REPLY TO PASINETTI AND ROBINSON

1. We must begin by recording our dismay that our long paper should end up appearing to Dr Pasinetti as primarily apologetics for a specific theory—the neo-classical theory of marginal productivity. We trust other readers will conclude otherwise.

It is true that we have succeeded in working out exhaustively, for the general neo-classical one-sector technology, the dynamic and statical implications of the Pasinetti hypothesis about different rates of saving out of pure-profit income and other incomes; thus, what Solow did a decade ago for the Harrod-Domar model, we have done definitively for the Pasinetti model, freeing our analysis from the earlier Cobb-Douglas assumptions of Meade and deriving stability conditions for the golden-age equilibria that are more definite than one ordinarily encounters in a model of this complexity. Just as Solow’s earlier work called for subsequent extension to two-or-more-sector canonical models—by Uzawa, Solow, Inada, Findlay, Drandakis, Burmeister, and many others—so does our one-sector model of Pasinetti saving invite subsequent extensions to such technologies.

No apologies are needed then for the detail of our investigation. Furthermore, even those more critical of the degree of empirical relevance of the neoclassical notions than we are, might be expected to welcome our elucidation of just how neoclassical notions fit in with Pasinetti’s earlier analysis, a task which he had only lightly touched upon (and in a set of equations blemished by a mathematical error). “Know thy enemy” is good advice for anyone, for to understand the full implications of a theory is to begin to understand its limitations.

2. None the less the major motivation for undertaking the paper was something quite different. It stemmed from our original perception that the Pasinetti golden-age equilibrium, instead of being the general one, had to be recognised as but one of two golden-age equilibria, being matched so-to-speak by what we called the Dual or Anti-Pasinetti equilibrium. (More neutral terminology would be Primal and Dual equilibria, a distinction that has essentially naught to do with the somewhat unfortunate earlier Meade terminology of neo-Keynesian and neo-classical.)
establish once and for all that symmetry does not depend on any simple diminishing returns assumptions of neo-classical type. The important thing is for the reader to understand the reasoning upon which the symmetry of the two regimes is based.

In providing this constructive demonstration, we shall also be able to isolate that special technological case, which if one believed it to be realistic, would provide considerable justification for concentrating on the Pasinetti regime to the exclusion of the Dual on the ground that the Dual regime is in the nature of a knife-edge solution. (And, dually, we shall be able to isolate that special technological case which makes the Pasinetti regime a rare and fortuitous occurrence of knife-edge type.)

Here we shall be examining only the question of existence of golden-age equilibrium, omitting completely all discussions of stability and instability and all consideration of the dynamic transient behaviour of the system when not in a golden age.

3. For simplicity assume a single consumption good (either simple or a composite market basket). Assume a finite number, large or small, of different blue-print pages, each corresponding to a different activity or capital process, calling different machines alpha, beta, gamma, ... in the usual Robinson fashion. As is well known, each profit or interest rate excludes many pages of the blue prints as not being competitively viable, leaving one or more sets of activities that can be viable at the given golden-age profit rate. We stress that nothing "well-behaved" is assumed about the technology other than that the factor-price frontier relating the real wage and the profit rate must be downward sloping. This fact is essential to the postulated equilibrium.
Wicksell effects; nor finally do we exclude any number of reswitching effects of the Cohen-curiousum type. (All this is in contrast with the Solow-Clark one-sector model involving neo-classical surrogate capital and smooth productivities, in which the AZ locus would be a simple ever-rising one.) Furthermore, a change in $n$ will shift the whole AZ locus, since the new weighting of the consumption and investment sectors would change capital-output ratios and factor shares in the absence of any special "equal factor-intensity" assumptions.

In Fig. 1 the $45^\circ$ line represents points where the interest rate and average product of capital are equal, leaving no return at all for labour: thus it is the locus of capital

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Figure 1

$AZ$ is the locus of Average Products corresponding to each profit rate, where $n$ has been fixed. At any point, like $Q$, the reciprocal of the slope of a ray from the origin to it measures capital's relative share, $\alpha_x$: since labour's share cannot be negative, only the area above the $45^\circ$ line is relevant. At $A$ where $r = 0$, labour gets all; at the point of maximum interest rate $Z$, capital gets all. Vertical segments, like $RS$ represent switch points; other segments where the locus slopes downward represent so-called perverse Wicksell effects and other admissible phenomena not found in one-sector well-behaved neoclassical models.

