More equal but less mobile?
Education financing and intergenerational mobility in Italy and in the US

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Abstract

A centralised and egalitarian school system reduces the cost of education for poor families, and so it should reduce income inequality and make intergenerational mobility easier. In this paper we provide evidence that Italy, compared to the USA, displays less income inequality, as expected given the type of school system, but also less intergenerational upward mobility between occupations and between education levels.

We explore some of the reasons which can explain this puzzling result and conclude that in a world in which family background is important for labor market success, a centralised and egalitarian tertiary education does not necessarily help poor children and may take away from them a fundamental tool to prove their talent and to compete with rich children.

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1. Introduction

The Italian school system can be characterised as a prevalently centralised and
public system financed by the government through taxation, which provides the same quality of education to everybody. The US system, instead, can be characterised as a prevalently decentralised and private system in the sense that public education is mainly financed at the local level and the share of students going to private school is substantially higher.

Given this characterisation, an Italian family at a low level of income (which can reflect a low level of acquired human capital) should have the same level of education available as a higher income family. A US low income (and low human capital) family, should instead have the additional disadvantage of a low expenditure in education decided by parents (as a result of a lower direct investment or because of locational choices in communities in which preferences are for lower tax rates and worse schooling institutions).\(^1\) Within this framework it would seem reasonable to predict for Italy a more compressed distribution of human capital investments (and therefore of incomes) matched by a higher likelihood of upward mobility for poor families.

The existing comparative empirical evidence on Italy and the US suggests that the first part of this prediction is supported by the evidence: Italy is indeed characterised by less income inequality than the US. In our study, we extend the comparison by looking jointly at the issues of income distribution and intergenerational mobility, and we find that the second part of the prediction is instead falsified. Standard measures of intergenerational mobility between occupations and between education levels indicate that in Italy poor and non-educated families are less likely to invest in the education of their children and to move up along the occupational ladder. In other words, the Italian system can be characterised as an offer of equal opportunities that has surprisingly not been accepted by the Italian poor families.

This is the puzzle that we would like to address in our paper. We would like to understand why the Italian school system, which is strongly egalitarian in the quality and cost of the education provided to rich and poor families, fails to generate at least the same degree of intergenerational mobility which prevails in the US, where the school system is instead highly decentralised and non-equalitarian.

We believe that this comparison between Italy and the US may suggest helpful improvements in the design of public education systems. Our theoretical model identifies some factors that can reduce the capacity of these systems to generate a sufficient amount of intergenerational mobility. We show that these factors are particularly strong when the individual effort is relatively more important than the quality of education for a successful accumulation of human capital. In this case,

\(^1\)In the absence of perfect financial markets, low income families are prevented from reaching the optimal level of investment in education (see Galor and Zeira, 1993; Banerjee and Newman, 1993). In addition, when education financing is provided locally, locational choices in communities, where preferences are for lower tax rates, provide worse schooling (see Benabou, 1996a,b).
even if the cost of schooling is low, the return to schooling is also low and the
offer of a better quality of education to poor families has little value to them. This
is instead the case in which a decentralised and private system does a better job in
raising the return to schooling, thereby making the investment in human capital
more attractive for poor families even if it is more costly.

We argue that this could be the case of the Italian public university system,
whose egalitarian and standardized quality does not attract the expected education-
al investment of poor families. Indeed, the Italian system does not offer a real
opportunity for children of lower income families to emerge and to keep the
returns of their educational investment.

The paper is organised as follows. The evidence on occupational and education-
al mobility is presented and discussed in Section 2. Section 3 shows why this
evidence represents a puzzle, and describes informally how we think it can be
explained. This explanation is then presented formally in Sections 4–6. Conclud-
ing remarks on the implications for the design of public education systems follow.

2. Evidence on the puzzle

2.1. Occupational mobility

Social mobility is defined and measured in many different ways in the literature.
Among economists, some authors focus on transitions between income classes or
between percentiles of the income distribution (Atkinson, 1980–81) while others
look at the speed of mean regression of incomes across generations (Becker and
Tomes, 1986; Solon, 1992; Zimmerman, 1992); among sociologists, instead, the
attention is concentrated on transitions between occupations ranked according to
social prestige (Treiman and Ganzeloom, 1990) or on the transitions between
social classes (Erickson and Goldthorpe, 1992). In general while economists tend
to study mobility in terms of incomes, sociologists are more likely to focus on
occupations.

Our approach can be characterised as a sort of intermediate third way that we
adopt partly because of data limitations but also because it offers some
advantages from the point of view of achieving a meaningful international
comparison and complements in a hopefully interesting way the existing literature.
Sociologists have been arguing for a long time that because of temporary income
fluctuations and measurement errors, yearly income changes are a misleading
upwardly biased indicator of mobility if the goal is to measure transitions between
long term economic status. Casting this argument in an econometric framework,
Solon (1992) and Zimmerman (1992) propose averages of individual incomes on
subsequent years as a measure of long term status, but we cannot follow their

See Appendix A.
suggestion because we do not have the necessary information for Italy. We take instead a road more familiar to sociologists and focus on occupations as indicators of economic status; but we also depart from the sociological literature because we do not rank occupations according to social prestige nor do we aggregate them according to subjectively defined social classes.

Given the information contained in our datasets, the concept of social mobility that we can measure is represented by mobility between occupations ranked according to the median income paid by each occupation in the generation of children in each country. The reader should therefore keep in mind that in this study, a dynasty is classified as mobile only if the occupation of the son is different from the occupation of the father. Take the case of a father and a son in the same occupation which is highly paid in relative terms when the father is observed, but which is paid less than average when the son is observed. According to our definition this dynasty is classified as immobile even if, in terms of individual incomes, it experiences downward mobility. Income changes that take place within the same occupation but across generations cannot be measured in our datasets and do not imply mobility according to our definition. Vice versa, the case of a father and a son possibly earning the same incomes but working in two different occupations is considered here as a case of intergenerational mobility. Therefore, intergenerational mobility in this study has to be interpreted as mobility between occupations even if occupations are ranked on the basis of incomes.

With this caveat in mind we begin our analysis with the evidence on inequality. The existence of greater labor income inequality in the US in comparison to Italy has already been documented in the literature and is confirmed in the datasets used in this study: as shown in Table 1, within each generation all the most common indicators of income inequality proposed in the literature are clearly larger in the US sample.

The comparative evidence on intergenerational social mobility for Italy and the US is, instead, less documented. Tables 2 and 3 present the matrices of transition between occupational income classes defined as proportions of equal size of the (log) difference between the highest and the lowest occupational incomes in the

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3 Cowell and Schluster (1998) suggest that the use of categorical data should increase the robustness of mobility measures. We also performed our analysis using sociological indexes of prestige to rank occupations, but our results concerning the relative performance of the two countries in terms of occupational mobility do not change. We present the evidence based on income ranking because it is less conventional from a methodological point of view and because it allows for an analysis of the relation between educational mobility and occupational mobility. Such analysis is impossible if occupations are ranked according to indicators of prestige constructed on the basis of educational achievements.

4 See, for example: Gottschalk and Smeeding (1997), Erickson and Ichino (1994) and Brandolini (1998).

5 For a description of these indicators see the appendix of the CEPR WP version of this paper (n. 1466, October 1996). Given that in each country occupational incomes for both generations are computed on the distribution of children, inequality differs across generations only because of changes in the distribution of each generation across occupations.
Table 1
Inequality measures for Italy and the US

<table>
<thead>
<tr>
<th>Measure</th>
<th>Italy Father</th>
<th>US Father</th>
<th>Italy Son</th>
<th>US Son</th>
</tr>
</thead>
<tbody>
<tr>
<td>90–10 percentile differential</td>
<td>140.6</td>
<td>164.3</td>
<td>131.5</td>
<td>150.3</td>
</tr>
<tr>
<td>Relative mean deviation</td>
<td>12.2</td>
<td>14.6</td>
<td>13.2</td>
<td>14.3</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>33.8</td>
<td>37.5</td>
<td>34.8</td>
<td>36.0</td>
</tr>
<tr>
<td>Standard deviation of logs</td>
<td>30.0</td>
<td>35.6</td>
<td>31.3</td>
<td>34.9</td>
</tr>
<tr>
<td>Gini coefficient</td>
<td>16.8</td>
<td>20.2</td>
<td>17.9</td>
<td>19.6</td>
</tr>
<tr>
<td>Atkinson (ε = 2)</td>
<td>8.7</td>
<td>11.8</td>
<td>9.3</td>
<td>11.4</td>
</tr>
<tr>
<td>Theil entropy</td>
<td>5.0</td>
<td>6.6</td>
<td>5.5</td>
<td>6.1</td>
</tr>
</tbody>
</table>

*All measures are expressed in % terms. Higher values imply greater inequality.

Table 2
Italy: interclass transition probabilities

<table>
<thead>
<tr>
<th></th>
<th>Son C1</th>
<th>Son C2</th>
<th>Son C3</th>
<th>Son C4</th>
<th>Abs. freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Father C1</td>
<td>21.8</td>
<td>50.4</td>
<td>22.3</td>
<td>5.4</td>
<td>367</td>
</tr>
<tr>
<td>Father C2</td>
<td>12.0</td>
<td>55.9</td>
<td>25.8</td>
<td>6.3</td>
<td>884</td>
</tr>
<tr>
<td>Father C3</td>
<td>5.9</td>
<td>27.0</td>
<td>51.6</td>
<td>15.5</td>
<td>341</td>
</tr>
<tr>
<td>Father C4</td>
<td>4.0</td>
<td>16.2</td>
<td>32.4</td>
<td>47.3</td>
<td>74</td>
</tr>
<tr>
<td>Abs. freq.</td>
<td>209</td>
<td>783</td>
<td>510</td>
<td>164</td>
<td>1666</td>
</tr>
</tbody>
</table>

*Each cell contains the row-to-column transition probability. C1–C4 are income classes defined as intervals of equal size of the (log) difference between the highest and the lowest occupational incomes.

Table 3
US: interclass transition probabilities

<table>
<thead>
<tr>
<th></th>
<th>Son C1</th>
<th>Son C2</th>
<th>Son C3</th>
<th>Son C4</th>
<th>Abs. freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Father C1</td>
<td>25.9</td>
<td>36.4</td>
<td>31.4</td>
<td>6.3</td>
<td>239</td>
</tr>
<tr>
<td>Father C2</td>
<td>22.5</td>
<td>37.7</td>
<td>29.7</td>
<td>10.1</td>
<td>337</td>
</tr>
<tr>
<td>Father C3</td>
<td>9.3</td>
<td>31.0</td>
<td>41.7</td>
<td>18.0</td>
<td>355</td>
</tr>
<tr>
<td>Father C4</td>
<td>4.2</td>
<td>15.1</td>
<td>42.0</td>
<td>38.7</td>
<td>119</td>
</tr>
<tr>
<td>Abs. freq.</td>
<td>176</td>
<td>342</td>
<td>373</td>
<td>159</td>
<td>1050</td>
</tr>
</tbody>
</table>

*Each cell contains the row-to-column transition probability. C1–C4 are income classes defined as intervals of equal size of the (log) difference between the highest and the lowest occupational incomes.

Two countries (see Table 4). According to this aggregation strategy, in each country these classes span over the same percentage increase in occupational incomes.

