## The Political Economy of Public Debt

We survey recent theories of public debt that incorporate political decision making in rich dynamic environments. These theories provide a new framework with which to interpret empirical evidence and to assess institutional reforms that may help control political inefficiencies. We discuss the inefficiencies that lead to overaccumulation of debt and their implications for the long-run distribution of debt.

# The Political Economy of Public Debt 

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## Introduction

- Since the world financial crisis began in mid-2008, public debt has increased rapidly in many countries.
- For many countries, however, the problem of high debt has a much earlier origin than the economic crisis of 2008.
- Structural imbalances in public finances, moreover, are expected to persist well past the end of the recession.
- These facts underscore the need for a general theory of public debt.
- Two approaches have dominated the economic literature.
- The macroeconomic literature has focused the analysis on the policies that would be chosen by a "benevolent planner". (Barro [1979], Stokey and Lucas [1983], Aiyagari et al. [2002]).
- Rich dynamic theories of fiscal policy: but ignoring the fact that policies are the result of political processes.
- This shortcoming has made it impossible for this literature to explain:
- the heterogeneity in fiscal policy that we observe,
- its dependence on political institutions
- and, of course, excess debt.
- The political economy and social choice literature has emphasized the need to study political institutions (Buchanan and Tullock [1962], Buchanan [2000, Svensson and Persson [1989], Alesina and Tabellini [1990]).
- These theories, however, have been developed in simple environments, abstracting from shocks:
- how does debt react in booms and recessions,
- how it evolves over time, and where it converges in steady state.
- This has made it difficult to verify these theories, empirically.
- In this two lectures I survey recent work attempting to bridge the gap between the two literatures:
- studying political distortions typical in the political economy literature;
- in rich dynamic frameworks typical of the macroeconomic literature.


## Plan for this week

- We start today with a simple dynamic model of fiscal policy.
- Standard neoclassical real business cycle framework;
- A legislature that chooses fiscal policy in each period by non cooperative bargaining.
- The public debt is a state variables, creating a dynamic linkage across policy-making periods.
- How do policies in a PE differ from a normative benchmark?
- In lecture 2 I propose an alternative model of public debt to study the relationship between fiscal policy, public debt and unemployment.
- In lecture 3 we investigate the role of public debt in a general equilibrium framework.
- In lecture 4 (if we have time) we study constitutional design:
- How should we choose the rules of the game?
- How should we interpret data coming from different institutions?
- Should we tie the hands of politicians? Is there a role for budget balance requirements?


# A Dynamic Theory of Public Spending, Taxation and Debt 

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- What determines fiscal policy in a dynamic economy?
- In an influential paper, Barro [1979] answered this question with a few simple ingredients:
- policies are chosen by a benevolent government;
- government spending needs fluctuate over time (wars, hurricanes, etc.);
- taxes are distortionary;
- deadweight costs are convex in the tax rate.
- The government should use budget surpluses and deficits as a buffer to prevent tax rates from changing too sharply.
- Empirical evidence supports this tax smoothing theory:
- historically the debt/GDP ratio in the U.S. and the U.K. tends to increase in periods of high government spending needs (such as wars) and decrease in periods of low needs (Barro (1979), (1986), and (1987))



## Problems

- In the absence of "ad hoc" limits on government bond holdings, the government wants to self insure:
- The government gradually acquires sufficient bond holdings to finance spending out of interest earnings.
- A steady state with zero taxes and huge public asset accumulation is obviously counter factual.
- How would a representative democracy manage public finance in a dynamic environment?
- Today we study a theory that attempts to address both problems.
- The theory has two ingredients:
- A neoclassical real business cycle framework with shocks on preferences;
- A legislature that chooses fiscal policy by non cooperative bargaining.
- Public debt is the state variable, creating a dynamic linkage across policy-making periods.
- We characterize the unique equilibrium, and compare the predictions with Barro’s tax smoothing approach.


## Results

1. The legislature smoothes taxation, but inefficiently: the MCPF is a submaringale.
2. Tax smoothing + Political Economy $=>$ counter cyclical theory of deficits (... and help explain empirical evidence)

## Plan for today

I. The model
II. The planner's solution
III. The political equilibrium

1. Equilibrium tax smoothing
2. The cyclical behavior of policies
IV. What type of tax smoothing do we observe?
V. Some empirical implications.

## I. The Model

## I. 1 The economy

- A continuum of infinitely-lived citizens live in $n$ identical districts. The size of the population in each district is normalized to be one.
- There are three goods - a public good $g$, private consumption $z$, and labor $l$.
- Each citizen's per period utility function is $z+A g^{\alpha}-\frac{l^{\left(1+\frac{1}{\varepsilon}\right)}}{\varepsilon+1}$.
- Discount factor: $\bar{\delta}$.
- Technology: $\mathrm{z}=w \mathrm{l}$ and $\mathrm{z}=p g$.
- The value of the public good varies across periods in a random way:

$$
A \square G(A) \text { with support }[\overline{\mathrm{A}}, \underline{\mathrm{~A}}]
$$

- There are markets for labor, the public good, and one period, risk free bonds.
- In a competitive equilibrium:
- price of the public good is $p$,
- the wage rate is $w$,
- and the interest rate is $\rho=1 / \delta-1$.


## I. 2 Public Policies

- The legislature can raise revenues in two ways: a tax on labor income (r) and borrowing (b).
- If the legislature borrows $b$ in period $t$ it must repay $(1+\rho) b$ in period $t+1$.
- The legislature can also hold bonds if it wants, so $b$ can be negative.
- Public revenues can be used to finance public goods or targeted district-specific transfers (non-distortionary pork).
- A policy choice is described by an $n+3$-tuple:

$$
\left\{r, g, x, s_{1}, \ldots, s_{n}\right\}
$$

- Define the net of transfer surplus to be:

$$
B(r, g, x ; b)=R(r)+x-p g-(1+\rho) b
$$

where $R(r)$ is the tax revenue function.

- The policy choice must satisfy the budget constraint:

$$
B_{\theta}(r, g, x ; b) \geq \sum_{i} S_{i}
$$

## Legislative policy-making

- Public decisions are made by a legislature of representatives from each of the $n$ districts.
- One citizen from each district is selected to be that district's representative.
- The legislature meets at the beginning of each period.
- The affirmative votes of $q<n$ representatives are required to pass legislation.
- One legislator is randomly selected to make the first policy proposal.
- If the proposal is accepted by $q$ legislators, the plan is implemented and the legislature adjourns until the next period.
- At that time, the legislature meets again with the only difference being that $b$ and (maybe) $A$ are different.
- If the first proposal is rejected, another legislator is chosen.
- There are $T$ such proposal rounds, each of which takes a negligible amount of time.


## I. 4 Equilibrium

- We look for a symmetric Markov-perfect equilibrium with stage-undominated strategies.
- In this type of equilibrium, representatives' proposals just depend upon the proposal round and the state variables.
- The problem has a recursive structure with state variables $b$ and $A$.
- An equilibrium is said to be well-behaved if the value function $v_{1}(b, A)$ is concave and continuous in $b$ for all $A$
- A well-behaved equilibrium exists and is unique.


## II. The Planner's Solution

- The planner's problem can be written in the recursive form:

$$
v^{\circ}(b)=\max _{r, g, x}\left\{\begin{array}{c}
u(r, g)+\frac{B(r, g, x ; b)}{n}+\delta E\left[v^{\circ}\left(x ; A^{\prime}\right)\right] \\
B(r, g, x ; b) \geq 0 \& x \leq \bar{x}
\end{array}\right\} .
$$

where $u_{1}(r, g)$ is the indirect utility function in state $\theta$ and $v^{\circ}(b)$ is the continuation value.

- The problem is one of "tax smoothing" (Barro 1979).
- Define the Marginal Cost of Public Funds (MCPF) at $b$ :

$$
-\frac{\frac{\partial u\left(r^{\circ}(b ; A), g^{\circ}(b ; A)\right)}{\partial r}}{\frac{\partial R\left(r^{\circ}(b ; A)\right)}{\partial r}}=\frac{1-r^{\circ}(b ; A)}{1-r^{\circ}(b ; A)(1+\varepsilon)},
$$

it is the "cost" of raising $1 \$$ in tax revenues.

