta\left(\frac{\sigma\gamma\psi}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{\gamma} h_{it}^{\gamma} \\
h_{it+1}^{\prime E} &= \theta\gamma\left(\frac{\sigma\gamma\psi}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{\gamma} h_{it}^{\gamma-1} \\
h_{it+1}^{\prime E}\left(h_{i\infty}^{E}\right) &= \gamma<1
\end{aligned}$$

Existence of different occupations at steady state:

$$V_{\infty}^{w}\left(h_{\infty}^{W}, w_{\infty}\left(\mu\right)\right) = \left(\frac{1-\mu_{\infty}}{\mu_{\infty}}\right)^{\sigma(1-\alpha)} \left(\frac{1-\sigma}{1-\sigma+\sigma\gamma}\right)^{1-\sigma} \alpha^{\sigma}\theta^{\frac{\sigma(\alpha+\psi)}{(1-\gamma)}} \left(\frac{\sigma\gamma\psi}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{\frac{\sigma\gamma\psi}{1-\gamma}} \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\frac{\sigma\gamma\alpha}{1-\gamma}}$$

Compute the value function: $V^{E}(h_{\infty}^{W}, w_{\infty}\left(\mu\right))$

$$\begin{aligned}
V^{E}(h_{\infty}^{W}, w_{\infty}\left(\mu\right)) &= \left(1-\alpha\right)^{\sigma}\left(\frac{1-\mu_{\infty}}{\mu_{\infty}}\right)^{-\sigma\alpha}\left(\frac{\left(1-\sigma\right)\left(1-\alpha\right)}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{1-\sigma}\theta^{\frac{\sigma(\alpha+\psi)}{(1-\gamma)}} \\
&\quad \left(\frac{\sigma\gamma\psi}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{\frac{\sigma\gamma\psi(1-\alpha-\gamma)}{(1-\gamma)(1-\alpha)}}\left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\frac{\sigma\gamma[(1-\alpha)\alpha+\gamma\psi]}{(1-\alpha)(1-\gamma)}}
\end{aligned}$$

thus

$$V^{W}(h_{\infty}^{W}, w_{\infty}\left(\mu\right)) > V^{E}(h_{\infty}^{W}, w_{\infty}\left(\mu\right)) \quad ssi$$

$$\frac{1-\mu_{\infty}}{\mu_{\infty}} > \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\frac{\gamma\gamma\psi}{(1-\alpha)(1-\gamma)}}\left(\frac{\sigma\gamma\psi}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{\frac{(-\gamma)\gamma\psi}{(1-\gamma)(1-\alpha)}}\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\left(1-\alpha\right)\left(1-\sigma+\sigma\gamma\right)}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{\frac{1-\sigma}{\sigma}}$$

$$\mu_{\infty} < \left[1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}{\psi\left(1-\sigma+\sigma\gamma\right)}\right)^{\frac{\gamma\gamma\psi}{(1-\gamma)(1-\alpha)}}\left(\frac{\left(1-\alpha\right)\left(1-\sigma+\sigma\gamma\right)}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{\frac{1-\sigma}{\sigma}}\right]^{-1}$$

Computing the value function of an entrepreneur (staying entrepreneur):

$$\begin{aligned}
V^{E}(h_{\infty}^{E}, w_{\infty}\left(\mu\right)) &= \left(\frac{1-\mu_{\infty}}{\mu_{\infty}}\right)^{-\sigma\alpha}\left(1-\alpha\right)^{\sigma}\left(\frac{\left(1-\sigma\right)\left(1-\alpha\right)}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{1-\sigma}\theta^{\frac{\sigma(\psi+\alpha)}{(1-\gamma)}}\left(\frac{\sigma\gamma\psi}{\left(1-\sigma\right)\left(1-\alpha\right)+\sigma\gamma\psi}\right)^{\frac{\sigma\gamma\psi}{(1-\gamma)}} \\
&\quad \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\frac{\sigma\alpha\gamma}{1-\gamma}}
\end{aligned}$$