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*We obtain similar results with different aggregation strategies, like for example the aggregation based on quartiles of the occupational income distribution (see the CEPR WP version of this paper, n. 1466, October 1996). We prefer the aggregation based on the income classes described in the text because, given the skewness of the income distribution, quartiles (in particular the fourth) may group together very dishomogeneous occupational incomes. Therefore similar transitions in terms of quartiles may mean very different transitions in terms of occupational incomes. Furthermore, the focus on absolute instead of relative transitions is consistent with the theoretical analysis presented in Section 4.*
Table 4
Income classes for the United States and for Italy

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>Minimum 100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Median 130</td>
<td>135</td>
</tr>
<tr>
<td></td>
<td>Maximum 139</td>
<td>144</td>
</tr>
<tr>
<td>Class 2</td>
<td>Minimum 148</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>Median 174</td>
<td>164</td>
</tr>
<tr>
<td></td>
<td>Maximum 215</td>
<td>216</td>
</tr>
<tr>
<td>Class 3</td>
<td>Minimum 215</td>
<td>219</td>
</tr>
<tr>
<td></td>
<td>Median 261</td>
<td>234</td>
</tr>
<tr>
<td></td>
<td>Maximum 314</td>
<td>318</td>
</tr>
<tr>
<td>Class 4</td>
<td>Minimum 322</td>
<td>331</td>
</tr>
<tr>
<td></td>
<td>Median 337</td>
<td>369</td>
</tr>
<tr>
<td></td>
<td>Maximum 463</td>
<td>474</td>
</tr>
</tbody>
</table>

*Statistics based on the distribution of sons’ incomes; results are similar for the distribution of fathers. Minimum occupational income normalized to 100. Income classes are defined as intervals of equal size of the (log) difference between the highest and the lowest occupational incomes.*

Differences between the two countries are apparent from the simple inspection of these transition matrices: in particular, the probabilities of persistence along the main diagonal are larger in Italy for the three upper classes. The fact that persistence in the first class is instead higher in the US may be interpreted as evidence on the role of ‘ghettos’ in this latter country. But the probability to reach the two highest classes from the bottom is higher in the US (37.7%) than in Italy (27.7%) while the probability of persistence in the top class is higher in this latter country (47.3% against 38.7%). If one computes on the basis of these matrices the most standard scalar indicators of mobility that have been proposed in the literature, the US appear unambiguously characterised by greater intergenerational mobility (see Table 5).

In order to investigate the statistical significance of the differences in intergenerational mobility in Italy and in the US, we aggregate the four income classes defined above in two groups and we estimate a probit model of the probability that the son is in the highest of these two groups. We define the highest group as the union of the classes 3 and 4 that were described in Table 4. Hence, the dependent variable of our probit models takes value 1 if the son is in income class 3 or 4, i.e. if his occupational income is greater than the income corresponding to half of the percentage difference between the maximum and the minimum of the distribution of occupational incomes. We estimate this probability as a function of a dummy

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8For a description of these indicators, see the appendix of the CEPR WP version of this paper (n. 1466, October 1996).
Table 5
Scalar indicators of mobility for interclass transition matrices

<table>
<thead>
<tr>
<th>Formula</th>
<th>Italy</th>
<th>US</th>
<th>Eq. opp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ML = 1 -</td>
<td>\lambda_2</td>
<td>$</td>
<td>0.55</td>
</tr>
<tr>
<td>$MT = \frac{k - \text{tr}(P)}{k - 1}$</td>
<td>0.74</td>
<td>0.85</td>
<td>1</td>
</tr>
<tr>
<td>$MD = 1 -</td>
<td>\text{det}(P)</td>
<td>^{1/(k-1)}$</td>
<td>0.79</td>
</tr>
<tr>
<td>$MB = \sum_{i} \sum_{j} f_{ij}</td>
<td>i - j</td>
<td>$</td>
<td>0.62</td>
</tr>
<tr>
<td>$MA = \sum_{i} \sum_{j} f_{ij}</td>
<td>W_i - W_j</td>
<td>$</td>
<td>22.44</td>
</tr>
</tbody>
</table>

$a |\lambda_2|$ is the modulus of the second greater eigenvalue; $\text{tr}(P)$ and $\text{det}(P)$ are, respectively, the trace and the determination of the interclass transition matrix $P$; $k$ is the number of classes; $f_{ij}$ is the joint frequency in cell $(i, j)$; the distance $|i - j|$ is the number of class borders crossed in the transition from $i$ to $j$. $|W_i - W_j|$ is the percentage difference between median incomes of class $i$ and $j$.

indicator for the income group of fathers (that takes value 1 if the father is in income class 3 or 4) and of two dummy indicators for the education levels of fathers and sons. In both generations and in both countries the education indicators take value 1 if the individual has a college degree. Age controls are also included in the regressions.

The results of this exercise are presented in Table 6 which reports, for each regression, the change in the probability that the son is in the highest group due to a change from 0 to 1 of each independent dummy variable. These effects are evaluated at sample averages. In model 1 only the family background variables are included as regressors: while the effect of the father’s education is equal in the two countries, the effects of the father’s income class is significantly larger in Italy.

In model 2 the education dummy for the son is introduced, and the effect of the father’s education disappears in both countries: this is a well known result in the literature and suggests that most of the effect of parental education on sons’ occupational achievements works indirectly through the effects on sons’ education. The effect of the occupational income class of fathers, however, remains significantly different from zero in both countries, and significantly larger in Italy than in the US. While in the US the effect of sons’ education is larger than the effect of parental income, in Italy the opposite is true. To put it more directly, in Italy it is better to... choose the right family than to obtain a college degree.

Coming to the comparison between models 2 and 3, in both countries a likelihood ratio test rejects the hypothesis that family background is irrelevant: adding parental characteristics to sons’ characteristics (i.e. going from model 3 to model 2) increases the predictive capacity (pseudo $R^2$) of the model by 150% in Italy; in the US the increase is much lower, being equal to just 19%.

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9 For the age controls the reported effect is that of an infinitesimal age increase.
10 Here and for the rest of this table, differences between coefficients have been tested using appropriately constructed $t$-tests; the null hypothesis of equal coefficients has been rejected with $P$-values smaller than 0.001.
11 See for example Treiman and Yip (1989).
Table 6
Determinants of the probability that a son is in income class 3 or 4*

<table>
<thead>
<tr>
<th></th>
<th>Italy</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 1</td>
<td>Model 2</td>
</tr>
<tr>
<td>Father in income class 3 or 4</td>
<td>0.37</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Father with college degree</td>
<td>0.18</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Son with college degree</td>
<td>0.31</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Father’s age</td>
<td>−0.001</td>
<td>−0.001</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Son’s age</td>
<td>−0.003</td>
<td>−0.03</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Observed prob.</td>
<td>0.427</td>
<td>0.427</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Predicted prob.</td>
<td>0.427</td>
<td>0.428</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Pseudo $R^2$</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>−939</td>
<td>−984</td>
</tr>
<tr>
<td>Sample size</td>
<td>1505</td>
<td>1505</td>
</tr>
</tbody>
</table>

*Maximum likelihood estimates of a probit model in which the dependent variable takes value 1 when the son is in income class 3 or 4. The table reports the probability effects, evaluated at the sample averages, due to a discrete change of each dummy independent variable. For the age controls the reported effects are those of an infinitesimal age change.
The probit estimates presented in Table 6 suggest that in both countries the occupational class of fathers is an important determinant of the occupational achievement of sons, but in Italy the effect is much stronger than in the US in absolute terms and relatively to the effect of sons’ education levels.

We turn now to the evidence on intergenerational mobility between education levels in which the relative lack of upward mobility in Italy appears even more striking.

2.2. Educational mobility

The comparison across countries of educational mobility patterns is certainly not an easy task given the enormous differences between national education systems.\(^\text{12}\) One strategy that seems reasonable to us consists of comparing the probabilities of reaching the highest educational degree offered by the schooling system of each country. Disregarding post graduate studies, which both in Italy and in the US concern a very small fraction of the population, we consider the college degree (laurea in Italy) as the relevant highest educational degree.\(^\text{13}\) We therefore begin our analysis of educational mobility by considering the probabilities of dynastic transitions between the following two educational categories: all the individuals without a college degree are classified as having low education, while those holding a college degree are in the high education group.

Table 7 presents the distribution across these educational categories in each generation and in each country. Italy is characterised in both generations by a lower fraction of college graduates,\(^\text{14}\) but experiences the largest percentage shift towards higher education from one generation to the other: while in the US the fraction of graduates increases by 69%, in Italy the same fraction increases by 200%. Yet not all Italian dynasties shared in the same way this greater opportunity to reach a college degree.

Tables 8 and 9 present, for Italy and the US respectively, the intergenerational transition probabilities between the educational categories that we have just considered.

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\(^{12}\)See Shavit and Blossfeld (1993).

\(^{13}\)We have classified in the high education group all those individuals holding a college degree or a Ph.D. degree in the US sample, or having obtained a laurea or a dottorato di ricerca in the Italian sample. This classification corresponds to the UNESCO classification ISCED 6 and ISCED 7, and requires 18 and 16 years of school attendance, respectively, in the two countries. People who attended some years of college without obtaining any degree were not considered as college degree holders. In the case of Italy we have also used an alternative classification scheme: in this case we have included in the high education group all those individuals holding at least a diploma di maturità degree i.e. a secondary school degree corresponding to ISCED 5 classification scheme; in such a case the minimum number of years of school attendance is 15.

\(^{14}\)Note that, according to OECD (1996), Italy has the lowest fraction of college graduates among all the OECD countries and in all the relevant age classes.
Table 7
Actual marginal and limiting distributions for education in Italy and the US

<table>
<thead>
<tr>
<th></th>
<th>Italy</th>
<th>Italy</th>
<th>Italy</th>
<th>Italy</th>
<th>US</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Father</td>
<td>0.97</td>
<td>0.03</td>
<td>0.92</td>
<td>0.08</td>
<td>0.84</td>
<td>0.16</td>
</tr>
<tr>
<td>Son</td>
<td>0.91</td>
<td>0.09</td>
<td>0.71</td>
<td>0.29</td>
<td>0.73</td>
<td>0.27</td>
</tr>
<tr>
<td>Limit</td>
<td>0.83</td>
<td>0.17</td>
<td>0.30</td>
<td>0.70</td>
<td>0.65</td>
<td>0.35</td>
</tr>
</tbody>
</table>

* Marginal and limiting distributions refer to the matrices of educational transition probabilities. Each limiting distribution is obtained under the assumption that the correspondent matrix describes a Markov process. For Italy: high education = college degree in column 1 and high school degree or more in column 2; for the US: high education = college degree.

Table 8
Italy: transition probabilities from ‘no college’ to ‘college’

<table>
<thead>
<tr>
<th></th>
<th>Son E1</th>
<th>Son E2</th>
<th>Abs. freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Father E1</td>
<td>92.9</td>
<td>7.1</td>
<td>1462</td>
</tr>
<tr>
<td>Father E2</td>
<td>34.9</td>
<td>65.1</td>
<td>43</td>
</tr>
<tr>
<td>Abs. freq.</td>
<td>1374</td>
<td>131</td>
<td>1505</td>
</tr>
</tbody>
</table>

* Each cell contains the row-to-column transition probability. E1, no college degree; E2, completed college degree.

Table 9
US: transition probabilities from ‘no college’ to ‘college’

<table>
<thead>
<tr>
<th></th>
<th>Son E1</th>
<th>Son E2</th>
<th>Abs. freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Father E1</td>
<td>79.2</td>
<td>20.8</td>
<td>870</td>
</tr>
<tr>
<td>Father E2</td>
<td>38.9</td>
<td>61.1</td>
<td>167</td>
</tr>
<tr>
<td>Abs. freq.</td>
<td>754</td>
<td>283</td>
<td>1037</td>
</tr>
</tbody>
</table>

* Each cell contains the row-to-column transition probability. E1, no college degree; E2, completed college degree.

In Italy, the probability that the son of a graduate is a graduate is higher than in the US (65.1% vs. 61.0%); vice versa the probability that the son of a non-graduate reaches a college degree is substantially lower in Italy than in the US (7.1% vs. 20.8%). The inspection of these transition probabilities clearly suggests that the opportunities of obtaining a college degree are more unequally distributed in Italy than in the US, even if Italy experiences a more substantial increase of the proportion of college graduates from one generation to the other.