- The MCPF obeys a martingale; that is,

$$
\frac{1-r^{\circ}(b ; A)}{1-r^{\circ}(b ; A)(1+\varepsilon)}=E\left[\frac{1-r^{\circ}\left(x^{\circ}(b ; A)\right)}{1-r^{\circ}\left(x^{\circ}(b ; A)\right)(1+\varepsilon)}\right] .
$$

- The tax rate obeys a supermartingale; that is,

$$
r^{\circ}(b ; A)>E\left[r^{\circ}(b ; A)\right]
$$

- In the short run, debt displays a counter-cyclical pattern and tax rates and public good spending are pro-cyclical.
- But in the long run the government accumulates sufficient assets to finance the public good at first best levels from the interest earnings. Tax rates are zero in the long run.


## III. The political equilibrium

- The proposer problem is:



. $\stackrel{s}{s}=v_{\tau+1}(b, A)-\left[u(w(1-r), g ; A)+\delta E v\left(x, A^{\prime}\right)\right]$,
- $u(w(1-r), g ; A)+B(r, g, x ; b)$
$+(q-1)\left[u(w(1-r), g ; A)+\delta E v\left(x, A^{\prime}\right)\right]-(q-1) v_{\tau+1}(b, A)$ $+\delta E v\left(x, A^{\prime}\right)$
- The proposer is effectively making decisions to maximize the collective utility of $q$ legislators.

$$
\begin{array}{rl}
\max _{r, g, x} & u(r, g)+\frac{B(r, g, x ; b, A)}{q}+\delta E v(x ; A) \\
\text { s.t. } & B(r, g, x ; b, A) \geq 0 \& x \leq \bar{x}
\end{array}
$$

- When $b$ is high and/or $A$ is high, pork is too expensive, so: $B(r, g, x ; b, A)=0$. Proposer's policy = Planner's policy
- When $b$ is low and/or $A$ is low, the opportunity cost of revenues is lower: $B(r, g, x ; b, A)>0$. There is pork.
- This diversion of resources, creates lower bounds on $r, b$, and an upper bound on $g$.
- Consider the case when b is low:

$$
\begin{gathered}
\max _{(r, g, x)} u(w(1-r), g ; A)+\frac{B(r, g, x ; b)}{q}+\delta E v_{1}\left(x ; A^{\prime}\right) \\
\text { s.t. } \quad x \in[\underline{x}, \bar{x}] \quad \text { and } \quad B(r, g, x ; b) \geq 0
\end{gathered}
$$

- We first ignore the budget constraint...focs are:
- for $r: \frac{1}{q}=\frac{1}{n}\left[\frac{1-r^{*}}{1-r^{*}(1+\varepsilon)}\right]$,
- for g: $\alpha A g^{*}(A)^{\alpha-1}=\frac{p}{q}$,

Marginal benefit of pork = marginal cost of taxation

Marginal benefit of public good $=$ marginal benefit of pork

$$
\text { for } x: \frac{1}{q} \geq-\delta E\left[\frac{\partial v_{1}\left(x^{*}, A^{\prime}\right)}{\partial x}\right]\left(=\text { if } x^{*}<\bar{x}\right) . \begin{gathered}
\text { Marginal benefit of } \\
\text { pork }=\begin{array}{c}
\text { marginal cost } \\
\text { of debt }
\end{array}
\end{gathered}
$$

- When budget is not binding, therefore optimal policy is:

$$
\left(r_{\tau}(b, A), g_{\tau}(b, A), x_{\tau}(b, A)\right)=\left(r^{*}, g^{*}(A), x^{*}\right)
$$

- When is it admissible to focus on this relaxed problem?
- Define:

$$
A^{*}(b, x)=\max \left\{A \in[\underline{A}, \bar{A}] \mid B\left(r^{*}, g^{*}(A), x ; b\right) \geq 0\right\}
$$

- Proposition 1: There exists a debt level $x^{*}$ such that:
- if $A \leq A^{*}\left(b, x^{*}\right) \quad$ The policy choice is

$$
\left(r_{\tau}(b, A), g_{\tau}(b, A), x_{\tau}(b, A)\right)=\left(r^{*}, g^{*}(A), x^{*}\right)
$$

Business as usual (BAU)
and there are pork transfers to a MWC.

- if $A \geq A^{*}\left(b, x^{*}\right)$ the optimal policy:
$\arg \max \left\{\begin{array}{l}u(w(1-r), g ; A)+\frac{B(r, g, x ; b)}{n}+\delta E v_{1}\left(x ; A^{\prime}\right) \\ \text { s.t. } B(r, g, x ; b)=0 \& x \in[\underline{x}, \bar{x}]\end{array}\right\}\left\{\begin{array}{c}\text { Responsible } \\ \text { policy } \\ \text { making } \\ (R P M)\end{array}\right.$
No pork transfers. Deliberations are unanimous.


## IV Equilibrium tax smoothing

$$
\operatorname{MCPF}(b ; A)=-\delta E\left[\frac{\partial v_{1}\left(x^{*}, A^{\prime}\right)}{\partial x}\right] .
$$

- If $A \leq A^{*}\left(x, x^{*}\right)$, a marginal increase in debt reduces only pork:

$$
\frac{\partial v_{1}(x, A)}{\partial x}=-\frac{1+\rho}{n} \quad\left(=-\frac{1}{\delta n}\right)
$$

- If $A \geq A^{*}\left(x, x^{*}\right)$, a marginal increase in debt increases taxes:

$$
\frac{\partial v_{1}(x, A)}{\partial x}=-\left(\frac{1-r_{1}(x, A)}{1-r_{1}(x, A)(1+\varepsilon)}\right)\left(\frac{1+\rho}{n}\right) .
$$

- Therefore:

$$
-\delta n E\left[\frac{\partial v_{1}(x, A)}{\partial x}\right]=G\left(A^{*}\left(x, x^{*}\right)\right)+\int_{A^{*}\left(x, x^{*}\right)}^{\bar{A}}\left(\frac{1-r_{1}(x, A)}{1-r_{1}(x, A)(1+\varepsilon)}\right) d G(A)
$$

- So combining with foc we have that $x^{*}$ must satisfy;

$$
\begin{aligned}
\operatorname{MCPF}(b ; A) & =G\left(A^{*}(x, b)\right)+\int_{A^{*}\left(x^{*}, x^{*}\right)}^{\bar{A}}\left(\frac{1-r_{1}\left(x, A^{\prime}\right)}{1-r_{1}\left(x, A^{\prime}\right)(1+\varepsilon)}\right) d G(A) \\
& \leq E\left[\operatorname{MCPF}\left(b ; A^{\prime}\right)\right]
\end{aligned}
$$

Proposition 2. The marginal cost of public funds is a sub martingale, strict for sufficiently low levels of b

## V. The Invariant distribution

- Define $\hat{A}(b, x)$ such that $x(b, \hat{A}(b, x))=x$
- Then the transition function can be derived from optimal policies as:

$$
H(b, x)=\left\{\begin{array}{l}
G(\bar{A}(b, x)) \text { if } x \in\left(x^{*}, \bar{x}\right] \\
G\left(A^{*}\left(b, x^{*}\right)\right) \text { if } x=x^{*}
\end{array}\right.
$$

- And the distribution of states defined inductively as:

$$
\psi_{t}(x)=\int_{b} H(b, x) d \psi_{t-1}(b)
$$

- Definition. $\psi^{*}(x)$ is an invariant distribution if

$$
\psi^{*}(x)=\int_{b} H(b, x) d \psi^{*}(b)
$$

Proposition 3. The equilibrium debt distribution converges to a unique invariant distribution whose support is $\left[x^{*}, \bar{x}\right]$. This distribution has a mass point at $x^{*}$ but is nondegenerate.

- So the planner's solution does not explain data: there is too much volatility, too much debt.
- Political economy explains why debt does not converges to levels compatible to self insurance.
- For the ID, the key variable is the lower bound on debt $\mathrm{x}^{*}$.

$$
\begin{aligned}
& \operatorname{MCPF}(b ; A)=\frac{n}{q} \\
& =G\left(A^{*}\left(x^{*}, x^{*}\right)\right)+\int_{A^{*}\left(x^{*}, x^{*}\right)}^{\bar{A}}\left(\frac{1-r_{1}\left(x^{*}, A\right)}{1-r_{1}\left(x^{*}, A\right)(1+\varepsilon)}\right) d G(A)
\end{aligned}
$$

- When $q=n$, we must have: $A^{*}\left(x^{*}, x^{*}\right)=\bar{A}$
- So when we reach the BAU we remain there forever.
- Corresponds to the planner's SS: perfect tax smoothing.
- When $q<n$, we must have: $A^{*}\left(x^{*}, x^{*}\right)<\bar{A}$
- Smoothing will be imperfect.