Value function of an entrepreneur becoming worker: $V^W(h_\infty^E, w_\infty(\mu))$

$$V^W(h_\infty^E, w_\infty(\mu)) = \left(\frac{1-\mu_\infty}{\mu_\infty}\right)^{(1-\alpha)\sigma} \alpha^\sigma \left(\frac{1-\sigma}{1-\sigma+\sigma\gamma}\right)^{1-\sigma} \theta^{\frac{\sigma(\alpha+\psi)}{(1-\gamma)}} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\sigma\gamma(\gamma+\psi)}{1-\gamma}} \left(\frac{\sigma\gamma}{1-\sigma+\sigma\gamma}\right)^{\frac{\sigma\gamma(\alpha-\gamma)}{(1-\gamma)}}$$

$$\begin{aligned} V^E(h_\infty^E, w_\infty(\mu)) &> V^W(h_\infty^E, w_\infty(\mu)) \quad \text{iff} \\ \mu_\infty &> \frac{1}{\left[\left(\frac{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}{\psi(1-\sigma+\sigma\gamma)}\right)^{\frac{\gamma\gamma}{1-\gamma}} \left(\frac{(1-\sigma+\sigma\gamma)(1-\alpha)}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{1-\sigma}{\sigma}} \frac{(1-\alpha)}{\alpha}+1\right]} \\ \mu_\infty &> \left[1+\frac{(1-\alpha)}{\alpha}\left(\frac{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}{\psi(1-\sigma+\sigma\gamma)}\right)^{\frac{\gamma\gamma}{1-\gamma}} \left(\frac{(1-\sigma+\sigma\gamma)(1-\alpha)}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{1-\sigma}{\sigma}}\right]^{-1} \end{aligned}$$

Mobility

$$\begin{aligned} w_\infty(\mu_\infty^-) &= \alpha\left(\frac{1}{\mu_\infty}-1\right)^{(1-\alpha)} \theta^{\frac{\alpha+\psi-1}{(1-\gamma)}} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\gamma}} \left(\frac{1-\sigma+\sigma\gamma}{\sigma\gamma}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}} \\ &= \alpha\theta^{\frac{\alpha+\psi-1}{(1-\gamma)}} \left(\frac{1-\alpha}{\alpha}\right)^{(1-\alpha)} \left(\frac{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}{\psi(1-\sigma+\sigma\gamma)}\right)^{\frac{\gamma\gamma(1-\alpha)}{1-\gamma}} \left(\frac{(1-\sigma+\sigma\gamma)(1-\alpha)}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} \\ &\quad \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\gamma}} \left(\frac{1-\sigma+\sigma\gamma}{\sigma\gamma}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}} \end{aligned}$$

and $w_\infty^- \equiv w_\infty(\mu_\infty^+)$

$$\begin{aligned} w_\infty(\mu_\infty^+) &= \alpha\left(\frac{1}{\mu_\infty}-1\right)^{(1-\alpha)} \theta^{\frac{\alpha+\psi-1}{(1-\gamma)}} \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\gamma}} \left(\frac{1-\sigma+\sigma\gamma}{\sigma\gamma}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}} \\ &= \alpha\left(\frac{1-\alpha}{\alpha}\right)^{(1-\alpha)} \theta^{\frac{\alpha+\psi-1}{(1-\gamma)}} \left(\frac{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}{\psi(1-\sigma+\sigma\gamma)}\right)^{\frac{\gamma\gamma\psi(1-\alpha)}{(1-\gamma)(1-\alpha)}} \left(\frac{(1-\alpha)(1-\sigma+\sigma\gamma)}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{(1-\sigma)(1-\alpha)}{\sigma}} \\ &\quad \left(\frac{\sigma\gamma\psi}{(1-\sigma)(1-\alpha)+\sigma\gamma\psi}\right)^{\frac{\gamma\psi}{1-\gamma}} \left(\frac{1-\sigma+\sigma\gamma}{\sigma\gamma}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}} \end{aligned}$$