Because of some missing information on school attendance among fathers, the number of son–father pairs reduces to 1505 observations for Italy and to 1037 for the US whenever the education of fathers is considered in the analysis.
Table 10
Scalar indicators of mobility for educational transition matrices*

<table>
<thead>
<tr>
<th></th>
<th>Italy</th>
<th>US</th>
<th>Italy</th>
<th>Eq. opp.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E2=coll.</td>
<td>E2=coll.</td>
<td>E2=HS or +</td>
<td></td>
</tr>
<tr>
<td>OR</td>
<td>24.6</td>
<td>6.0</td>
<td>27.3</td>
<td>1</td>
</tr>
<tr>
<td>MT</td>
<td>0.42</td>
<td>0.60</td>
<td>0.34</td>
<td>1</td>
</tr>
<tr>
<td>MB</td>
<td>0.12</td>
<td>0.27</td>
<td>0.14</td>
<td></td>
</tr>
</tbody>
</table>

* OR is the odds ratio; in a 2×2 matrix the indexes MT, MD and ML defined in Table 5 are all equal; tr(P) is the trace of the interclass transition matrix P; k is the number of classes; $f_{ij}$ is the joint frequency in cell (i, j); the distance $|i - j|$ is the number of borders crossed in the transition from i to j.

The odds ratios for the two transition matrices, reported in Table 10, show that the odds of obtaining a college degree in Italy are almost 25 times higher if the father has a college degree, while in the US having a graduate father increases the odds only by 6 times. Hence, both countries do not ensure a situation of equal opportunities in the transitions between education levels, but Italy appears to be more distant than the US from such a situation. This is confirmed also by the other 16 scalar indicators contained in Table 10.

One might argue that a college degree means more in Italy than in the US in terms of human capital acquisition. Indeed at least one additional year of schooling is required in Italy to obtain a laurea and in some disciplines, like engineering or medicine, the laurea involves educational curricula that in the US are required for postgraduate studies only. Therefore, as far as Italy is concerned, we provide evidence also for a different classification of educational categories according to which the high education group includes all the individuals who have obtained a high school degree or more. Table 7 shows that with this alternative classification Italy is characterised by an even larger increase of the fraction of highly educated dynasties (262%); furthermore, among sons, the proportion of highly educated individuals in Italy (high school or more) becomes similar to the proportion of highly educated individuals in the US (college or more). Yet even with such a favourable classification, the opportunities of reaching the higher educational category are more unequally distributed in Italy than in the US (see Tables 10 and 11). The odds of reaching a high school degree or more are now even larger if the father is in the same educational category (the odds ratio is 27.3) and the distance

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16 When we estimate a probit model for higher education (not reported here — see the CEPR WP version of this paper, n. 1466, October 1996), we find that the coefficient on father’s income and education are higher in Italy than in the US.
Table 11
Italy: transition probabilities from ‘less than high school’ to ‘high school or +’.

<table>
<thead>
<tr>
<th></th>
<th>Son E1</th>
<th>Son E2</th>
<th>Abs. freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Father E1</td>
<td>75.9</td>
<td>24.1</td>
<td>1389</td>
</tr>
<tr>
<td>Father E2</td>
<td>10.3</td>
<td>89.7</td>
<td>116</td>
</tr>
<tr>
<td>Abs. freq.</td>
<td>1066</td>
<td>439</td>
<td>1505</td>
</tr>
</tbody>
</table>

*Each cell contains the row-to-column transition probability. E1, less than high school; E2, completed high school or more.

from a situation of equal opportunities increases with respect to the previous classification (see the indicator MT in Table 10).\(^{17}\)

2.3. How robust is this evidence?

While there are several international comparisons of income inequality which confirm our ranking of Italy and the US,\(^{18}\) comparative studies of intergenerational mobility which include Italy are very rare. It is nevertheless reassuring that, to our knowledge, the existing studies confirm the essence of our findings. For example, using matrices of transitions between social classes defined according to the prestige of occupations, Erickson and Goldthorpe (1992) find that Italy displays less mobility. Similar results are obtained by Schizzerotto and Bison (1996) applying the methodology of Erickson and Goldthorpe to the same dataset that we analyse in this paper. In a study of educational attainment across cohorts, Shavit and Blossfeld (1993) find a decline of the impact of fathers’ occupational status on sons’ educational achievements in the US, whereas the opposite trend is observed in Italy.

It may be argued that all these studies, as well as ours, face the problem that Italy and the US are compared at quite different stages of development. In our case, while the individuals in the generation of sons grew up approximately in the same years in the two countries,\(^{19}\) a large fraction of the Italian fathers were already alive in the 19th century while none of the US fathers was born before the year 1900.\(^{20}\) Even assuming that the two countries were at the same stage of development in the generation of sons it would be hard to make the same claim for the generation of fathers. Most of the Italian fathers went to school in a prevailing

\(^{17}\)Only the Bartholomew index of movement \(MB\) indicates more mobility for Italy with this alternative educational classification, but this should not be surprising given that \(MB\) is an indicator of movement not an indicator of equality of opportunities (see the appendix of the CEPR WP version of this paper, n. 1466, October 1996); its value is driven by the structural shift towards higher education that characterised Italy in the post-war period, but it hides the existence of unequal opportunities.

\(^{18}\)For example the ones quoted in footnote 4.

\(^{19}\)Between 1920 and 1960 in Italy and between 1931 and 1961 in the US.

\(^{20}\)The range of variations of birth years of fathers is comprised between 1863 and 1939 for Italy and between 1900 and 1947 for the US.
agricultural society where only 5 years of education were compulsory. On the contrary, American fathers were brought up in a considerably more industrialized society, where at least 10 years of education were compulsory almost everywhere. The transformations experienced by the Italian society between the two generations have certainly been more profound than the transformations experienced by the American society.

Although this feature of our data may appear as a problem we believe that it actually enhances the robustness of our results. Indeed, despite the deeper structural changes experienced by Italy during the period of observation (consider for example the much larger fraction of sons who abandoned the agricultural occupations of their fathers, or the postwar extension of compulsory education), we find less occupational and educational mobility in Italy than in the US.

Furthermore, since our goal is to measure the degree of occupational mobility as perceived by the sons, it should not be perceived as a problem the fact that in both countries we rank also fathers’ occupations according to the distribution of incomes in the generation of sons (see Appendix A). This distribution simply provides a unit of measurement which reflects the criteria most likely to be used by sons to evaluate the direction and distance of the occupational change with respect to their fathers.

Another potential source of bias in our data could originate from the fact that American sons are on average 11 years younger than the Italian ones (see Table 12). However, we think that also this feature of the data should reinforce our results. Since Italian children are on average older, they must have had more time to get rid of the effects of an unfavorable family background. Vice-versa, family background should be more important in the US where children are observed earlier in their careers. This is because we expect family networking to be more important at the beginning of a career than at the end. Nevertheless, despite the fact that the younger age of sons should make family background more important in the US, we find that it matters more in Italy.

More problematic are the potential biases generated by the different sampling and data collection procedures for the two countries. The Italian survey (see

<table>
<thead>
<tr>
<th>Country</th>
<th>Father/son</th>
<th>Av. age</th>
<th>S.D.</th>
<th>Min. age</th>
<th>Max. age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>Father</td>
<td>47</td>
<td>7</td>
<td>31</td>
<td>83</td>
</tr>
<tr>
<td>N=1666</td>
<td>Son</td>
<td>44</td>
<td>11</td>
<td>25</td>
<td>65</td>
</tr>
<tr>
<td>United States</td>
<td>Father</td>
<td>47</td>
<td>7</td>
<td>27</td>
<td>74</td>
</tr>
<tr>
<td>N=1050</td>
<td>Son</td>
<td>33</td>
<td>5</td>
<td>25</td>
<td>59</td>
</tr>
</tbody>
</table>

*Italian data refer to 1666 father-son pairs; sons were interviewed in 1985, and information regarding their fathers refers to the year in which sons were 14 years old. Source: Indagine nazionale sulla mobilità sociale. US data refer to 1050 father-son pairs; information on sons refers to 1990, while information on fathers refers to 1974. Source: Panel Study of Income Dynamics.
Appendix A) is designed to provide a nationwide representative sample of the population of sons and precludes the possibility of attrition between generations because the information on fathers is obtained from sons’ recollections. In the PSID, instead, both generations are directly observed, but poor households are over-represented and attrition implies that for some dynasties the information on sons is not available.

The over-representation of poor households should bias downward the measurement of mobility for the US, since intergenerational persistence is higher in low income families. However, if attrition is more likely among poor households, mobility may instead be overestimated in the same country. The fact that for Italy information on fathers is obtained from sons’ recollections while for the US it is directly observed, should instead reinforce unambiguously our conclusions, in as much as sons’ recollections are more subject to measurement error.

Our datasets certainly do not offer the best possible experimental situation one would like to have in order to compare intergenerational mobility across countries. Nevertheless we believe that most of the sources of bias which we can identify tend to reinforce our conclusion. To the best of our knowledge, this conclusion is also not contradicted by the existing literature. It seems fair to conclude that family background is a more important determinant of individual social fortunes in Italy than in the US.

3. Is this a puzzle?

There are natural objections to our posing the empirical evidence described above as a puzzle. These potential objections fall into two different groups. The first group denies some of the basic premises of our reasoning: in particular that the Italian schooling system should be expected to induce more mobility than the US one. The second group provides instead alternative simple explanations of the lower mobility in Italy, which do not take into consideration the schooling system. In particular, it has been suggested that barriers to entry into/exit from certain occupations could explain the lack of occupational mobility in Italy.

While we are not aware of hard evidence suggesting that institutional barriers to entry and exit from occupations should be higher in Italy than in the US, in this section we show that the differences in the two education systems are instead substantial. Furthermore, all the elements of this comparison suggest that the decentralized and non-egalitarian US school system should not have generated more equality of educational and occupational opportunities than the centralized.

\[^{21}\text{See Mulligan (1997) and Lillard (1998).}\]
\[^{22}\text{See, for example, Cobalti and Schizzerotto (1994) and Schizzerotto and Bison (1996).}\]
\[^{23}\text{Just to give an example, both countries regulate in very similar ways the access to liberal professions like those of medical doctors, lawyers or architects.}\]
and egalitarian Italian system. Even if access to the higher-status occupations were substantially more restricted in Italy than in the US, the characteristics of the two education systems should have at least partially compensated for the different incentives to upward mobility induced by the labor market in the two countries. On the contrary, we observe that educational mobility (in particular upward mobility) is abnormally lower in Italy than in the US.

For these reasons, we believe that the objections described above are not entirely convincing. Further research on the comparison of mobility in different countries, and on the reasons for the differences, is certainly necessary, but at the present state of knowledge our evidence seems indeed to represent a puzzle. The rest of the paper will be devoted to describing our proposed explanation of the puzzle, which we will first sketch informally at the end of this section. We should also say from the outset that we do not view our proposed explanation as an alternative to others. While we do not deny the possibility of other explanations of the lack of mobility in Italy (like, for example, the non-competitiveness of labor markets), we do think that these other explanations cannot account alone for the entire evidence and that our explanation is at least a necessary complement.

3.1. Education in the two countries: centralization vs. decentralization

A first fundamental difference is that while both countries spend a similar fraction of GNP on public education, the sources of public funding are very different. In Italy, 83.1% of public expenditures for primary and secondary education comes from the central government as opposed to local authorities, whereas in the US only 7.9% of this expenditure is centrally financed at the federal level and as much as 44.3% is financed instead at the local level (city or county). This is a crucial difference from the point of view of this paper: in the US, independently of how much funding comes from private sources, public education should also increase the role of family background as a determinant of educational decisions because of the effect of parental locational choices in communities characterised by different combinations of local tax rates, housing prices and quality of schooling institutions. As we have shown above, however, the role of family background is instead surprisingly more important in Italy where education is not only financed mainly out of public sources but these sources are also strictly controlled by the central government.