1 In these neoclassical cases, $r = f'(k)$, $f'' < 0$. If the elasticity of substitution, $\sigma$, is always 1, the Cobb-Douglas case, $AZ$ is a straight line through the origin with slope greater than one; for the CES function with $\sigma$ a constant below unity, it is a convex curve going out from the origin and arching above the $45^\circ$ line until it hits that line at the maximum $r$; if $\sigma$ is a constant above unity, $AZ$ begins at a point on the $45^\circ$ line, accelerating above the line toward infinity like a parabola; for $\sigma = \infty$, $AZ$ is a vertical curve above the $45^\circ$ line; for $\sigma = 0$, $AZ$ is a horizontal curve from the vertical axis to the $45^\circ$ line; if $f''(k)$ is asymptotic to both axes, with a passing through unity, $AZ$ rises out from the origin but not.
share equal to unity, or \( \alpha_k = 1 \). At any point on the locus, capital's share \( \alpha_k \) is read off as the reciprocal of the slope of a ray out from the origin to the point on \( AZ \), by virtue of the arithmetical relations,

\[
\alpha_k = \frac{r}{A} = \frac{\text{(profit/capital)}}{\text{(output/capital)}} = \text{profit/output}.
\]

Since wages can never be negative, we never have use for the part of the diagram below the 45° line.\(^1\)

Thus far all has been technology and competitive cost minimization. Fig. 2 now introduces saving behaviour of the Pasinetti type. On the horizontal axis is plotted \( s_c \), the fraction of profit permanently saved by those who earn only profit; on the vertical axis is \( s_w \), the fraction of income of whatever sort permanently saved by those who work \textit{and} own capital from previous accumulation. From any point on Fig. 2, like \( q \) with its specified \((s_c, s_w)\) values, and with \( n \) given, our task is to go back to Fig. 1 and find the corresponding golden-age equilibrium point or points. It is our contention that this can be done for a general blue-print technology; and when it is done, we find that there are two symmetrical regions, the one marked Dual and the one marked Pasinetti in Fig. 2.\(^2\)

Let us review the requirement for golden-age equilibrium that \textit{any} technical model must satisfy once \( n \) and \((s_c, s_w)\) are specified. An \((r, A)\) point on \( AZ \) of Fig. 1 can be a golden-age solution \((r^\infty, A^\infty)\) if and only if it satisfies the following four conditions:

(i) It must be on the \( AZ \) locus \( A(r; n) \), or \( A^\infty = A(r^\infty, n) \), to be competitively viable

(ii) \( r^\infty \leq n/s_c \), so that \( K_c \) does not grow faster than \( L \)

(iii) \( A^\infty \leq n/s_w \), so that \( K_w \) does not grow faster than \( L \)

(iv) At least one of (ii) and (iii) must hold as an equality, or \((r^\infty - n/s_c)(A^\infty - n/s_w) = 0\), so that \( K \) grows as fast as \( L \).

Note that (i)-(iv) hold for \textit{any} technology.

There are two regimes satifying (i)-(iv), Pasinetti and Dual.

**Pasinetti:**

\[
r^\infty = r^* = n/s_c \\
A^\infty = A(r^*; n) < n/s_w, \text{ or } s_w < \frac{r^\infty}{A^\infty} s_c = \alpha(r^*; n)s_c.
\]

**Dual:**

\[
A^\infty = A^{**} = n/s_w \\
r^\infty = r^{**} < n/s_c, \text{ or } s_w > \frac{r^\infty}{A^\infty} s_c = \alpha(r^{**}; n)s_c.
\]

Of course, there is the borderline knife-edge where

\[
r^\infty = r^* = r^{**} = n/s_c \text{ and } A^\infty = A^{**} = A^* = n/s_w
\]

and we are simultaneously in both regimes, but with \((k_c/k)^\infty = 0\).

After this review, we can relate Figs. 2 and 1. To demonstrate all this, first concentrate on a point like \( q \) in Fig. 2, which represents high rentier thrift relative to worker

\(^1\) The vertical axis below \( A \) is shown as part of the boundary in Fig. 1, which is debatable in that it assumes that capital goods representing excess capacity can be accumulated viably at a zero interest rate (perhaps being costlessly stored). If capital goods are subject to exponential depreciation independently of use and storage methods—say at a rate of 5 per cent per year—AZ would have to be extrapolated into the second quadrant of negative interest rates, with a vertical line down to the axis stemming from the \(-0.05\) terminus. If money is costlessly storable and prices are not rising at a 5 per cent or greater annual rate, it might be impossible to effectuate such a negative real interest rate. When we come to discuss regions of possible "oversaving", issues of effective demand and liquidity traps will depend upon which of the various assumptions concerning \( A \)'s position is made.