## IV. What type of smoothing do we observe?

- Barro (1979) conjectured that tax rates should obey a martingale and this inspired an large empirical literature.
- In general, the planner's solution implies that the MCPF is a martingale (and the tax rate a supermartingale).
- We can test our prediction that the MCPF is a submartingale.

USA



JPN


FRA


ITA


GBR



CAN


## VI. Conclusion

- We have developed a political economy theory of the behavior of fiscal policy over the real business cycle.
- Legislative bargaining induces (imperfect) taxation smoothing.
- Taxation smoothing induces an "increasing" cost of taxation.
- Debt is too high and too volatile.
- Empirical evidence supports these predictions;
- This is a neoclassical theory: efficient market, inefficient taxation, no unemployment.


# Fiscal Policy and Unemployment 

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## Introduction

- During the "Great recession," countries have pursued a variety of fiscal strategies - tax cuts, public works projects.
- Recent experience reveals that the willingness to use fiscal policy is tempered by the cost of high levels of debt.
- All this suggests an interesting and potentially important interaction between fiscal policy and unemployment.
- Fiscal policy has the potential to mitigate unemployment;
- The desirability of stimulus policies, on the other hand, depend on the country's debt position.
- This paper explores this interaction between fiscal policy and unemployment.
- It constructs a dynamic economic model with a private and a public sector.
- Unemployment can arise because of sticky wages but can be mitigated by tax cuts and spending increases.
- The model allows government to finance stimulus activities by issuing debt.
- This model is used to explore the levels of unemployment that arise in steady state and the way in which fiscal policy is used.
- The paper consider both outcomes with a benevolent government and with policies determined in each period by legislative bargaining.
- Our work differs from the prevailing literature with sticky wages in which fiscal policy is assumed exogenous (or drastically simplified by assuming no public debt).
- With a benevolent government:
- In the long run, there is no unemployment.
- The mix of public and private outputs are optimal.
- The way in which a benevolent government achieves this outcome is by accumulating bond holdings.
- The earnings from these assets are used to finance unemployment mitigation when the private sector experiences negative shocks.
- Main lesson: In the long run a benevolent government employs fiscal policy to circumvent market inefficiencies.
- When fiscal policy is endogenous, it is not possible to model the dynamics of unemployment without modeling how fiscal policy is chosen.
- This motivates our introduction of political decision making.
- With legislative bargaining, when the private sector experiences negative shocks, unemployment arises:
- In these recessions, government mitigates unemployment with debt-financed stimulus plans.
- The stimulus plans typically involve both tax cuts and public production increases.
- When choosing such plans, the government balances the benefits of reducing unemployment with the costs of distorting the output mix.
- When the private sector is not experiencing negative shocks:
- the government reduces debt until it reaches a floor level.
- The existence of this floor level prevents asset accumulation as in the benevolent government solution.
- When there is unemployment, the larger is government's debt level, the larger is unemployment.
- Main lesson: With political decision-making, the model delivers an appealing positive theory of fiscal policy and unemployment.


## Plan for today

I. The model
II. The optimal fiscal policy

- Static analysis
- Dynamic analysis
III. The political equilibrium
IV. The equilibrium stimulus plans
V. Conclusion


## I. The model

## I. 1 The economy

- We consider an infinite horizon economy with:
- Two final goods, a private good $x$ and a public good $g$;
- Two inputs: labor l, and, a natural resource z (say, oil)
- There are two types of citizens, workers and entrepreneurs:
- A mass $n_{w}$ of workers, endowed with 1 unit of labor each period which they supply inelastically.
- A mass $n_{e}$ of entrepreneurs produce the private good by combining labor and oil with their own effort.
- The public good is produced by the government using labor: $g=l$
- Each entrepreneur produces with the Leontief production technology $x=A \cdot \min \{l, \epsilon, Z\}$ where $\epsilon$ represents the entrepreneur's effort.
- Workers' per period payoff function is $x+\gamma \cdot \ln g$, where $Y$ measures the relative value of the public good.
- Entrepreneurs' per period payoff function is $x+\gamma \cdot \ln g-\xi \epsilon^{2 / 2}$ where the third term represents the disutility of providing entrepreneurial effort.
- All individuals discount the future at rate $\beta$.
- There are markets for the private good, oil and labor.
- The private good is the numeraire, and the wage rate is $\omega$.
- We assume that $\omega \geq \underline{\omega}$.
- This friction is the source of unemployment.
- The natural resource $z$ is provided by foreign suppliers and has an exogenous but variable price $p_{\theta}$.
- Each period, $\operatorname{Pr}\left(p_{\theta}=p_{H}\right)=\alpha$ and $\operatorname{Pr}\left(p_{\theta}=p_{L}\right)=1-\alpha$.
- We will sometimes say that the economy is in the high cost state when $\theta=H$ and the low cost state when $\theta=L$.
- There is also a market for risk-free one period bonds: $\rho=1 / \beta-1$.


## I. 2 Public policies

- The government can raise revenues in two ways: a tax on profits ( T ) and borrowing (b).
- If the legislature borrows $b$ in period $t$ it must repay $(1+\rho) b$ in period $t+1$.
- The legislature can also hold bonds if it wants, so $b$ can be negative.
- Public revenues are used to finance public goods. Surplus revenues are distributed to citizens by lump transfers.


## I. 3 Market equilibrium

- Assume the state of the economy is $\theta$ and that the tax rate is T and the public good level is $g$.
- Entrepreneur choose $l, z$ and $\epsilon$ to maximize:

$$
\max _{(l, z, e)}(1-\tau)\left(A \min \{l, z, \varepsilon\}-p_{\theta} z-\omega_{\theta} l\right)-\xi \frac{\varepsilon^{2}}{2} .
$$

- Setting demand of $l$ equal to supply, we obtain:

$$
\omega_{\theta}=\left\{\begin{array}{c}
\underline{\omega} \quad \text { if } A_{\theta} \leq \underline{\omega}+\xi\left(\frac{n_{w}-g}{n_{e}(1-\tau)}\right) \\
A_{\theta}-\xi\left(\frac{n_{w}-g}{n_{e}(1-\tau)}\right) \quad \text { if } A_{\theta}>\underline{\omega}+\xi\left(\frac{n_{w}-g}{n_{e}(1-\tau)}\right)
\end{array}\right.
$$

where $A_{\theta}=A-p_{\theta}$.

- ...so when $A_{\theta}$ is small (relative to T and g ) we may have unemployment:

$$
u_{\theta}=\left\{\begin{array}{c}
\frac{n_{w}-g-n_{e}(1-\tau)\left(A_{\theta}-\underline{\omega}\right) / \xi}{n_{w}} \quad \text { if } A_{\theta} \leq \underline{\omega}+\xi\left(\frac{n_{w}-g}{n_{e}(1-\tau)}\right) \\
0 \quad \text { if } A_{\theta}>\underline{\omega}+\xi\left(\frac{n_{w}-g}{n_{e}(1-\tau)}\right) .
\end{array}\right.
$$

- Note: $t \uparrow u \uparrow$ and $g \uparrow u \downarrow$
- From these expressions we obtain the indirect utility functions:

$$
\begin{gathered}
v_{e \theta}(\tau, g)=\frac{\left(A_{\theta}-\omega_{\theta}\right)^{2}(1-\tau)^{2}}{2 \xi}+\gamma \ln g \\
v_{w \theta}(\tau, g)=\left(1-u_{\theta}\right) \omega_{\theta}+\gamma \ln g
\end{gathered}
$$

- Substituting in the expression for the equilibrium wage, we see that the private sector output is:

$$
x_{\theta}=\left\{\begin{array}{c}
n_{e} A(1-\tau)\left(A_{\theta}-\underline{\omega}\right) / \xi \quad \text { if } A_{\theta} \leq \underline{w}+\xi\left(\frac{n_{w}-g}{n_{e}(1-\tau)}\right) \\
A\left(n_{w}-g\right) \quad \text { if } A_{\theta}>\underline{\omega}+\xi\left(\frac{n_{w}-g}{n_{e}(1-\tau)}\right)
\end{array}\right.
$$

- Note that T affects the private sector output only when the minimum wage constraint is binding.
- This is because labor is inelastically supplied and as a consequence the wage adjusts to ensure full employment.
- Given this, the public policies must satisfy the budget constraint:

$$
R_{\theta}\left(\tau, \omega_{\theta}\right)-\omega_{\theta} g \geq b(1+\rho)-b^{\prime}
$$

- The upperbound on debt is:

$$
b \leq \bar{b}=\max _{\tau} R_{H}(\tau, \underline{\omega}) / \rho
$$

## I. 4 Politics

- We assume that the economy is divided into $N$ identically sized political districts, each a microcosm of the economy as a whole.
- In each period, policy decisions are made by a legislature consisting of $N$ representatives, one from each district.
- Each representative maximizes the welfare of his/her own district
- The budget surplus can be divided among the districts in any way the representatives choose.
- The affirmative votes of $Q<N$ representatives are required to pass legislation.