\footnote{In 1993, the incidence of public expenditure for education on GNP was 5.0% in Italy and 5.1% in the US. (Data from OECD, 1996.) In this section we are only able to present figures for recent years, but as far as we can tell from the available scarce statistics for previous years, these figures are qualitatively representative of the school systems that the generation of sons were facing in the two countries. Note also that despite a never ending post-war parliamentary debate, the basic structure of the Italian education system is still the one of the reform designed in 1924 by Giovanni Gentile, the Minister of Education of the fascist regime.}

\footnote{Data from OECD (1996).}
In addition to this fundamental difference, several other institutional features of the two systems emphasize centralisation in Italy and decentralisation in the US. For example, the age of compulsory education which is determined by a law at the parliamentary level in Italy, while in the US is dictated at the state level, ranging between 8 and 13 years, with an average of 10.05 years and a standard deviation of 1.19 years. Furthermore, in Italy the types of educational curricula available in both private and public schools are established by a parliamentary law at the central level. For each type and level of schooling parliament establishes also the subjects that have to be taught, the outlines of teaching programs for each subject, the textbook prices (for compulsory education), the evaluation and grading methods and even the daily time of entrance and exit from school and vacation periods. Therefore, for example, a parliamentary vote is in principle needed to authorise a school not to teach a given subject or to teach a different new one. At a different but still centralised level, the Minister of Education issues ~600 documents (circolari ministeriali) each year in which additional instructions are given to teachers and headmasters with the precise goal of making the education system as uniform as possible over the entire country. As a result, for each level and type of school, final exams are uniformly defined, and in particular for the highschool degree the written exam questions are identical for all students and administered in the same day over the entire country. Note that private schools also have to obey these laws and regulations if they want to obtain legal value for the degrees that they offer.

The recruitment of teachers is also completely centralised in Italy, with uniform requirements for each type and level of education: aspirant teachers have to compete in national competitions and to pass similar final exams in order to be authorised to teach (this happens also for university professors). The teachers’ salaries are centrally determined on the basis of seniority and of level of schooling, with basically no room for individually based differentiation.

At the other extreme, the US public education system is far from featuring a similar effort aimed at centralising and making as uniform as possible any aspect of the educational process. Where the US system comes closer to centralisation is in the requirements of standard uniform exams for admission to higher levels of schooling. However, these exams are not imposed by any law and, paradoxically, they are probably the most explicit indication of the degree of decentralisation and diversity of the educational curricula offered by US schools.

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26 Two years of compulsory education were initially introduced in Italy in 1859, subsequently raised to three in 1877 and to 6 in 1904. The actual obligation of 8 years was introduced in 1962 (Law n.1859, 31/12/1962). In the case of the US, Bowles and Gintis (1976) report that a wide movement in favor of raising compulsory education to the secondary level occurred in the US during the 1920s and the 1930s; however regional differences persisted much longer. Figures on actual compulsory ages in the US are taken from the US Education Department (1995).

27 See, for example, the Italian Law DL 297 16/4/94, ‘Testo unico delle disposizioni legislative vigenti in materia di istruzione, relative alle scuole di ogni ordine e grado’.
The decentralisation of public education financing in the US makes the quantity and moreover the quality of public education available to a child heavily dependent on the locational choices and on the income of the family of origin. However, in addition to the possibility of choosing the quality of public education ‘with their feet’, US families have also the option of a well-established private education system particularly at the university level. The proportions of students enrolled in private schools in Italy are 8.1%, 7.8% and 3.5%, respectively, for primary, secondary and tertiary education; in the US the analogous proportions are higher, being equal, respectively to 12.0%, 9.1% and 21.8%. The difference is particularly striking for tertiary education. Furthermore, in the US the proportion of public sources in the expenditures for tertiary education is only 51.7% (in 1993) while in Italy it reaches 89.8%.

Therefore in addition to the crucial effect of the decentralisation of public education funding and regulations, the differentiation of educational curricula in the US is strengthened by the greater diffusion of private schools. It is of course difficult to measure how much the decentralisation of funding for public education and the greater diffusion of private schools result in an effectively more dishomogeneous quality of education provided by the US system. It may be indicative, however, to observe that while the coefficient of variation across the 20 Italian regions of the pupil-to-teachers average ratios is 6.5% (for primary and secondary education) the correspondent coefficient of variation across US states is 13.2%. The standard errors of (comparable across countries) textscores for reading and narrative capabilities are, respectively, 3.4 and 3.6 in Italy and 4.8 and 4.9 in the US.

This evidence, albeit certainly not conclusive, is consistent with the view that the centrally funded and centrally administered Italian public education system provides a quality of education that is more uniform and less expensive than the quality provided by the decentralised and more largely private US system. Yet, as we have seen, while the Italian system succeeds in generating lower income inequality, it fails to generate more intergenerational mobility and more equality of opportunities. In Section 4 we present a model which builds on the existing literature on private and public education systems in order to shed some light on this empirical puzzle and, more generally, on the relationship between income

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28Data for the US refer to year 93/94 and for Italy to year 94/95. See US Education Department (1995) and ISTAT (1995).
29Note that while the incidence of public education expenditure on GNP is similar in the two countries (see footnote 24), the incidence of total expenditure differs, being equal to 5.1% in Italy and 6.8% in the US. The discrepancy is due to the dimension of private expenditure in the US. (Data from OECD, 1996).
30Our computations are based on US Education Department (1995) and on ISTAT (1995).
31US Education Department (1995). Unfortunately similarly comparable figures for mathematical textscores, which would suffer less from the biases due to different linguistic backgrounds in the US and different dialects in Italy, are not available.
inequality and intergenerational mobility.\textsuperscript{32} First, however, we want to sketch informally the story that our model proposes.

3.2. A possible explanation: the role of incentives

We build in particular on Glomm and Ravikumar (1992) but we add an important element: people have talent, which is an essential requirement in the acquisition of human capital.\textsuperscript{33} The consideration of talent is what makes the problem of mobility interesting from an economic point of view: without mobility a society may assign high talented people to low education groups, and people with low talent to high education groups; beyond reasons of fairness, this is an undesirable feature of an immobile society.\textsuperscript{34}

Talent is transmitted from father to son with some persistence and cannot be directly observed. Talent should be interpreted as the combination of the genetic and environmental transfers from parents to children; so the assumption of persistence is plausible independently of any belief on genetic transmission.\textsuperscript{35} Talent realizations are restricted for simplicity to two, ‘high’ and ‘low’. The only test for talent is performance at school. If someone attempts to acquire education, and succeeds, he has a high talent; while, if he fails, he has a low talent. Therefore, school as a sorting mechanism only works for those who choose to invest in human capital. Since talent is imperfectly observable, each person can only try to make some inference about it from family history.

So the most important decisions, in particular those determining the investment in human capital, are taken on the basis of the belief that each person has on his own talent. The higher this belief, the more likely a person is to invest in education: in fact we shall see that the rational decision is to invest in education if and only if the subjective belief of having the necessary talent is higher than a critical threshold. We refer to this as the self-confidence factor, although we have

\textsuperscript{32}This relation has been surprisingly somewhat neglected in the literature. An important exception is represented by the work of Atkinson (in particular Atkinson, 1980–81, 1983) who takes up the challenge posed in Pen (1971) to ‘build a bridge between the figures on vertical mobility and income distribution’. More recently, also the model proposed by Galor and Tsiddon (1996) in which, inequality and intergenerational mobility are positively correlated and driven by the pace of technological innovations.

\textsuperscript{33}Note that, by focusing on the role of talent and self-confidence as determinants of human capital investment decisions, our model adds to Glomm and Ravikumar (1992) the consideration of mobility which they do not address. In their model the predicted mobility is necessarily zero, since a dynasty which has an income higher than another in the initial period has a higher income forever. The reason for the difference is clear: in the model of Glomm and Ravikumar there is no talent, persistent or i.i.d.

\textsuperscript{34}We are speaking loosely here on purpose: the full analysis of the implications of our model for welfare and efficiency is beyond the goals of the present paper.

\textsuperscript{35}As we will see, without persistence the problem of mobility becomes trivial and our model features perfect mobility independently of the schooling system.
to remember that it is a perfectly rational consideration, since this belief summarizes all the information a person has about his own talent. This belief becomes an important way in which family background affects the decision of a child. A family may be stuck at low levels of education for a sequence of periods because the previous family experiences have given its members a low confidence. Therefore, a fraction of the population has high talent, but does not use it, because of the adverse belief. We say that a society is more mobile if a larger fraction of the people in the low income group makes an effort to increase personal income through an educational investment. The key issue that we analyze in this stylized framework is which institutional set-up for schooling (centralized and funded through taxation vs. decentralized and funded privately) makes a society more mobile in the above sense, and why. Given this characterization of mobility, it is desirable to increase it if one wants to reduce the probability that talented individuals remain stuck with low human capital.

In a public school system in which a uniform education quality is offered to everyone, the combination of taxes and educational expenditures transfers revenues from high income families to low income families, and makes more education available to the latter, at no additional cost. In a private school system a higher income makes the choice of schooling easier; so income inequality tends to be more persistent in this context. The transfer of resources induced by the public system and commonly quoted in its support, creates indeed an important incentive for low income families to increase their human capital and tends to raise the degree of mobility induced by public education.

There are however other factors, which go in the opposite direction. First, in a private system, a higher parental income directly increases the amount of resources available for the education of the son, while it does not in a public system. If people are altruists, this adds to the attractiveness of a higher education because one knows that if the investment in human capital is successful one will be able to transfer more resources to the next generation; in a public system, the educational transfer to the next generation is centrally determined independently of parental income. Second, a single tax rate may force some parents to a rate of expenditure in education lower than they would desire, thereby making less likely an otherwise attractive investment in education for their sons. Finally, the fact that the tax rate is unique makes useless any information that a person may acquire on his and his son’s personal abilities, because he cannot adjust the expense in education for the son according to this information.

Even if they do not prevail, these three factors certainly reduce the capacity of a

\[\text{Empirical evidence on the role of self-confidence is limited. In an NLS sample Lillard (1998) finds a significant effect of ‘. . . family dummy variables measuring whether or not the son expects ‘much’ help from his parents to pursue higher education and how much his parents encouraged him to pursue higher education’ (p. 17). These dummies are significant in predicting both school performance and earnings.}\]
public, centralized and egalitarian system to increase intergenerational mobility. With our model we want to attract attention to the conditions which may reinforce these factors, in order to avoid them and to improve the design of public education systems. We will explore these conditions with the help of numerical computations after the formal presentation of the model.

4. The model

4.1. Human capital and wages

The economy has a sequence of different generations. Population is a continuum, each person lives for two periods and is productive only in the second. His production depends on his human capital, which is described by a real number $h$. He earns a wage equal to $h$. There are infinitely many periods; in each period $t$ the distribution of human capital is denoted by $G_t$; the total human capital is therefore:

$$H_t = \int h dG_t(h)$$  \hspace{1cm} (4.1)

4.2. The technology for human capital

Each person has a basic working ability, of quality normalised to 1, and a natural talent, which has no direct productive use, but is critical in acquiring additional human capital.

Talent is denoted by $a \in \{L, H\}$; it is transmitted from father to son with some persistence. More precisely, talent follows a first order Markov process:

$$P(a_{t+1} = H|a_t = H) = P(a_{t+1} = L|a_t = L) = 1 - \alpha$$

with $\alpha \in (0, 1/2)$. Talent is not always known exactly; we denote by $\nu_t$ the belief that the talent of the member born at $t$ of the dynasty is $H$. It will be clear in the sequel who holds this belief. A higher human capital can be produced by the combination of a learning effort, the help of an educational system, and the direct or indirect contribution of the human capital of the father. We assume that this is possible only if the talent of the person is of the high type.

The technology has (as in Glomm and Ravikumar, 1992) a Cobb Douglas functional form. More precisely,

$$h_{t+1} = \begin{cases} 
1 & \text{if } a_{t+1} = L; \\
(1 - n_t)\theta e_t h_t^a & \text{if } a_{t+1} = H;
\end{cases}$$

where $n_t$ is the leisure enjoyed, $e_t$ is the quality of education, and $h_t$ is the human capital of the father.

Talent cannot be directly observed; the only way to determine it is to put it to
the test of the education system. If the person decides to go to school, and fails, then he knows his talent was low; on the contrary if he succeeds he knows that it was high.