\(^2\) When the mapping is done properly, one recognises that the shaded area in Fig. 1 corresponds in a certain definite sense to the shaded Dual area in Fig. 2, and likewise for the unshaded Pasinetti regions.
thrift. In Fig. 1, \( Q \) will turn out to be the only equilibrium corresponding to \( q \) of Fig. 2, and will represent Pasinetti equilibrium. Why? Because on Fig. 1 we locate \( q \)'s vertical solution and its horizontal \( Q \) line that intersect in the \( Q \) image point on Fig. 1.
a point on the $AZ$ locus itself (namely $Q$): both its vertical $n/s_e$ and horizontal $n/s_w$ lines are fully relevant and hence both Dual and Pasinetti equilibrium criteria are simultaneously satisfied.

Now we can show how a Dual equilibrium point is determined. Consider in Fig. 2 the point $q''$, which is due west of the boundary point $q'$. If we plot its $n/s_w$ horizontal, that will of course go through $Q$ in Fig. 1 because its $q''$ is at $q'$'s latitude. But because its $s_e$ is smaller than that of $q'$, its vertical $n/s_e$ will be east (east, not west) of $Q$, corresponding to a still higher interest rate. Now it is the vertical which is irrelevant, and so $q''$ does go over into $Q$—but now with a Dual equilibrium. Similarly every point due west of $q'$ gives rise to the same Dual equilibrium at $Q$, as the reader should verify.¹

It should be clear that the $zstuq'gh; boundary in Fig. 2 corresponds in precise detail to the $AZ$ locus in Fig. 1, the only difference being that the reciprocals of the variables are taken and scale changes are made corresponding to the growth rate $n$. If we had plotted the capital-output ratio and the reciprocal of $r$ (i.e. the "number of years purchase") in Fig. 1, the diagrams would have been exactly the same except for scale factor $n$. If both figures had been plotted on double-log paper, one locus would simply be the upside-down reversed image of the other. The equation of Fig. 2's boundary is simply given in terms of 1's $A(r; n)$, in accordance with our general criterion:

$$s_w = \alpha_k(r; n)s_e = \frac{r}{A(r; n)} s_e = \alpha_k(n/s_e; n) s_e.$$

This last expression was called $\alpha^*_k s_e$ in our paper.

We must warn against a possible, but quite unnecessary confusion. In Fig. 2 one can be at any point whatsoever; in Fig. 1, one is at equilibrium only on the $AZ$ locus. When you are at any point in the Pasinetti region of Fig. 1, $s_w$ is deduced to be less than the golden-age $H/Y$ ratio. When one is at any point in the Dual region, $s'$ is deduced

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saving on the part of workers can lead to a Dual equilibrium at $A$ where the rate of interest has been bid down to zero. Needless to say it is somewhat paradoxical if people should persist in wanting to save a positive fraction of their incomes when there is no useful purpose served by further capital goods of any kind. Still, as Keynes emphasized in 1936, in a monetary economy where people might try to save or hoard against some future need, there could arise "liquidity-trap" and other effective-demand problems. It is noteworthy that the liquidity-trap-at-zero-$r$ problem arises only in the Dual equilibrium. This is because, in the Pasinetti equilibrium, reduction of profits to zero will automatically take care of any saving out of profits. (However, if one thinks it realistic to fear "liquidity-profit traps" at higher-than-zero rates of return, because of risk aversions and other factors, then the Pasinetti regime can also run into effective demand troubles.) We put "oversaving" between inverted commas because we realize that various devices to tax money to create a negative interest rate or various assumptions excluding money or any hoardable asset might do away with some of these effective demand problems. However, this is not the place to grapple with such issues.

We relegate to a footnote the elucidation of the areas in the two figures marked with 2's and 3's, simply remarking here that reversals of direction in the $A(r)$ locus inevitably bring in the possibility of multiple Dual equilibria.

In concluding this taxonomic discussion, note that in the neoclassical case with a monotonic $AZ$ going out forever from the origin, the "oversaving" region of Fig. 2 shrinks away to nothing and the undersaving region to the point at the origin, leaving us with simple Pasinetti and Dual regions with a well-behaved boundary between them.