## II. Optimal fiscal policy

## II. 1 The static case

- Consider the budget constraint:

$$
R_{\theta}\left(\tau, \omega_{\theta}\right)-\omega_{\theta} g \geq b(1+\rho)-b^{\prime}
$$

- We start by fixing debt, so:

$$
R_{\theta}\left(\tau, \omega_{\theta}\right)-\omega_{\theta} g \geq r
$$

- The problem becomes:
$\max _{(\tau, g)}\left\{\begin{array}{c}R_{\theta}\left(\tau, \omega_{\theta}\right)-\omega_{\theta}(\tau, g) \cdot g-r+n_{e} v_{e \theta}(\tau, g)+n_{w} v_{w \theta}(\tau, g) \\ \text { s.t. } R_{\theta}\left(\tau, \omega_{\theta}\right)-\omega_{\theta}(\tau, g) \cdot g \geq r\end{array}\right\}$.
- It can be rewritten as:

$$
\max _{(\tau, g)}\left\{\begin{array}{r}
x_{\theta}(\tau)\left(\frac{A_{\theta}}{A}\right)-n_{e} \xi \frac{\left(\frac{x_{\theta}(\tau)}{A_{e}}\right)^{2}}{2}+\gamma \ln g-r \\
\text { s.t. } R_{\theta}(\tau, \underline{\omega})-\underline{\omega} g \geq r \& g+\frac{x_{\theta}(\tau)}{A} \leq n_{w}
\end{array}\right\},(*)
$$

- $x_{\theta}(\tau)\left(\frac{A_{\theta}}{A}\right)-n_{e} \xi \frac{\left(\frac{x_{(\tau)} A_{r_{e}}}{}\right)^{2}}{2}+\gamma \ln g-r$ is the surplus produced
- $R_{\theta}(\tau, \underline{\omega})-\underline{\omega} \geq r \quad$ is the budget constraint
- $g+\frac{x_{\theta}(\tau)}{A} \leq n_{w} \quad$ is the resource constraint
- Problem (*) can be graphically analyzed.


When $r \leq r_{\theta}^{o}=R_{\theta}\left(\tau_{\theta}^{o}, \underline{\omega}\right)-\underline{\omega} g_{\theta}^{o}$, the solution is $g_{\theta}{ }^{o}, \mathrm{~T}_{\theta}{ }^{o}$, independent from $r$. This allocation is efficient.


- When $r>r_{\theta}^{o}$ the efficient allocation is unfeasible.
- When $r \leq r_{\theta}^{*}$, the solution is at a kink: $g_{\theta}^{-}>g_{\theta}^{o}, \tau_{\theta}^{-}>\tau_{\theta}^{o}$.
- The output mix is distorted in favor of the public good.


When $r>r_{\theta}^{*}$, we have unemployment. Further increases in $r$ induce a reduction in $g$ and $T$.

- In summary:
- If $r \leq r_{\theta}^{o}$, the solution involves full employment with no distortions.
- If $r \in\left(r_{\theta}^{o}, r_{\theta}^{*}\right]$, the solution involves full employment with distortions: $g_{\theta}^{-}>g_{\theta}^{o}, \tau_{\theta}^{-}>\tau_{\theta}^{o}$.
- If $r>r_{\theta}^{*}$, the solution involves unemployment.
- What have we learned?
- The government trades-off distorting the mix of public and private outputs with minimizing unemployment:
- When $r \in\left(r_{\theta}^{o}, r_{\theta}^{*}\right]$, the government finds it optimal to increase $g$ and T to keep full employment.
- When $r>r_{\theta}^{*}$, the government accepts unemployment, the higher is $r$, the lower $g$, the higher T .
- What do we still need to know? Revenue requirements are endogenous. What is the relevant range of $r$ ?
- We need to endogenize public debt.


## II. 2 The dynamic case

- The dynamic problem can be written as:

$$
V_{\theta}(b)=\max _{\left(\tau, g, b^{\prime}\right)}\left\{\begin{array}{c}
b^{\prime}-b(1+\rho)+x_{\theta}(\tau)\left(\frac{A_{\theta}}{A}\right)-n_{e} \xi \frac{\left(\frac{x_{\theta}(\tau)}{A n_{e}}\right)^{2}}{2}+\gamma \ln g+\beta E V_{\theta^{\prime}}\left(b^{\prime}\right) \\
\text { s.t. } R_{\theta}(\tau, \underline{\omega})-\omega_{\theta} g \geq b(1+\rho)-b^{\prime}, \quad g+\frac{x_{\theta}(\tau)}{A} \leq n_{w}
\end{array}\right\} .
$$

- Policies now are functions of $b$ and $\theta: \tau_{\theta}(b), g_{\theta}(b), b_{\theta}^{\prime}(b)$
- The revenue requirement is now endogenous:

$$
r_{\theta}(b)=(1+\rho) b-b_{\theta}^{\prime}(b)
$$

- From the previous analysis, the economy converges to full employment with no distortions iff $r_{\theta}(b)<r^{o}{ }_{\theta}$.

Proposition. In any solution to the government's problem,

$$
\operatorname{Pr}\left(\lim _{t \rightarrow \infty} r_{\theta_{t}}\left(b_{t}\right) \leq r_{\theta_{t}}^{o}\right)=1,
$$

so the economy converges to full employment with no distortions.

- Intuition: The benevolent government finds it optimal to accumulate resources to self insure against the labor market distortions.
- The result is related to the steady state in the tax smoothing model. But here there is no tax smoothing!
- The analysis suggests there is an intimate connection between how fiscal policy is chosen, and unemployment, even (and especially) when there are market imperfections.
- Market imperfections and political distortion in policymaking are needed to explain unemployment.


## II. The political equilibrium

- One legislator is randomly selected to make the first policy proposal.
- If the proposal is accepted by $Q$ legislators, the plan is implemented and the legislature adjourns until the next period.
- At that time, the legislature meets again with the only difference being that $b$ and (maybe) $\theta$ are different.
- If the first proposal is rejected, another legislator is chosen.
- The proposer is forced to internalize the welfare of $Q$ districts.
- The proposer's problem can be written as:
$\max _{\left(\tau, g, b^{\prime}\right)}\left\{\begin{array}{c}b^{\prime}-(1+\rho) b+x_{\theta}(\tau)\left(\frac{A_{\theta}}{A}\right)-n_{e} \xi \frac{\left(\frac{x_{\theta}(\tau)}{A_{e}}\right)^{2}}{2}+\gamma \ln g+\beta E V_{\theta^{\prime}}\left(b^{\prime}\right) \\ +(q-1)\left(R_{\theta}(\tau, \underline{\omega})-\underline{\omega} g+b^{\prime}-(1+\rho) b\right) \\ \text { s.t. } R_{\theta}(\tau, \underline{\omega})-\underline{\omega} g \geq b(1+\rho)-b^{\prime}, \quad g+\frac{x_{\theta}(\tau)}{A} \leq n_{w}\end{array}\right\}$.
where $q=N / Q>1$.
- This problem can be studied graphically as before.
- Since $q>1$, politicians put more weight on tax revenues and primary surplus: the indifference curve is steeper in the $g, T$ space.
- In the short run, politicians trade-off distorting the mix of public and private outputs with minimizing unemployment.
- Now the trade off favors tax revenues to finance targetable transfers.
- As before, unemployment is zero if and only if:

$$
r_{\theta}(b)=(1+\rho) b-b_{\theta}^{\prime}(b) \leq r_{\theta}^{*} .
$$

Proposition. In any equilibrium, public debt converges to a stationary distribution such that $\operatorname{Pr}\left(r_{\theta}(b)>r^{*}{ }_{\theta}\right)>0$, and so there is unemployment with positive probability.