4.3. Preferences

The utility of each person depends on leisure of the first period, consumption of the second period $c_{i+1}$, and a term which describes the expected utility from the quality of the education which is left to the son. The expectation is taken with respect to the belief $v_{i+1}$ that the person has on his son’s talent, which is not known with certainty.\footnote{A standard procedure to model altruism is to assume that individual utility positively depends on bequests (see Glomm and Ravikumar, 1992; Galor and Zeira, 1993; Banerjee and Newman, 1993). Other authors have conditioned current utility on offsprings’ future incomes (Becker and Tomes, 1986) or consumption (Mulligan, 1997, ch. 3). Here we take the unconventional route of conditioning current utility on educational resources, which in our context are worth something only when the child is talented.}

Formally:

$$U(n_i, c_{i+1}, v_{i+1}, e_{i+1}) = \log n_i + \log c_{i+1} + v_{i+1} \log e_{i+1}$$

(4.2)

The budget constraint of each person will depend on the institutional arrangement for the provision of education: so we shall deal with it in the next section.

4.4. Two institutions for education financing

As in Glomm and Ravikumar (1992) we consider two different possible institutional arrangements for the provision of education, that is in the context of our model, for the determination of the quantity $e_i$.

The first is a purely private regime, where $e_i$ is decided by the father, and paid out of his income. The second regime is a pure state school system. The quality of education provided to each child is the same, and is decided as follows. A tax rate $\tau \in [0, 1]$ is voted in each period, and chosen according to majority rule. The tax rate applied to the total income gives an amount spent on the collective education:

$$E_i = \tau_i H_i$$

(4.3)

We can now state the budget constraint formally. In the case of a private school system, the individual is facing the two constraints:

$$n_i \leq 1; \quad c_{i+1} + e_{i+1} \leq h_{i+1};$$

while in the case of the state school system, with tax rate $\tau_{i+1}$, we have:
\[ n_t \leq 1; \quad c_{t+1} \leq h_{t+1}(1 - \tau_{t+1}). \]

4.5. The timing

The life of each person lasts for only two periods. A person born at date \( t \) knows the history of attempts to get an education and of successes and failures of former members of his dynasty. In the private school system, he also knows the amount that the father has devoted to his education; while in the state school system he knows the prevailing level of educational quality of the system.

On the basis of the history of his dynasty he now computes his belief on his own talent, denoted by \( n \). He then decides whether or not to go to school, a choice which is denoted as the choice between a \( Y \) or an \( N \) respectively. If he decides \( Y \), he also decides the amount of effort he devotes to the learning activity. He then goes to school, and this is the end of the first period.

At time \( t + 1 \) the talent of the person is revealed and \( h \) is determined. In the state school system the tax rate \( \tau \) is then voted by the old generation. Then the remaining income is consumed and taxes are paid, or, in the private school system, the amount \( e \) of funds for the education of the son is provided. Then the son is born and the life of the older generation ends. Note that, to simplify notation, generations do not overlap in this model, but in each calendar period both generations are alive: the oldest in the first part and the youngest in the second part of the period.

To summarise, and to clarify the informational restrictions for the agents: the decision about education (that is, whether to go to school, and if so how much effort to spend in education) is taken without knowledge of the talent of the person; the vote on taxes, the consumption decision, and the amount for the education of the son, are decided after the additional information on the talent of the person has been obtained.

4.6. Learning about talent

Consider a person with an initial belief \( n \) in his own talent. If he decides to go to school and he is successful, he will change to 1 the belief in himself while the

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\(^{39}\)Note that at the moment of deciding about schooling, each person learns about his talent from his family history, but not from his performance in the early stages of his education. This is clearly an extreme assumption. We have two reasons to defend it. The first is that some of the important decisions about schooling are taken at the very early stages of the education. For instance, the quality of the elementary education is important, and has sometimes decisive influence on future choices. The second reason is that we can easily think of a richer model where, say, each agent makes successive choices in education, and receives at each step a signal correlated with his talent from his performance. This model would yield the same qualitative results as ours (provided, of course, that these signals are not too precise).

\(^{40}\)For a discussion of the paradox of voting within this framework, see the CEPR WP version of this paper, n. 1466, October 1996.
belief on the talent of his son will be $1 - \alpha$. After a failure in school, instead, these two beliefs will be, respectively, 0 and $\alpha$.

If the person decides not to go to school, then he will gather no information about his own talent and will have a belief

$$\hat{\nu} = \alpha + (1 - 2\alpha) \nu$$  \hspace{1cm} (4.4)

in the talent of the son. We denote by $\hat{\nu}^i$ the $i$th iterate of the function defined in (4.4). This is a function increasing in $\nu$, and its iterates converge to the value 1/2 independently of the initial value.

In particular, since the belief of the first member that follows a failure in school is $\nu = \alpha$, this function tells us the belief of the $i$th member of a dynasty which has not attempted schooling after the failure. This value, given by the function in (4.4) computed at $\nu = \alpha$, is equal to

$$\hat{\alpha}^i = 0.5(1 - (1 - 2\alpha)^{i+1})$$  \hspace{1cm} (4.5)

4.7. The optimal policies

We begin with the case of the private school system. The optimal policy is decided by backward induction from the second period, after the decision between $Y$ or $N$ has been taken (and, in the case of a decision $Y$, the amount of leisure $n_t$ has been chosen). In the second period we have therefore three possible cases: $Y$ and a success, $Y$ and a failure, and $N$. In each of these cases the problem of the agent is to maximize for a given human capital $h_{t+1}$ and belief $\nu_{t+1}$ on the talent of the son:

$$\max_{(c_{t+1}, e_{t+1})} \log c_{t+1} + \nu_{t+1} \log e_{t+1}, \text{ subject to } c_{t+1} + e_{t+1} \leq h_{t+1}$$

which has an optimal $e_{t+1}$ equal to:

$$\frac{\nu_{t+1}}{1 + \nu_{t+1}} h_{t+1}$$

and value:

$$(1 + \nu_{t+1}) \log h_{t+1} + L(\nu_{t+1})$$

where the function $L$ is defined in Appendix B.1.

So the optimal expense in case of a $Y$ decision and a success is $e_{t+1} = (1 - \alpha)/ (1 + (1 - \alpha))h_{t+1}$; in the case of $Y$ and failure we have: $e_{t+1} = \alpha/(1 + \alpha)$; and finally, if the decision has been $N$, and the belief on his own talent was $\nu$, then: $e_{t+1} = \hat{\nu}(1 + \hat{\nu})$.

In the case of the state system, the important decision in the second period is the

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\footnote{A related learning process is in Piketty (1995) although in that model people learn about a parameter that is social and not dynastic.}
one about voting, since consumption is a pure residual from income after payment of taxes. The optimal tax rate $\tau_{t+1}$ is

$$\frac{v_{t+1}}{1 + v_{t+1}}.$$

So in the three cases corresponding to the one described above for the private system case we have: $\tau_{t+1} = (1 - \alpha)/(1 + (1 - \alpha)); \tau_{t+1} = \alpha/(1 + \alpha)$; and $\tau_{t+1} = \hat{\theta}/(1 + \hat{\theta})$, respectively.

We can now solve the problem of deciding in the first period the pair $(Y_n, n)$ (go to school, with effort $n$), versus $N$. Leaving the details to Appendix B.2, in order to understand the optimal policies in the two systems it may be helpful to focus on three generations, each one living for two periods: the grandfather, born at $t - 1$, the father, born at $t$, who is the agent whose two periods decisions are being modelled, and the son, born at $t + 1$.

In the private system the optimal choice of expenditure for education of the father is a function of the father’s belief on the son’s talent, and of the father’s realised human capital; we denote this function by $e_{t+1}(v_{t+1}, h_{t+1})$. Furthermore, the father’s optimal choice of $Y$ versus $N$, and of effort in school, is a function of the human capital of the grandfather and of the available quality of education (decided by the grandfather); we denote this function, which will have to be positive for a father to go to school, by $D_{t+1}(v_n, e, h)$. Similarly in the state system, the optimal father’s vote on taxes is a function of the father’s belief in the son’s talent; we denote this function with $\tau^x_{t+1}(v_{t+1})$. Furthermore, the fathers’ optimal choice of $Y$ versus $N$, and of effort in school, is a function of the human capital of the grandfather and of the average quality of education available to the father in the state system, $e^{s}_t$. We denote this function, which will have to be positive for a father to go to school, by $D^s_{t+1}(v_n, e^{s}_t, h_t)$.

Both functions $D_{t+1}(v_n, e, h)$ and $D^s_{t+1}(v_n, e^{s}_t, h_t)$ are crucial to determine mobility in the two systems. A detailed discussion of this issue, and of the two functions, is developed in Section 5.

4.8. The typical history of a dynasty

To get some intuition about the way in which the model works we can follow the typical path of a dynasty. After a failure in school of a given member, his son will have a belief $\alpha$ in his own talent and a human capital equal to 1. Now for a

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42Remember that in each calendar period two generations are alive, but they do not overlap: the oldest lives in the first part and the youngest in the second part of each period.

43Note that in general the quality of education available to the father depends on the aggregate human capital and on the median voter preferred tax rate in the generation of grandfathers, but in steady state it will be identical for all generations.
sequence of periods the members of the dynasty will choose not to go to school because their self confidence is too low.

During these periods, however, the belief on talent grows (by the fact that the iterates of the updating rule (4.5) are increasing) until it reaches a critical level at which the corresponding member of the dynasty decides to go to school. For convenience we shall denote this critical level \( \nu^p \) in the private school system and \( \nu^s \) in the state school system case. This critical level, or, equivalently, the length of this initial sequence of periods will depend of course on the institutional arrangement and on the equilibrium; we discuss later how to characterise it, and the various additional factors that influence such critical level in the two systems.

In case of success in school and until a new failure occurs (in which case the cycle we have just described starts all over again) the dynasty goes through a sequence of increasingly better periods. In each of these periods the members go to school, acquire human capital in an increasing quantity and keep the belief to a high level. In the private school system the members devote an increasing amount of income to the education of their children; while in the state school system they vote for large tax rates in support of education. Eventually, however, a failure occurs and the cycle starts over.

4.9. Equilibria and steady state distributions

In this paper we shall concentrate our attention on the long run property of equilibria; and they can be easily studied by considering the invariant distribution on the relevant variables: human capital, beliefs over talent, investment in education and so on.

From our previous discussion of the typical history of a dynasty it should be clear that only certain beliefs over talent are possible in the long run, for a given critical belief. Each dynasty experiences a failure with certainty over an infinite time horizon. After this, the belief of the member of the dynasty in the next generation over his own talent at the moment of deciding about his schooling effort is \( \alpha \) (i.e. the probability of being different from his parent). The following members update their beliefs \( \alpha^k, \ k = 1, 2, \ldots \) using (4.5) without going to school until the critical level is reached. At that point the corresponding member of the dynasty goes to school, talent is revealed and the belief can only go back to \( \alpha \) (in case of failure in school) or to \( 1 - \alpha \) (in case of success); from this last belief the only transitions possible are either to \( 1 - \alpha \) again (success) or to \( \alpha \) (failure).

If the critical level is above \( 1/2 \) there are countably many beliefs possible; if it is below, then there are only finitely many. In both cases, however, they are a subset of the countable set \( \{ \alpha, \hat{\alpha}, \hat{\alpha}^2, \ldots, 1 - \alpha \} \). Note that, in turn, this will produce a countable set of possible human capital level, and of possible expenditures in education and of tax rates voted.
In order to examine the structure of the invariant distribution, the first step is the definition of the appropriate state space:

**Definition 4.1.** The state space of the process is the product space $\mathcal{B} \times H = [0, 1] \times \mathbb{R}^+$ of beliefs over $\{H, L\}$ and of human capital values.