5. By now we think the reader will agree with us that we have shown the complete symmetry between the Primal and Dual equilibria. In general, neither is more general than the other.

There are, however, special cases in which one becomes more restricted than the other. Because Professor Meade has already touched upon elements of this in connection with his 1966 one-sector discussion, we shall be very brief.

Suppose the $AZ$ curve is always a strict horizontal line. That is a constant capital-output ratio with a vengeance. (And it can strictly occur only if there are completely fixed coefficients in both the consumption and other sectors and if in addition there are strong "equal-intensity" or "organic composition of capital" assumptions made. Otherwise a shift in $n$ would shift and twist the $AZ$ locus, permitting it to be horizontal for at most one $n$ and then only because of fortuitous and singular Wicksell effects. If $AZ$ is to be "approximately constant", we must be "approximately near to" the fixed coefficient case just described.)

In this special case, as Fig. 3a shows, the Dual equilibrium region is completely swallowed up by the regions of undersaving and "oversaving". Only the Pasinetti region is free of these pathologies. So after all, detailed analysis does demonstrate what we had suspected all along—that the primacy of the Pasinetti equilibrium to the exclusion of the Dual equilibrium is not a general feature of these systems but rather is true only in

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1 Any point in the region of Fig. 2 marked with 3's has its horizontal $n/s_w$ line intersecting $AZ$ in three places, while at the same time its vertical $n/s_r$ line is irrelevantly off to the east of all of them. Hence any of the three points represents Dual golden-age equilibrium. It would take us beyond our present task to discuss their local and global stabilities. (Note that horizontal segments $UV$ and $vw$ give rise to an infinity of alternative Dual equilibria.) Any point in the region marked with 2's has its horizontal $n/s_w$ line with the above triplet property; but now its vertical $n/s_r$ lies west of one of the intersections and thus invalidates it, leaving us with two valid Dual equilibria and one valid Pasinetti equilibrium due south of the point in question. These multiple Dual equilibria, with varying interest rates show that our heuristic rule that "the system tries to find the lowest interest rate" does not really represent true teleology. The system doesn't really care if it ends up at a Dual equilibrium with a higher interest rate rather than with a lower one.

Multiple equilibria for the Pasinetti case occur only on the vertical loci, $RS$ and $sr$. Of course the multiplicity there does not affect the uniqueness of the profit rate, which is everywhere the same on any such vertical; but it does affect the observable capital-output ratios, which are in indifferent equilibrium along segments of such a vertical stretch.
the special case which involves a fixed capital output ratio or borders on it. Those whose studies of modern technology suggest to them that such constancy is realistic are welcome to the hypothesis. Our reading of experience and engineering points quite otherwise. But in any case our earlier discussion of the fixed-coefficient model, far from being an exercise in polemics, should be welcomed by those who believe in that model's relevance. In any case, let no one claim that from the general laws of arithmetic and logic comes any primacy of one of the equilibria as against its Dual. (Remember the Primal is Dual to the Dual, as usual.)

These show the complete symmetry of the singular special cases in which one of the two equilibria turns out to be more restricted than the other. In 3a we have the fixed coefficient case where the only
the Dual area looks smaller than that of the Primal. And if we had used double-log charts both areas would be infinite in size! Obviously the economic relevance of the two kinds of equilibrium would have to depend upon the empirical probability density with which it is reasonable in a modern mixed economy to expect the saving propensities in Fig. 2 to fall. To Dr Pasinetti's critical remarks on our econometric estimates of these quantities we now briefly turn. (But in doing so, we first gather some important fruit from the present qualitative exercise. We now know that the observable capital-output ratios and factor shares can be summarized completely on the $AZ$ locus of Fig. 1; and that every such observable point can belong to a point on the boundary in Fig. 2 or to an infinity of points in either region which share its latitude or longitude. Hence, without independent measurement of at least one of $s_h$ or $s_m$ one cannot in general hope to infer, even in principle.
Professor Joan Robinson. As she says, we did put the rabbit into the hat in full view of
the audience before drawing it out again. i.e. our logical theorems do follow correctly
from our axiomatic conditions, a fact for self-congratulation not apology. Her further
implication—that our logical proofs of stability and existence are so transparently obvious
if to involve a trivial waste of time—reveals more what she considers, we think, or