- Intuition: the legislature can not commit to save sufficient resources to fight unemployment in bad times.
- What type of unemployment dynamics can we observe? It depends on the severity of the political distortions.

Proposition. There is a threshold $q^{*}$ such that:
-If $q>q^{*}$, the economy is persistently in a state of unemployment.
-If $q \leq q^{*}$, the economy cycles between three states:

- Good times: the economy is at full employment, $g$ and t maximize the utility of the mwc.
- Tough times: the economy is at full employment, but $g$ and $\tau$ are distorted to stimulate the economy.
- Bad times: unemployment is positive, $g$ and $\tau$ are distorted.
- Unemployment is increasing in b in the high cost state, and in the low cost state when $b$ is sufficiently large.
- For given b, unemployment is higher in the high than the low cost state.


## III. Equilibrium stimulus plans

- How does a government react to a high cost state?
- Assume here for simplicity that $q>q^{*}$ (so we always have unemployment).

Proposition: In the steady state of an equilibrium:

- The government reacts to a $H$ state by increasing the primary deficit (lowering taxes and increasing g).
- When $b$ is sufficiently low, $g$ will be lower than the employment maximizing level.
- When $b$ is sufficiently high, $g$ will be higher than the employment maximizing level.


This observation may help understand equilibrium multipliers.

- The multipliers are computed as:

$$
\begin{aligned}
& M_{g}=\Delta G D P / \text { cost of measure } g \\
& M_{T}=\Delta G D P / \text { cost of measure } T
\end{aligned}
$$

- An important literature is devoted to their measurement, seeing them as a measure of the effectiveness of policies.
- The implicit assumption is that the government should equalize the multipliers across instruments.


Proposition: In the steady state of an equilibrium the multiplier is not equalized across measures:

- When $b$ is low: $M_{g}>M_{T}$;
- When b is high: $M_{g}<M_{T}$.
- Suppose you are advising a legislator, and you are benevolent: Could you choose a superior policy mix? Should you change the size of the stimulus plan?
- No: In the $H$ state, legislators choose the optimal policy mix given the state and the equilibrium continuation value.
- Only planner who could commit to a policy could improve welfare.


## IV. Conclusion

- This paper has explored the interaction between fiscal policy and unemployment.
- We have argued that when fiscal policy is endogenous, assuming market imperfections is not sufficient to obtain a theory of unemployment.
- We have proposed a political economy model that delivers an appealing theory of fiscal policy and unemployment.
- The theory provides a new perspective to evaluate and interpret fiscal policy.



$$
\because \frac{A_{\phi}}{2}\left[n_{w}<\frac{n_{e} A_{\phi}}{2}\right] .
$$

$$
\gamma \geq \frac{A_{\theta}}{2}\left[n_{w}-\frac{n_{e} A_{\theta}}{2 \xi}\right] .
$$

# The political economy of public debt: a general equilibrium approach 

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## Introduction

- Debt affects the economy in two ways:
- Directly by allowing the government to "smooth" taxation over time and (potentially) across states;
- By affecting interest rates and therefore equilibrium savings.
- In the previous lecture we ignored the second effect.
- In this lecture we study how the second effect changes the problem.
- We provide a sharp characterization of equilibrium policies that generalizes previous results:

$$
\operatorname{MCPF}(b)=\frac{1}{1-\varepsilon_{\rho, b^{\prime}}(b)}\left[\operatorname{MCPF}\left(b^{\prime}\right)+\Phi(b ; q, e)\right]
$$

- We compute calibrated versions of the model.
- Who "pays" for the political distortions?
- With GHH utilities, it leads to a too large government, as measured by T .
- With KPR utilities, taxes decrease over time: government is too small.


## Plan for the talk

I. The model
II. The planner's optimum
III. The political equilibrium:
I. No political conflict, no debt
II. The effect of political conflict
IV. Extensions and discussion

## I. The Model

## I. 1 The economy

- A continuum of infinitely-lived citizens live in $n$ identical districts. The size of the population in each district is normalized to be one.
- There are $n+2$ goods: private consumption $z$, and labor $l$, and $n$ local public goods $g^{i}$
- Each citizen's per period utility function is

$$
u_{i}\left(c, l, g_{i}, g_{-i}\right)=u(c, l)+f\left(g^{i}, \sum_{j} g^{j}\right)
$$

- Discount factor: $\bar{\delta}$.
- Two examples will be useful:
- The Greenwood-Hercowitz-Huffman (GHH) utility:

$$
u_{i}\left(c, l, g_{i}, g_{-i}\right)=\frac{1}{1-\sigma}\left(c_{t}-\psi_{0} \frac{l_{t}^{1+\psi}}{1+\psi}\right)^{1-\sigma}+f\left(g^{i}, \sum_{j} g^{j}\right)
$$

- And the King-Plosser-Rebelo (KPR) utility:

$$
u_{i}\left(c, l, g_{i}, g_{-i}\right)=\frac{1}{1-\sigma}\left(c_{t}\left(1-l_{t}\right)^{\psi}\right)^{1-\sigma}+f\left(g^{i}, \sum_{j} g^{j}\right)
$$

- Linear technology: $z=w l$ and $g=z / p$.
- There are markets for labor and the public good.
- There is also a market in risk-free, one period bonds. Both citizens and the government have access to this market.
- Assets held by an agent in district $i$ in period $t$ are $a^{i}{ }_{t}$.
- In a competitive equilibrium:
- price of the public good is $p$,
- the wage rate is $w$,
- the interest rate is denoted as $\rho$.


## I. 2 Public Policies

- The legislature can raise revenues in two ways: a tax on labor income (T) and borrowing (b).
- If the legislature borrows $b$ in period $t$ it must repay $\rho b$ in period $t+1$.
- The legislature can also hold bonds if it wants, so $b$ can be negative.
- Public revenues can be used to finance local public goods.
- A policy choice is described by an $n+2$-tuple:

$$
\left\{r, b^{\prime}, g^{1}, \ldots, g^{n}\right\}
$$

- The policy choice must satisfy the budget constraint:
- Public good provision must be non-negative: $g^{l} \geq 0$.
- Debt must be feasible: $x \leq \bar{x}$


## I. 3 The private sector

- In a symmetric equilibrium we have $a_{t}^{i}=a_{t}$ :

$$
a_{t} a_{t \square 1} / \nleftarrow \frac{1}{n} \boldsymbol{B}_{t} b_{t \square} / \nrightarrow
$$

- We can therefore express the citizens' choices as a function of current public policies only. In the GHH case we have:
- This give us an indirect utility function:

$$
u\left(\tau, \sum_{j} g^{j}\right)=\frac{1}{1-\sigma}\left(w\left[\frac{w}{\psi_{0}}\left(1-\tau_{t}\right)\right]^{\frac{1}{\psi}}-\psi_{0} \frac{\left[\frac{w}{\psi_{0}}\left(1-\tau_{t}\right)\right]^{\frac{1+\psi}{\psi}}}{1+\psi}-\frac{p \sum_{j} g^{j}}{n}\right)^{1-\sigma} .
$$

- The interest rate is:



## I. 4 Legislative policy-making

- Public decisions are made by a legislature of representatives from each of the $n$ districts.
- One citizen from each district is selected to be that district's representative.
- The legislature meets at the beginning of each period.
- The affirmative votes of $q<n$ representatives are required to pass legislation.
- One legislator is randomly selected to make the first policy proposal.
- If the proposal is accepted by $q$ legislators, the plan is implemented and the legislature adjourns until the next period.
- At that time, the legislature meets again with the only difference being that $b$ is different.
- If the first proposal is rejected, another legislator is chosen and the process repeats.