This state space has to be understood as follows. For the pair $(n, h)$, $n$ is the belief of a person in his own talent, at the moment in which he decides the schooling effort $n$; and $h$ is the human capital that the same person has at the end of the schooling period. The following Lemma describes formally the transition probabilities over this state space: let $\hat{a}$ be such that the belief $\hat{a}$ is the critical belief, $\nu_\#^\hat{a}$ or $\nu_H^\hat{a}$. Then:

**Lemma 4.2.** The transition probabilities over $\mathcal{B} \times H$ are as follows (wp; with probability):

- from $(\hat{a}^{k-1}, 1)$ to $(\hat{a}^k, 1)$ for $k = 0, \ldots, i - 1$, wp 1;
- from $(\hat{a}^{i-1}, 1)$ to $(\hat{a}^i, h_0)$ wp $\hat{a}$, and to $(\hat{a}^i, 1)$ wp $1 - \hat{a}$;
- from $(\hat{a}^i, 1)$ and $(1 - \alpha, 1)$ to $(\alpha, 1)$ wp 1;
- from $(\hat{a}^i, h_0)$ to $(1 - \alpha, h_1)$ wp $1 - \alpha$, and to $(1 - \alpha, 1)$ wp $\alpha$;
- from $(1 - \alpha, h_i)$ to $(1 - \alpha, h_{i+1})$ wp $1 - \alpha$, and to $(1 - \alpha, 1)$ wp $\alpha$.

The above transition probabilities imply that, after a failure and if it does not go to school, a dynasty moves with certainty across states characterized by a human capital equal to 1 and by subsequent updates of the belief in talent. When the dynasty reaches the critical level of self confidence it goes to school. Since the initial belief after a failure is correct, the updated belief on talent is equal to the true probability of being talented. Therefore, with probability $\hat{a}$ the decision to go to school is successful and $h_0$ human capital is accumulated; with probability $1 - \hat{a}$, instead, the member of the dynasty is untalented and human capital remains equal to 1. If the dynasty keeps being successful no more updating is needed because each subsequent member knows he is the offspring of a talented parent. Therefore, with probability $1 - \alpha$ the dynasty continues to be successful and accumulate increasing human capital, while with probability $\alpha$ it fails, human capital falls to 1 and the story starts all over.

The definition and the computation of the invariant distribution for these transition probabilities is reported in the Appendix B.3. We discuss instead, in the next section, how the probabilities in the transition matrix, and therefore intergenerational mobility, depend on the type of school system.

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See the CEPR WP version of this paper (n. 1466, October 1996), for a proof that this state space is a sufficient description of the process in the sense that the fact that a dynasty is in state $x \in X$ at time 0 provides sufficient information to describe the future conditions of the dynasty.
5. Mobility

As we have seen, even on the reduced state space \( \mathcal{B} \times H \) the transition matrices are infinite: so we have to find some simple index of the different degrees of mobility in the two educational systems. The simplest is the transition probability among two different classes of human capital.

We divide the total population in two classes: those who have a human capital equal to 1, the minimum value, and those who have a higher value. The first class will be denoted by \( C_* \), the second by \( C_2 \). We can then compute the transition matrix between these two classes, say \( p_{ij}, i = 1, 2; j = 1, 2 \), where \( p_{ij} \) is the probability that a dynasty transits from \( C_i \) to \( C_j \); we have that:

**Lemma 5.1.** The matrix of transition probability across classes is:

\[
\begin{pmatrix}
(1 - \frac{\hat{\alpha}^i}{i + 1}) & \frac{\hat{\alpha}^i}{i + 1}
\end{pmatrix}
\begin{pmatrix}
\alpha
\frac{1}{i + 1}
\end{pmatrix}
\]

The term \( \frac{\hat{\alpha}^i}{i + 1} \) is a decreasing function of \( i \).

The proof is in Appendix B.4. Note that \( \frac{\hat{\alpha}^i}{i + 1} = \alpha \) when \( i = 0 \).

The value of \( \frac{\hat{\alpha}^i}{i + 1} \) can be considered an index of mobility at the steady state equilibrium of the system: the higher this value the more mobile the society is. Note that it is inversely related to the integer \( i \), the number of periods a dynasty remains ‘discouraged’ after a failure. We summarise this as our definition of mobility:

**Definition 5.1.** A society is more mobile, the shorter the period in which a discouraged dynasty does not attempt to acquire education; that is, the lower the value of the critical \( i \) (i.e. the lower the level of self-confidence needed to go to school).

We now turn to a discussion of this critical value and of how it is influenced by the institutional setting for education financing.

5.1. Why does mobility differ in the two institutional settings?

The critical value of \( i \) is the first time after failure that the expected utility from a \( Y \) decision is higher than the expected utility of an \( N \) decision. In the private school system, for a father with belief \( \nu \) in his own talent and available quality of
education \( e \), the difference between these two expected utilities is given by the function:

\[
D^p(\nu, e) = \nu \beta [1 + (1 - \alpha)] L \left( \frac{1}{\nu \beta [1 + (1 - \alpha)]} \right) \\
+ \nu [1 + (1 - \alpha)] \log(\theta e^\gamma) + V(\nu) = \\
\max_{n \in [0,1]} \left( \log n + \nu [1 + (1 - \alpha)] \log[\theta e^\gamma (1 - n)^\beta] + V(\nu) \right) \\
(5.6)
\]

where the term \( V(\nu) \) is equal to:

\[
V(\nu) = \nu L(1 - \alpha) + (1 - \nu) L(\alpha) - L(\bar{\nu}). \\
(5.8)
\]

and the function \( L \) is defined in Appendix B.1.

In the state school system, for the father with belief \( \nu \) on his own talent and available quality of education \( e \), the difference between the expected utilities of the \( Y \) and \( N \) decisions is given by the function:

\[
D^s(\nu, e) = \nu \beta L \left( \frac{1}{\nu \beta} \right) + \nu \log(\theta e^\gamma) = \\
\max_{n \in [0,1]} \log n + \nu \log[\theta (\tau H)^\gamma (1 - n)^\beta]. \\
(5.9)
\]

Mobility under the two systems differs whenever, everything else being equal, the first critical generation \( i \) for which \( D^p \) becomes positive is different from the first critical generation \( i \) for which \( D^s \) becomes positive. It is, therefore, crucial to consider how the two functions differ for each given \( i \).

One important difference is that a state school system transfers revenues from high income families to low income families and makes a better education available to the latter at no additional cost. This effect of a state system, which we label transfer of resources, is commonly quoted as the main reason for which public education should raise intergenerational mobility.

But other factors, highlighted by our framework, point in the opposite direction making it possible for a private system to induce more mobility. First a father in the private system who decides his effort in the production of his own human capital also takes into account the fact that in case of success the higher income available to him will also positively affect his son. In the state system instead a higher income will not have this effect, since the expense in education comes from a common fund, and the contribution of each person to it is negligible. Everything else being equal, this makes the value of the \( Y \) choice higher in the private system.

\[45\] This is the function that was introduced in the section in which optimal first period policies were described. Here the human capital of the grandfather does not appear as an argument of the function \( D^p \), and analogously for \( D^s \) below, because it is equal to 1 for the critical generation.
as reflected by the coefficient $\nu[1 + (1 - \alpha)]$ rather than $\nu$ in front of $\log h_{i+1}$ in the two expressions (5.7) and (5.10); and it increases the effort spent on education in the state system (as is clear from the Eqs. (B.14) and (B.17) in the appendix). We call this factor effective altruism.

Furthermore, for a given $i$, the median tax rate in the state system is different from the preferred tax rate according to which the critical parent would like to finance education for his son. In general the latter is larger than the former and this factor, which we label rate of expenditure, tends to reduce the transfer of resources factor and the capacity of a state system to increase mobility.\(^{36}\)

Finally, the fact that in the state system the tax rate is unique makes any information that a person may acquire on his and his son’s personal abilities useless, because he cannot adjust the expense in education for the son according to this information. Formally this effect can be related to the presence of the term $V(\nu)$ in the expression for $D^\circ$; this term is absent, instead, in the expression for $D^\circ$ because in the state system the tax rates in the three events $Y$ and a success, $Y$ and a failure, and $N$ are the same. The opposite is true for a father in the private system as reflected in the term $V(\nu)$ in the expression for $D^\circ$. We may call this term the value of information, which is due to the information acquired by going to school versus not going. If he goes to school, the father will know if his talent is high or low: hence he will know if the talent of the son is more likely to be high (with probability $1 - \alpha$) or more likely to be low (with probability $\alpha$). If he does not go, he will only have the information contained in his updated belief $\hat{\nu}$. But the function $L$ in Eq. (5.8) is convex; so we conclude that the value of information is always non-negative and therefore increases the desirability of human capital investment in the private system.

We can now summarise our comparison of the two functions $D^\circ$ and $D^\circ$, i.e. of the factors that determine the critical decision to acquire human capital in the two systems. We have seen four factors that affect this critical decision. Three of them, the effective altruism, the rate of expenditure and the value of information, tend to make the private school system more mobile. The first makes a higher income even more attractive for the father in the private system, thanks to the direct positive effect on the son. The second induces lower mobility in the state system by forcing a common lower tax rate, chosen by the median voter, on the critical voter. The third simply adds in the private system an additional reason to go to school: acquiring information on talent.

On the other side there is the transfer of resources factor. This factor captures

\(^{36}\)To see why, let’s call the critical voter the voter in the state system whose son is the first agent to go to school. We can compare his position to the position of the median voter. Observe that the proportion of unskilled individuals is larger than half when $i \neq 0$. (The proof of this statement is in Appendix B.4.) Then the median voter is always unskilled if $i \neq 0$, as a result the tax rate for the median voter is always lower than the optimal tax rate for the critical voter.
the fact that taxation in state systems transfers revenues from higher to lower income dynasties, increasing the quality of education available to the latter.

In the next section, with the help of numerical computations, we explore which conditions reinforce the different set of factors outlined above, making intergenerational mobility higher under one or the other stylised education system.

6. Numerical computations

We use the model described above to generate numerically two paradigmatic cases: one in which a centralised and egalitarian education system induces more mobility and one in which the opposite is true. Both outcomes are possible depending on parameter values.47

In Table 13 we present the relevant indicators that describe the performance of each education system in the two different paradigmatic cases. In both these cases the parameter $\alpha$, which measures the persistence in the transmission of talent, has been set equal to 0.1 while the scale parameter $\theta$ in the production function of human capital has been set equal to 2.8.48 The two paradigmatic cases differ instead for the values of the parameters $\beta$ and $\gamma$. These parameters measure, respectively, the elasticity of human capital accumulation with respect to effort $(1 - n_t)$ and with respect to the available quality of education $e_t$.

Part A of Table 13 shows that the main results of Glomm and Ravikumar (1992) hold also in our model. In both Case 1 and Case 2, the state system features a lower degree of inequality but also a lower total human capital (i.e. lower income) and a lower total expenditure in education. The median income in the upper class, that is a measure of inequality because income in the lower class is equal to 1 for everybody, is in fact larger in the private system independently from $\gamma$ and $\beta$. The counterpart of this greater inequality is the larger accumulation of human capital and the larger expenditure in education that the private system can generate, thanks to the fact that fathers are free to spend what they prefer for the education of their sons on the basis of their income and their beliefs on talent. In the state system, instead, where the total quality of education is determined by the common tax rate decided by the median voter and by the aggregate amount of human capital, the total expenditure in education is lower.

A common argument in defence of public schools is that they offer a better quality of education to poor dynasties that, in a private system, would otherwise spend too little for the education of their children. The last column of Part A in Table 13 confirms this intuition: the critical expenditure in education $e_t$, reported

\footnote{For the procedure followed in these numerical computations see Appendices B.5 and B.6.}

\footnote{Note that $\alpha = 0.5$ implies that the talent of the son is independent of the talent of the father; therefore $\alpha = 0.1$ implies a relatively high inheritability of talent. We will mention later how the results change in relation to the values of $\alpha$ and $\theta$.}
Table 13
Steady state performance indicators of the two systems

<table>
<thead>
<tr>
<th></th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>School system</th>
<th>Tax rate</th>
<th>Median income upp. class</th>
<th>Total human capital</th>
<th>Total expenditure in education</th>
<th>Critical expenditure in education</th>
</tr>
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<tbody>
<tr>
<td><strong>Part A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>0.3</td>
<td>0.1</td>
<td>State</td>
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<td>3.40</td>
<td>1.66</td>
<td>0.47</td>
<td>0.47</td>
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<td></td>
<td></td>
<td>Private</td>
<td></td>
<td>7.30</td>
<td>3.19</td>
<td>1.37</td>
<td>0.15</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.1</td>
<td>0.6</td>
<td>State</td>
<td>0.47</td>
<td>2.52</td>
<td>1.69</td>
<td>0.80</td>
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<td></td>
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<td>8.85</td>
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<tr>
<td>Case 1</td>
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<td>0.07</td>
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</table>

*All the indicators are computed at the steady state for: \( \alpha = 0.1 \) and \( \theta = 2.8 \). The median income of the upper class is a measure of inequality in these economies given that all the individuals in the lower class have an income equal to 1. Total human capital is defined as in Eq. (4.1). Total expenditure in education is the sum of what each father spends for the education of his son in the private system, while in the state system is given by Definition B.2. The critical expenditure in education is the education available to the generation that goes to school: it is equal to total expenditure in the state system because of the normalization of population. The proportion of unskilled is equal to \( \rho(1 + i) \) as in Section B.3. The probability of upward mobility is equal to the term \( \hat{a}'/(i + 1) \) in Lemma 5.1. The critical beliefs are the beliefs \( \nu^s \) or \( \nu^p \), respectively, for the private and the state system, that dynasties have to reach after a history of no schooling in order to decide to make an investment in education. The first generation in school after a failure is the value of the critical \( i \) as characterized, for example, in Lemma 4.2.

in this column, is what the fathers of the first generation going to school spend for the education of their children. Table 13 shows that in both Case 1 and Case 2 the state system offers a better quality of education to this critical generation and this is an implication of the transfer of resources factor that we mentioned in the previous section. The reader will recall that this is indeed the factor that tends to favour mobility in a state system.\(^9\)

However, the reader will also recall that other factors point in the opposite direction. Part B of Table 13 shows indeed that the provision of a better quality of education to poor families does not necessarily make the state system more mobile than the private system. The paradigmatic case in which the state system fails to generate more mobility is Case 1 in which \( \beta = 0.3 \) and \( \gamma = 0.1 \). Table 13 shows

\(^9\)Note that given that the population is normalised to 1, the total expenditure in education in the state system is equal to the expenditure for each individual including the critical one.
that in this case the probability of upward mobility is higher in the private system (0.09) than in the state system (0.05). A greater level of self confidence (i.e. the critical belief) is needed in the state system in order to go to school (0.42 versus 0.18) and seven generations (instead of one in the private system) wait after a failure without going to school before self confidence becomes sufficiently high to try the human capital investment.

In this case the public offer of equal educational opportunities is not sufficient to ensure more social mobility because the relative weight $\gamma$ of the quality of education in the production function for human capital is too low. As a result the transfer of resources effect, which tends to increase mobility in a state system, is dominated by the other three factors, mentioned in the previous section, that tend to increase mobility in a private system: effective altruism, the rate of expenditure and the value of information. On the contrary, in Case 2, when $\beta = 0.1$ and $\gamma = 0.6$, the quality of education is so important for the accumulation of human capital that the state system is capable of inducing greater mobility: the reason is that this is precisely the situation in which the public offer of a better education to poor families makes the investment in human capital convenient.

Increasing the values of the parameters $\alpha$ and $\theta$ (that is, making the transmission of talent more random and increasing everything else being equal, the accumulation of human capital in case of success in school) makes mobility more likely in both systems but does not change their qualitative relative performance in relation to the values of $\beta$ and $\gamma$. This is clear from our characterisation of the mobility matrix in Section 5: when the talent of the child is independent of the talent of the parent, this matrix has identical rows, irrespective of the values of the parameters and of the schooling system.

To summarise the results of our numerical computations, in order for the transfer of the resources factor to prevail, making the state system more mobile, two main conditions have to be met. First, redistribution of educational resources from rich to poor dynasties has to be high enough to ensure a sufficiently better quality of education for poor dynasties; and this is the common argument supporting the idea that state systems should generate more upward mobility. And second, the educational process must be such that individual effort is relatively less important than the quality of education for a successful accumulation of human capital.

7. Conclusions

If one of the goals of a public education system is to favour equal opportunities of social mobility, the Italian school system failed to achieve this goal. The centralised and public structure of education financing in Italy has indeed ensured a substantial uniformity of the quantity and quality of education offered to both rich and poor families; but despite this offer of equal opportunities Italy, in
comparison to the US, displays lower intergenerational mobility not only in terms of occupations but also in terms of education levels.

The failure of the Italian public system is certainly not a failure of public education per se. Germany, for example, where education is mainly public, has recently been shown to feature higher intergenerational mobility than the US.\footnote{See Checchi (1997) and Couch and Dunn (1997).} Note, however, that the German state school system is highly selective and diversified, in particular at the higher levels. Indeed, an indication of our model is that universities represent the level at which it is risky to offer public education in a uniform and egalitarian way because such an offer may decrease the incentive of poor families to invest in the accumulation of human capital even if education is free. Primary education, on the contrary, is the level at which individual effort is relatively less important than the quality of schools in the educational process, and a state school system certainly generates the conditions for a larger human capital investment of poor families.

Our comparison between Italy and the US also suggests that a decentralized and non-standardized school system allows a better tailoring of the available educational opportunities to the needs of the demand and supply of labor. Everything else being equal, the possibility to choose among a larger variety of investment opportunities should increase the attractiveness of an investment in human capital. Any form of consulting or screening capable of helping the children of disadvantaged families to acquire more information on their talents and attitudes should improve their capacity to make the best schooling decisions.

To conclude, in a world in which family background is important for labor market success, an excessively centralized and uniform quality of education, particularly at the university level, does not necessarily help poor children and may take away from them a fundamental tool to prove their talent and to compete with rich children.

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Appendix A. Empirical appendix

The Italian data come from a national survey conducted in 1985: the *Indagine Nazionale sulla Mobilità Sociale*. A representative sample of 5016 individuals aged between 18 and 65 was interviewed on their working life, their social attitudes and their family background. From this file, we extracted information concerning the status of the respondent in 1985 and his/her family when he/she was 14. Therefore, while respondents are observed in the same year (1985), their parents are observed in different years, ranging in principle from 1934 to 1981.

From the original sample we excluded all individuals not belonging to the labor force or whose occupation was unknown. In addition, for comparison with the US sample (see below), we excluded all women and all individuals younger than 25; this latter restriction is justified by the fact that we want to allow for the possibility of completing university curricula. With these restrictions the original sample reduces to 1666 son–father couples; their age distribution is reported in Table 12.

US data comes, instead, from the Panel Study of Income Dynamics (PSID), which consists of a longitudinal sample of families interviewed for the first time in 1968 and then followed on a yearly basis. The subsample that we use is an extract of the original sample containing information on 1050 father–son couples, whose occupation was known and whose age was greater than 25 at the time of the interview.

An important difference between the two datasets is that US data are based on direct interviews with both sons and fathers, while Italian data on fathers are based on sons’ recollections. Information on US sons was collected in 1990, while information on corresponding fathers refers to 1974. Because of the short interval between the two interviews, US sons are on average considerably younger than their fathers, as shown in Table 12.

In each country we consider the median income paid by each occupation as the indicator of individual long term economic status. We have not found a single classification of elementary occupations applicable to both countries, nor a conversion table from the national classifications into a common international one. For Italy our dataset is based on the occupation classification developed by DeLillo and Schizzerotto (1985), who grouped 13,000 elementary occupations into 97 basic groups, characterised by a similar degree of social desirability (as measured by the ranking obtained in sample interviews). For the US, we rely on the classification scheme developed by Duncan (1961). In this case the classification scheme includes 96 basic groups.

As far as occupational median incomes are concerned, for the US sample we have information about the earnings of both generations. On the contrary, in the
Italian sample, we do not have any direct information about incomes. We therefore merged occupational income data from another source according to the following procedure. We started with incomes taken from the 1987 wave of the *Indagine sui Bilanci delle Famiglie Italiane* run by the Bank of Italy. Since this survey reports net incomes, we have estimated the corresponding gross incomes on the basis of the relevant fiscal legislation for 1987.\footnote{The Italian system of personal income taxation is step-wise progressive and allows for tax deductions based on household composition. It is therefore possible to reconstruct for each individual his/her gross income starting from his/her net income. Note that preliminary versions of this paper have circulated with evidence based on net incomes.} We then estimated an earnings function using gross incomes. Regressors in the earning function were: age, six education dummies, nine qualification dummies, 11 sector dummies and five geographic dummies. We used the estimated parameters to predict incomes for the individuals in our main sample. From these predicted individual incomes we constructed the occupational ranking based on the median income of each occupation. This procedure could of course be used only for the generation of sons. Therefore we were forced to use also for fathers the occupational ranking constructed for sons. In order to allow for a meaningful comparison, we imposed the same restriction on the US dataset as well. But in this dataset we have been able to check that the ranking of occupations in terms of median incomes is fairly stable across generations: the correlation between occupational incomes constructed on the distribution of sons and on the distribution of fathers is equal to 0.78.

Appendix B. Theoretical appendix

B.1. A useful function

The following optimization problem appears repeatedly in our paper:

$$\max_{y \in [0, x]} \log(x - y) + z \log y.$$ 

Its solution is $y = (z/(1 + z))x$, and the value is:

$$(1 + z) \log x + L(z), \quad (B.1)$$

where we have denoted:

$$L(z) = z \log z - (1 + z) \log (1 + z). \quad (B.2)$$

In order to lighten the presentation, we often refer to this function in the paper.
B.2. First period optimal policies

We begin with the private school system. The agent born at \( t \) is comparing the maximum between two quantities. The first is the expected maximum utility from the choice \((Y, n_t)\) today, assuming that in the following period the agent will make the optimal choice (of consumption and expenditure on education for the son) conditional on the new information about his own and the son’s talent. With belief \( \nu_t \) on his own talent the first choice gives a success with probability \( \nu_t \) and failure with probability \( 1 - \nu_t \). If we substitute the values of the second period in the utility function (4.2) and write the maximisation problem for the first period we get:

\[
\max_{n_t \in [0,1]} \log n_t + \nu_t [1 + (1 - \alpha)] \log h_{t+1} + L(1 - \alpha) + (1 - \nu_t) L(\alpha) \tag{B.3}
\]

The optimal choice of leisure is

\[
\frac{1}{1 + \nu \beta [1 + (1 - \alpha)]} \tag{B.4}
\]

and the value is

\[
\nu \beta [1 + (1 - \alpha)] L\left(\frac{1}{1 + \nu \beta [1 + (1 - \alpha)]}\right) + \nu [1 + (1 - \alpha)] \log (\theta e^{\gamma h}) + \nu L(1 - \alpha) + (1 - \nu) L(\alpha). \tag{B.5}
\]

The second quantity we need to consider is the expected maximum utility from a choice \( N \) today. The effort does not affect the human capital, so the optimal choice of leisure is 1; the belief on the son will be \( \hat{\nu}_t \); and the corresponding value has the very simple form:

\[
L(\nu_{t+1}) = L(\hat{\nu}_t) \tag{B.6}
\]

The reasoning in the case of the state school system is similar. The agent solves:

\[
\max_{n_t \in [0,1]} \log n_t + \nu_t [1 - \tau] \log h_{t+1} + (1 - \alpha) \log (\tau H) + (1 - \nu) (\log (1 - \tau) + \alpha \log (\tau H))
\]

where the tax rate \( \tau \) is the prevailing tax rate (and not the tax rate chosen in the second period by the agent). The optimal choice of leisure is

\[
\frac{1}{1 + \nu \beta} \tag{B.7}
\]

and the value is
\[ v \beta \left( \frac{1}{\nu \beta} \right) + \nu \log(\nu e^\tau) + \log(1 - \tau) + [(1 - \alpha) \nu + (1 - \nu)L(\alpha)] \log(\tau H). \]  
(B.8)

B.3. The invariant distribution

In this section we provide the values of the invariant distribution over the state space \( \mathcal{B} \times H \), for a given value \( \hat{\alpha}' \) of the critical belief.