## II. The Planner's Solution

- The planner's problem can be written as:
$\left.\max _{\substack{\left(\tau_{t}, g_{t}, b_{t}, b_{t=0}^{\prime}\right.}}^{\sum_{t=0}^{\infty} \delta^{t}\left[\frac{1}{1-\sigma}\left(w\left[\frac{w}{\psi_{0}}\left(1-\tau_{t}\right)\right]^{\frac{1}{\psi}}-\psi_{0} \frac{\left[\frac{\left.w_{0}\left(1-\tau_{t}\right)\right]^{\frac{1+\omega}{w}}}{1+\psi}\right.}{1+\psi}-p g_{\tau}\right)^{1-\sigma}+\varphi\left(g_{\tau}\right)\right]} \begin{array}{c}\text { s.t. } \rho_{t}\left(\tau_{t}, g_{t}, \tau_{t+1}, g_{t+1}\right)\left[b_{t}+p n g_{t}-\tau_{t} n w l_{t}\left(\tau_{t}\right)\right]-b_{t}^{\prime} \leq 0 \\ b_{t}^{\prime} \leq \bar{x}, g \geq 0, \tau \in[0,1] .\end{array}\right\}$
where $\rho_{t+1}\left(\tau_{t}, g_{t}, \tau_{t+1}, g_{t+1}\right)$ is the endogenous interest rate. and $\varphi(g)=f(g, n g)$.
- This problem cannot be expressed recursively in the usual way.
- If we fix the interest rate, the planner would like to smooth taxation and the benefit of $g$ uniformly over time.
- By changing fiscal policy, however, the planner can manipulate interest rates.
- Consider period $t=0$. Ignore $g$, lets focus on $T$ :
- Marginally reduce $T_{0}$ and increase $T_{1}$.
- This increases the $u_{c}$ at $t+1$.
- Ceteris paribus, interest rates $\downarrow$ to clear the market.
- If $b_{0}>0$, we spend less on interest: this is good because we save deadweight loss of taxation.
- What happens at $t>0$ ?
- An increase in $b_{t+1}$ may reduce the interest rate at $t$, but it will have a symmetric and opposite effect at $t-1$.
- In equilibrium, the planner at $t=0$ internalizes both costs and benefits and does not find it optimal to manipulate $\rho$ anymore.

Proposition 1: In a well behaved planner's problem:

- if bo=0, then $T_{t}, g_{t}, b_{t}$ are constant in all periods and the steady state level of debt is bo.
- If bo$=0$, then:
- $T_{t}, g_{t}, b_{t}$ are constant at the steady state $\tau^{0}, g^{\circ}, b^{\circ}$ for $t \geq 1$.
- $\tau_{0}, g_{0}, b_{0}$, on the other hand may be higher or lower than the steady state $\tau^{\circ}, g^{\circ}, b^{\circ}$.
- At $t=0$, debt may go up or down. Why?
- Assume that taxes are constant.
- Consider now a marginal increase in $g_{1}$ financed by a marginal reduction in $g o$.
- The effect on the interest ate is:

$$
\rho_{t+1}\left(b, b^{\prime}\right)=\frac{1}{\delta}\left[\frac{w l\left(\tau^{*}\right)-c\left(l\left(\tau^{*}\right)\right)-p g_{t}(b)}{w l\left(\tau^{*}\right)-c\left(l\left(\tau^{*}\right)\right)-p g_{t+1}\left(b^{\prime}\right)}\right]^{-\sigma}
$$

- So now a decreases in debt reduces the interest rate.
- What should we expect when we put the two effects together?
$\sigma, \omega=1 ; \psi=0.4, g^{*}=20 \%$ of GDP

| $b(0)$ | -0.050 | 0.000 | 0.050 | 0.250 | 0.500 |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $b(1)$ | -0.066 | 0.000 | 0.044 | 0.177 | 0.312 |  |
| $g(0)$ | 0.13 | 0.11 | 0.11 | 0.08 | 0.06 |  |
| $g(1)$ | 0.12 | 0.11 | 0.11 | 0.11 | 0.10 |  |
| $\tau(0)$ | 0.26 | 0.20 | 0.17 | 0.12 | 0.10 |  |
| $\tau(1)$ | 0.19 | 0.20 | 0.20 | 0.22 | 0.23 |  |
| $R$ | 1.41 | 1.05 | 0.93 | 0.73 | 0.64 |  |
| $\mathrm{~g}(0)$ | 0.47 | 0.57 | 0.62 | 0.72 | 0.78 |  |
| $y(1)$ | 0.58 | 0.57 | 0.57 | 0.54 | 0.52 |  |
| $b(0) / y(0)$ | -0.11 | 0.00 | 0.08 | 0.35 | 0.64 |  |
| $b(1) / y(1)$ | -0.11 | 0.00 | 0.08 | 0.33 | 0.60 |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |


$\sigma, \psi=1 ; \omega=0.5, g^{*}=20 \%$ of GDP

| $b(0)$ | -0.050 | 0.000 | 0.050 | 0.250 | 0.500 |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $b(1)$ | -0.049 | 0.000 | 0.047 | 0.223 | 0.421 |  |
| $g(0)$ | 0.17 | 0.16 | 0.15 | 0.13 | 0.11 |  |
| $\mathrm{~g}(1)$ | 0.16 | 0.16 | 0.16 | 0.16 | 0.15 |  |
| $\tau(0)$ | 0.20 | 0.20 | 0.20 | 0.19 | 0.17 |  |
| $\tau(1)$ | 0.20 | 0.20 | 0.20 | 0.21 | 0.22 |  |
| R | 1.08 | 1.05 | 1.03 | 0.96 | 0.90 |  |
| $\mathrm{y}(0)$ | 0.80 | 0.80 | 0.80 | 0.81 | 0.83 |  |
| $\mathrm{y}(1)$ | 0.80 | 0.80 | 0.80 | 0.79 | 0.78 |  |
| $\mathrm{~b}(0) / \mathrm{y}(0)$ | -0.06 | 0.00 | 0.06 | 0.31 | 0.60 |  |
| $\mathrm{~b}(1) / \mathrm{y}(1)$ | -0.06 | 0.00 | 0.06 | 0.28 | 0.54 |  |
|  |  |  |  |  |  |  |
| $b(0)-b(1)$ | -0.001 | 0.000 | 0.003 | 0.027 | 0.079 |  |

$\sigma, \omega, \psi=1 ; \mathrm{g}^{*}=20 \%$ of GDP

| $b(0)$ | -0.050 | 0.000 | 0.050 | 0.250 | 0.500 | $b(0)$ | -0.050 | 0.000 | 0.050 | 0.250 | 0.500 |
| :--- | ---: | ---: | ---: | ---: | ---: | :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| $b(1)$ | -0.051 | 0.000 | 0.050 | 0.235 | 0.446 | $b(1)$ | -0.050 | 0.000 | 0.050 | 0.250 | 0.498 |
| $\mathrm{~g}(0)$ | 0.16 | 0.16 | 0.15 | 0.14 | 0.12 | $\mathrm{~g}(0)$ | 0.20 | 0.20 | 0.20 | 0.19 | 0.19 |
| $\mathrm{~g}(1)$ | 0.16 | 0.16 | 0.16 | 0.16 | 0.15 | $\mathrm{~g}(1)$ | 0.20 | 0.20 | 0.20 | 0.20 | 0.19 |
| $\tau(0)$ | 0.20 | 0.20 | 0.19 | 0.19 | 0.17 | $\tau(0)$ | 0.20 | 0.20 | 0.20 | 0.21 | 0.22 |
| $\tau(1)$ | 0.20 | 0.20 | 0.20 | 0.21 | 0.23 | $\tau(1)$ | 0.20 | 0.20 | 0.20 | 0.21 | 0.23 |
| R | 1.07 | 1.05 | 1.00 | 0.98 | 0.93 | $R$ | 1.05 | 1.05 | 1.05 | 1.05 | 1.05 |
| $\mathrm{y}(0)$ | 0.80 | 0.80 | 0.81 | 0.81 | 0.83 | $\mathrm{y}(0)$ | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 |
| $\mathrm{y}(1)$ | 0.80 | 0.80 | 0.80 | 0.79 | 0.77 | $\mathrm{y}(1)$ | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 |
| $\mathrm{~b}(0) / \mathrm{y}(0)$ | -0.06 | 0.00 | 0.06 | 0.31 | 0.60 | $\mathrm{~b}(0) / \mathrm{y}(0)$ | -0.05 | 0.00 | 0.05 | 0.26 | 0.51 |
| $\mathrm{~b}(1) / \mathrm{y}(1)$ | -0.06 | 0.00 | 0.06 | 0.30 | 0.58 | $\mathrm{~b}(1) / \mathrm{y}(1)$ | -0.05 | 0.00 | 0.05 | 0.26 | 0.51 |
|  |  |  |  |  |  |  |  |  |  |  |  |
| $b(0)-b(1)$ | 0.001 | 0.000 | 0.000 | 0.015 | 0.054 | $b(0)-b(1)$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.002 |