We denote by \( \Pi \), respectively \( \Sigma \), the transition matrix in the private, respectively state, system; \( \Pi(x, x') \) is the probability of the transition from \( x \) to \( x' \). An equilibrium invariant distribution is a probability \( F^* \) that reproduces itself, when each person makes the optimal choice. More formally we say:

**Definition B.1.** A steady state equilibrium distribution for the private school system is a probability measure \( F^*_p \) over the product space \( \mathcal{B} \times H \) such that

1. \( F^*_p = F^*_p \Pi \).
2. Each member of each dynasty is choosing effort and school expenditure optimally, according to the functions \((D^p, e^p)\) of Section 4.

Similarly we say:

**Definition B.2.** A steady state equilibrium distribution for the state school system is a triple \((\tau^*, e^*, F^*_S)\) of a tax rate, an average education quality and a probability measure \( F^*_S \) over the product space of beliefs and human capital such that \((F^*_S, H)\) is the marginal of \( F^*_S \) over \( H \):

1. \( F^*_S = F^*_S \Sigma \);
2. \( \tau^* \int hdF^*_S,h(h) = e^* \);
3. \( \tau^* \) is the median voter tax rate for \( F^*_S \);
4. Each member of each dynasty is choosing effort and votes on tax rate optimally, according to the functions \((D^S, \tau^S)\) of Section 4.

The integer \( i \) is the only factor determining this distribution. Therefore, in an invariant distribution, for each integer \( k = 0, 1, \ldots, i - 1 \) there is a corresponding fraction \( p_k \) of the population in state \((\hat{\alpha}^k, 1)\), a fraction \( p_{i-1} (1 - \hat{\alpha}') \) in state \((\hat{\alpha}', 1)\), and a fraction \( p_i \hat{\alpha}' \) in state \((\hat{\alpha}', h_0)\). It is immediate from the transition matrix that:

\[ p_0 = p_1 = \cdots = p_{i-1} = p. \]  
(B.9)

It will be useful now to use the following notational device: the state \((1 - \alpha, 1, \ldots, 1)\)
is the state of a person with belief \((1 - \alpha)\) in the first period of his life, coming after \(j\) consecutive successes in his dynasty, and who fails at school. Now denote by \(q_j\) and \(r_j\) respectively the fraction of the population in state \((1 - \alpha, h_j)\) and \((1 - \alpha, 1)\) we have:

\[
q_0 = p_{i-1} \hat{\alpha}^i = p \hat{\alpha}^i; \quad r_0 = p_{i-1} (1 - \hat{\alpha}^i) = p (1 - \hat{\alpha}^i);
\]  

\(q_{j+1} = (1 - \alpha) q_j, \quad r_{j+1} = \alpha q_j, \quad j = 0, 1, 2, \ldots \quad (B.10)\)

But now observing that:

\[
p = \sum_{j=0}^{\infty} r_j
\]

we may write:

\[
p_0 + \cdots + p_{i-1} + \sum_{j=0}^{\infty} r_j = (i + 1)p
\]

but also:

\[
p_0 + \cdots + p_{i-1} + \sum_{j=0}^{\infty} r_j + \sum_{j=0}^{\infty} q_j = 1
\]

and also from (B.11)

\[
\sum_{j=0}^{\infty} q_j = \frac{q_0}{\alpha}
\]

Using the equations above we get:

\[
p(1 + i) + \frac{1}{\alpha} q_0 = 1,
\]

which we can solve to get finally:

\[
p = \frac{\alpha}{\alpha(i + 1) + \hat{\alpha}^i}; \quad q_0 = \frac{\alpha \hat{\alpha}^i}{\alpha(i + 1) + \hat{\alpha}^i}; \quad q = \frac{\hat{\alpha}^i}{\alpha(i + 1) + \hat{\alpha}^i}; \quad (B.12)
\]

where \(q = \sum_{j=0}^{\infty} q_j\) is the fraction of the population with human capital greater than 1 and \((i + 1)p\) is the fraction of the population with human capital equal to 1.

B.4. Proofs

**Proof of Lemma 5.1.** Let \(F\) be an invariant distribution for the process described by the matrix \(I\). From the ergodic theorem, the measure of the set of dynasty histories with two consecutive values of 1 of human capital is given by:
\[ \sum_{(\nu, h): h = 1} F(\nu, h) \left( \sum_{(\nu', h'): h' = 1} I((\nu, h), (\nu', h')) \right). \]

From our computation of the invariant distribution we derive that the above quantity is equal to:

\[ p(i + 1) - 2p\hat{\alpha}^i + qa; \]

while the total fraction of population with human capital 1 is \( p(i + 1) \). Taking ratios and using the value for \( p \) and \( q \) in the Appendix B.3 we get the result. The proof for the other row is obvious.

Recall now that \( \hat{\alpha}^i = 0.5[1 - (1 - 2\alpha)^i] \); calculus applied to the function \( (1 - (1 - 2\alpha)x^{-1} \) proves the second claim.

**Proof** that the proportion of unskilled individuals is larger than half when \( i \neq 0 \) (see footnote 46).

The statement is equivalent to \( (i + 1)p > 1/2 \) which in turn is equivalent to:

\[ \frac{\hat{\alpha}^i}{(i + 1)\alpha} < 1. \]

But \( \hat{\alpha}^i = 1/2[1 - (1 - 2\alpha)^{i+1}] \); so this is equivalent to:

\[ (1 - 2\alpha)^{i+1} > 1 - 2\alpha(i + 1); \quad (B.13) \]

Call \( 2\alpha = x \) and \( i + 1 = n \) to simplify; and observe that

\[ f(x) = (1 - x)^n \]

has derivative at zero equal to \(-n\), and is strongly convex. Then since \( f(x) > f(0) + f'(0)x \) for every strongly convex function, and the above expression is exactly \( (B.13) \).

**B.5. Procedure for the numerical computations**

In this appendix we describe the procedure to compute the long run equilibrium. We begin with the private school system. The procedure checks for each integer \( i \) if the corresponding belief \( \hat{\alpha}^i \) is the critical belief of an equilibrium distribution. Recall that a critical belief is the least belief such that the member of a dynasty with that belief decides to go to school.

In the previous section we have determined the steady state equilibrium proportion of the population for the different beliefs. Note that there are several types of people having the belief \( 1 - \alpha \); namely, those whose dynasty has had a sequence of one, two, and so on successes. These types will have different levels of human capital. We now proceed to determine these levels and the corresponding proportions. Let us begin with the first. After the critical level \( \hat{\alpha}^i \) is reached, the member of the dynasty goes to school. The father had a human capital equal to 1,
a belief in his own talent equal to $\hat{\alpha}^{i-1}$, and has invested $e = \hat{\alpha}^i/(1 + \hat{\alpha}^i)$ in the education of his son.

The son invests the optimal amount of effort given these characteristics, and succeeds with probability $\hat{\alpha}^i$. If he does, he has a human capital of

$$h_0 = \theta \left( \frac{\hat{\alpha}^i \beta [1 + (1 - \alpha)]}{1 + \hat{\alpha}^i \beta [1 + (1 - \alpha)]} \right)^{\beta} \left( \frac{\hat{\alpha}^i}{1 + \hat{\alpha}^i} \right)^\gamma.$$

Similar arguments give that the dynasties with $j$ consecutive successes in the past have a level of human capital that follows the difference equation

$$h_j = \theta \left( \frac{(1 - \alpha) \beta [1 + (1 - \alpha)]}{1 + (1 - \alpha) \beta [1 + (1 - \alpha)]} \right)^{\beta} \left( \frac{1 - \alpha}{1 + (1 - \alpha)} \right)^\gamma h_{j-1}^{(\gamma + \delta)}$$

for $j = 1, \ldots$.

We have conjectured so far that the integer $i$ determines a critical belief $\hat{\alpha}^i$. The last step of the procedure is to verify this conjecture. If it is true, we have found a steady state equilibrium; if it is not, we proceed to the next integer. To verify the conjecture we have to check that the belief $\hat{\alpha}^i$ is indeed the least one for which people go to school. But the difference in expected utility between the two choices $Y$ and $N$ for a person with belief $\nu$ in his own talent, expenditure $e$ decided by the father and human capital $1$ of the father is given by the function $D^\nu$. The final step is now obvious: find the least integer $i$ such that

$$D^\nu \left( \hat{\alpha}^i, \frac{\hat{\alpha}^i}{1 + \hat{\alpha}^i} \right) \geq 0.$$

The procedure to determine the steady state equilibrium for the state school system is similar, and we provide here the main lines. In this case too we check if $\hat{\alpha}^i$ is the critical belief of the equilibrium, for every $i$. Recall now that the preferred level of taxes only depends on the belief of the father at the moment of voting. A simple computation now determines the median voter in this population, and the winning tax rate $\tau(\hat{\alpha}^i)$. Also arguments like the one given above give the human capital for generations with $j$ successes. The equations are now:

$$h_0 = \theta \left( \frac{\hat{\alpha}^i \beta}{1 + \hat{\alpha}^i \beta} \right)^{\beta} e^{\gamma};$$

and

$$h_j = \theta \left( \frac{(1 - \alpha) \beta}{1 + (1 - \alpha) \beta} \right)^{\beta} e^{\gamma} h_{j-1}^b,$$

for $j = 1, \ldots$. The $e$ in the formulas for human capital above is for the moment a parameter to be determined. Taking into account that the proportion of population with $h_0$ is $p \alpha^i$, and the proportion of population with $h_j$ is $q \alpha (1 - \alpha)^j$ for every
We can now determine the aggregate human capital and therefore the aggregate income, this last as a function of $e$ (besides $i$), $H(i, e)$ say. Now solving for $e$

$$e = \tau(\hat{e})H(i, e)$$

determines a value of the education quality level in the state school system $e(i)$, say. The final step is, as before, the determination of the integer $i$ for which indeed the belief $\hat{\alpha}$ is the critical level. The function giving the difference between the expected utility of the $Y$ and the $N$ decision, for a person whose father has a human capital equal to 1 is now given by: the function $D'$, and as before we conclude by determining the least integer $i$ such that $D'(\hat{\alpha}', e(i)) \geq 0$.

### B.6. A borderline case

The support of the invariant distribution is a countable set. In the computation of the median voter we begin to add from the lower tax rate, adding at each step discrete quantities corresponding to the different types of voters. It may happen therefore that one of these sums corresponds exactly to half of the voters. This is typically an unlikely event; there is one case however that is particularly important, and requires a detailed discussion.

Suppose that the critical $i$, i.e. the first time after a failure in which a dynasty tries to go to school, is zero. In the invariant distribution exactly half of the population would be unskilled, with a preferred tax rate equal to $\alpha/(1 + \alpha)$, and exactly half would be skilled, with most preferred tax rate equal to $(1 - \alpha)/(1 + (1 - \alpha))$. In this case the equilibrium in voting does not exist.

In the numerical computations, we present however the results for the case in which the critical $i$ is zero, and the tax rate is equal to $(1 - \alpha)/(1 + (1 - \alpha))$. We think the values we present are significant for the following reason.

Consider an economy in which the value of the parameters are such that with $i = 0$ exactly half of the population prefers the tax rate $(1 - \alpha)/(1 + (1 - \alpha))$ to the rate $\alpha/(1 + \alpha)$. This is not, at the corresponding stationary distribution, an equilibrium, because the proportion of population voting for the higher tax rate is not strictly larger than half. Consider however a path where the proportion of the population with human capital higher than 1 is larger than half, say $\mu_0$. Along the path the values of aggregate human capital and the distribution of human capital and belief converge to the values of an economy with tax rate equal to $(1 - \alpha)/(1 + (1 - \alpha))$. The transition is the one described in the previous Lemma 5.1; so the fraction of the population with belief higher or equal to $1 - \alpha$ is equal to $\mu^*_0$ in period $n$, a proportion strictly larger than half.

So along any such path, in every period, the economy is in an equilibrium in which the values of average human capital, its distribution among the population,
and so on are close to the values that we report for the case of the critical $i$ equal to 0, and tax rate equal to $(1 - \alpha)/(1 + (1 - \alpha))$.

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