| $b(0)$ | -0.050 | 0.000 | 0.050 | 0.250 | 0.500 | $b(0)$ | -0.050 | 0.000 | 0.050 | 0.250 | 0.500 |
| :--- | ---: | ---: | ---: | ---: | ---: | :--- | :--- | :--- | ---: | ---: | ---: | ---: |
| $b(1)$ | -0.054 | 0.000 | 0.053 | 0.260 | 0.504 | $b(1)$ | -0.056 | 0.000 | 0.050 | 0.217 | 0.382 |
| $\mathrm{~g}(0)$ | 0.16 | 0.16 | 0.16 | 0.16 | 0.15 | $\mathrm{~g}(0)$ | 0.17 | 0.16 | 0.15 | 0.14 | 0.12 |
| $\mathrm{~g}(1)$ | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | $\mathrm{~g}(1)$ | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 |
| $\tau(0)$ | 0.20 | 0.20 | 0.20 | 0.19 | 0.17 | $\tau(0)$ | 0.21 | 0.20 | 0.19 | 0.16 | 0.14 |
| $\tau(1)$ | 0.20 | 0.20 | 0.20 | 0.22 | 0.24 | $\tau(1)$ | 0.20 | 0.20 | 0.20 | 0.21 | 0.23 |
| $R$ | 1.06 | 1.05 | 1.05 | 1.02 | 0.99 | $R$ | 1.11 | 1.05 | 1.00 | 0.87 | 0.77 |
| $\mathrm{y}(0)$ | 0.80 | 0.80 | 0.80 | 0.81 | 0.83 | $\mathrm{y}(0)$ | 0.79 | 0.80 | 0.81 | 0.84 | 0.86 |
| $\mathrm{y}(1)$ | 0.80 | 0.80 | 0.80 | 0.78 | 0.76 | $\mathrm{y}(1)$ | 0.80 | 0.80 | 0.80 | 0.79 | 0.77 |
| $\mathrm{~b}(0) / \mathrm{y}(0)$ | -0.06 | 0.00 | 0.06 | 0.31 | 0.61 | $\mathrm{~b}(0) / \mathrm{y}(0)$ | -0.06 | 0.00 | 0.06 | 0.30 | 0.58 |
| $\mathrm{~b}(1) / \mathrm{y}(1)$ | -0.07 | 0.00 | 0.07 | 0.33 | 0.67 | $\mathrm{~b}(1) / \mathrm{y}(1)$ | -0.07 | 0.00 | 0.06 | 0.28 | 0.49 |
| $b(0)-b(1)$ | 0.004 | 0.000 | -0.003 | -0.010 | -0.004 | $b(0)-b(1)$ | 0.006 | 0.000 | 0.000 | 0.033 | 0.118 |

$\omega, \psi=1 ; \sigma=2, \mathrm{~g}^{*}=20 \%$ of GDP
$\sigma, \omega=1 ; \psi=10, \mathrm{~g}^{*}=20 \%$ of GDP
0.0 .0 .118

## III. The political equilibrium

- We look for a symmetric Markov-perfect equilibrium in weakly stage-undominated strategies.
- The state variable is $b$.
- An equilibrium can be formally defined by:
- a collection of policy proposals $T(b), b^{\prime}(b), g(b), g^{c}(b)$;
- a value function $v(b)$;
- and an interest rate function $\rho\left(b^{\prime}, T, \sum g^{i} ; b\right)$.
- We focus, without loss of generality, on equilibria with immediate agreement.
- In equilibrium, there is a reciprocal feedback between the policy proposals, $v(b)$ and $\rho\left(b^{\prime}, \tau, \sum g^{i} ; b\right)$.
- Given $v, b$ and $\rho$, the prescribed policy proposals must maximize the proposer's payoff:
$\max _{\left(\tau, g, g^{c}, b\right)}\left\{\begin{array}{c}u\left(\tau, \sum_{j} g^{j}\right)+f\left(g, \sum_{j} g^{j}\right)+\delta v\left(b^{\prime}\right) \\ \text { s.t. } b^{\prime}-\rho\left(b^{\prime}, \tau, \sum_{j} g^{j} ; b\right)\left[b+p \sum_{j} g^{j}-n w \tau l^{*}(\tau)\right] \geq 0 \\ u\left(\tau, \sum_{j} g^{j}\right)+f\left(g^{c}, \sum_{j} g^{j}\right)+\delta v\left(b^{\prime}\right) \geq v(b) \\ b^{\prime} \leq \bar{b}, g \geq 0, \tau \in[0,1]\end{array}\right\}$
- On the other hand, $v$ and $\rho$ are themselves determined by the equilibrium policy proposals.
- We have:

$$
\begin{aligned}
v(b) & =u\left(\tau(b), \sum_{j} g^{j}(b)\right) \\
& +\frac{f\left(g(b), \sum_{j} g^{j}(b)\right)}{n}+\frac{q-1}{n} f\left(g^{c}(b), \sum_{j} g^{j}(b)\right) \\
& +\frac{n-q}{n} f\left(0, \sum_{j} g^{j}(b)\right)+\delta v\left(b^{\prime}\right)
\end{aligned}
$$

- We say that an equilibrium is well-behaved if $v(b)$ is concave and differentiable in $b$, and the policy functions are differentiable in $b$.
- We have:

Proposition 3: There is $a \sigma^{*}$ such that a well behaved equilibrium exists if $\sigma>\sigma^{*}$.

## III. 1 Characterization

- Consider the incentive compatibility constraint:

$$
\begin{aligned}
& u\left(\tau(b), \sum_{j} g^{j}(b)\right)+f\left(g^{c}, \sum_{j} g^{j}\right)+\delta v\left(b^{\prime}\right) \\
& \geq u\left(\tau(b), \sum_{j} g^{j}(b)\right)+\left[\begin{array}{l}
\frac{f\left(g(b), \sum_{j} g^{j}(b)\right)}{n} \\
+\frac{q-1}{n} f\left(g^{c}(b), \sum_{j} g^{j}(b)\right)
\end{array}\right]+\delta v\left(b^{\prime}\right)
\end{aligned}
$$

(from now on, for simplicity: $f\left(0, \sum_{j} g^{j}\right)=0$ )

- We can write:

$$
\left.f\left(\frac{g_{C}}{g}, 1+(q-1) \frac{g_{C}}{g}\right)\right)=\left[\frac{1}{n-q+1}\right] f\left(1,1+(q-1) \frac{g_{C}}{g}\right)
$$

Lemma 1: There is a constant $k^{*}(q)<1$ such that the proposer does pays $k^{*}(q) g$ to a MWC. The fraction $k^{*}(q)$ is increasing in $q$ and $\kappa$; and equal to 1 when $q=n$.
-The proposer's problem is:


- The expected value function is:

$$
v(b)=u(\tau, K(q) g)+\left[\begin{array}{l}
\frac{1}{n} \cdot \frac{f\left(1,1+(n-1) \kappa^{*}\right)}{f(1, n)} \\
+\frac{q-1}{n} \frac{f\left(\kappa^{*}, 1+(n-1) \kappa^{*}\right)}{f(1, n)}
\end{array}\right] \varphi(g)+\delta v\left(b^{\prime}\right)
$$

where

$$
v(b)=u\left(\tau, \sum g\right)+\varphi(g)+\delta v\left(b^{\prime}\right)
$$

is the objective function of a utilitarian planner.

- How does this differ from a benevolent planner's problem?
- There are 3 differences.
- The first is static inefficiency. We can write:

$$
v_{p}(b)=u\left(\tau, \frac{\sum g}{n}\right)+\frac{f\left(\frac{n}{1+(n-1) \kappa^{*}}, n\right)}{f(1, n)} \varphi\left(\frac{\sum g}{n}\right)+\delta v\left(b^{\prime}\right)
$$

The proposer overweighs the benefit of transfers.

- The second and third differences have to do with dynamic inefficiency.
- The second is a dynamic inconsistency of preferences.
- We can write:

$$
\begin{aligned}
v(b) & =v_{p}(b)+\frac{1}{n}\left[\begin{array}{l}
\frac{(q-1) f\left(\kappa^{*}, 1+(n-1) \kappa^{*}\right)}{f(g, n g)} \\
-\frac{(n-1) f\left(1,1+(n-1) \kappa^{*}\right)}{f(g, n g)}
\end{array}\right] \cdot \varphi(g(b)) \\
& =v_{p}(b)+\Phi(b)
\end{aligned}
$$

So there is an extra benefit in increasing debt.

- The third difference is the manipulability of interest rate:
- In the social planner's solution the planner is tempted to manipulate interest rates only at $t=0$.
- This because at $t=0$ he internalizes effect at $t$ and at $t+1$ of a change in $b_{t}$.
- In a political equilibrium there is no commitment, so this temptation will be present in every period.


## III. 2 The case with no political conflict

- It is useful to introduce the $\operatorname{MCPF}(b)$ : the monetary transfer necessary to compensate a representative agent for the increase in taxes required to marginally reduce the debt level.
- This is:
where $\lambda(b)$ is the Lagrange multiplier of the budget constraint.
- Let $\rho\left(b^{\prime}, b\right)$ be the interest rate; and let $\rho(b)$ be the equilibrium interest rate $\rho\left(b^{\prime}(b), b\right)$.
- The elasticity of the interest rate with respect to b’ evaluated at $b$ is:

To isolate the role of conflict, we first eliminate it:

$$
f\left(g^{i}, \sum_{j} g^{j}\right)=f\left(\sum_{j} g^{j}\right)
$$

The political equilibrium coincides to the case of a benevolent planner with no commitment.

We have:
Proposition. With a unanimous constituency, the Euler equation can be written in terms of the $\operatorname{MCPF}(b)$ as :

$$
\operatorname{MCPF}(b)=\frac{1}{1-\varepsilon_{\rho, b^{\prime}}(b)} \cdot \operatorname{MCPF}\left(b^{\prime}(b)\right) .
$$

- This representation has a straightforward interpretation:
- The planner equates the marginal benefit of debt to the marginal cost weighted by a markup factor:

$$
\frac{1}{\left(1-\varepsilon_{\rho, x^{\prime}}(x)\right)}
$$

- The planner behaves as a particular type of monopolist who internalized the price effects of supply.
- How important is the assumption of an exogenous g?
- With g exogenous: $\varepsilon_{\rho, b^{\prime}}(b)<0$, so:

$$
\operatorname{MCPF}(b)<\frac{1}{1-\varepsilon_{\rho, b^{\prime}}(b)} \cdot \operatorname{MCPF}\left(b^{\prime}(b)\right)
$$

Proposition. If $g$ is exogenous, there is no stable steady with an interior level of $b$.

- Lets now assume taxes are constant.
- Since:

$$
\rho_{t+1}\left(b, b^{\prime}\right)=\frac{1}{\delta}\left[\frac{w l\left(\tau^{*}\right)-c\left(l\left(\tau^{*}\right)\right)-p g_{t}(b)}{w l\left(\tau^{*}\right)-c\left(l\left(\tau^{*}\right)\right)-p g_{t+1}\left(b^{\prime}\right)}\right]^{-\sigma}
$$

- At the margin, an increase in b' implies a reduction in $g_{t+1}\left(b^{\prime}\right)$ and so an increase in $\rho_{t+1}\left(b, b^{\prime}\right)$. When debt is positive, this implies that $\varepsilon_{\rho, b}(b)>0$ so:

$$
\operatorname{MCPF}(b)=\frac{M C P F\left(b^{\prime}(b)\right)}{1-\varepsilon_{\rho, x^{\prime}}(x)}>M C P F\left(b^{\prime}\right)
$$



In this case debt converges to zero and zero is a stable steady state.

- Which effect dominates?
- We have computed calibrated versions of the model to see if we can explain debt without political conflict.

Figure 1. The Dynamics of the Economy, No PE distortion ( $\sigma=\omega=0.9, \delta=0.96$ )


Taxes, $\tau(\mathrm{b})$


Public good, g(b). \% of GDP


Interest rate


Figure 1a. Evolution of the Economy. No PE distortion

Public debt, \% of GDP


Public good, \% of GDP


Taxes


GDP. \% of Period 1


## III. 2 The political equilibrium

- In a political equilibrium, the temptation to reduce debt is mitigated by the political bias.
- Proposition. In a political equilibrium, the Euler equation can be written in terms of the $\operatorname{MCPF}(b)$ as:
$\operatorname{MCPF}(b)=\frac{1}{1-\varepsilon_{\rho, b^{\prime}}(b)}\left[\operatorname{MCPF}\left(b^{\prime}(b)\right)+\Phi(b ; q, e)\right]$. where $\Phi(b ; q, e)<0$.
- What should we expect?


So now debt can be positive in the steady state

- Is this effect significant?
- Here too we have computed calibrated versions of the model.

Figure 2. The Dynamics of the Economy. PE distortion $=0.45(\sigma=\omega=0.9, \delta=0.96)$

Next period public debt, $\mathrm{H}(\mathrm{b})$, and $45^{\circ}$ line



Taxes, $\tau(b)$


Public good. g(b). \% of GDP


Next period public debt, \% of current GDP


Figure 3. Political Economy Distortions and Steady State Fiscal Policies


Figure 4. Evolution of the Economy with PE distortion

Public debt, \% of GDP


Public good, \% of GDP


Taxes


GDP, \% of Period 1


## The KPR utility

- Does the utilty function matters for the dynamics?
- Consider the KPR utility:

$$
u_{i}\left(c, l, g_{i}, g_{-i}\right)=\frac{1}{1-\sigma}\left(c_{t}\left(1-l_{t}\right)^{\mu}\right)^{1-\sigma}+f\left(g^{i}, \sum_{j} g^{j}\right)
$$

- This utility has two features:
- Constant intertemporal elasticity of substitution in consumption.
- Constant elasticity of the marginal utility of leisure with respect to consumption.

Figure 1. The Dynamics of the Economy. No PE distortion ( $\sigma=\omega=0.9, \delta=0.96$ )



Next period public debt, \% of current GDP


Taxes, $\tau(b)$


Public good. g(b). \% of GDP


Interest rate


Figure 1a. Evolution of the Economy. No PE distortion

Public debt, \% of GDP


Public good, \% of GDP


Taxes


GDP. \% of Period 1


Figure 2. The Dynamics of the Economy. PE distortion $=0.45(\sigma=\omega=0.9, \delta=0.96)$


Figure 2a. Evolution of the Economy with PE distortion


Public good, \% of GDP


GDP. \% of Period 1


Figure 3. Political Economy Distortions and Steady State Fiscal Policies



Steady state public good, \% of GDP


| Table 2. The Impact of PE Distortions: GHH vs KPR |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | GHH |  |  | KPR |  |  |
|  | No PE | $\mathrm{PE}=0.85$ |  | No PE | $\mathrm{PE}=0.85$ |  |
| debt (\% of GDP) | 0 | 9 | $\uparrow$ | 0 | 77 | $\uparrow$ |
| g (\% of GDP) | 20 | 21 | $\uparrow$ | 20 | 16 | $\downarrow$ |
| $\tau$ | 20 | 21.5 | $\uparrow$ | 20 | 18 | $\downarrow$ |
| GDP | 100 | 98.2 | $\downarrow$ | 100 | 97.5 | $\downarrow$ |

## Conclusion

- We have developed a general equilibrium theory of public debt.
- Under the most plausible assumptions, with no political conflict high levels of debt are difficult to justify.
- Political economy provides an natural way to explain public debt, its effect on the size of government depends on the economic environment.
- Possible extensions: shocks, endogenous growth, etc.


## In Summary

- There was a time....
- In these lectures we have provided a simple and tractable framework to study political economy in dynamic economies.
- We have overviewed a number of political systems and economic applications.
- The framework may (hopefully) prove useful to study a number of other issues:
- Endogenous growth
- Fiscal capacity, ...etc


# Analyzing the Case for a Balanced Budget Amendment to the U.S. Constitution 

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## Introduction

- The desirability of a budget balance rule (BBR) is a recurrent debate in American politics:
- In 1995 the House approved a BBR by 300-132;
- There are bills pending in the $110^{\text {th }}$ congress;
- Many U.S. states currently have BBR.
- The trade-off is clear:
- A disciplinary effect on policy makers;
- A flexiblility cost, due to the restricted policy space.
- How do these two effects shape fiscal policy? What are their welfare implications?
- In this paper we study the impact of a BBR in the political economy model of Battaglini and Coate [2008].
- Though simply an upper-bound on deficits, BBR induce debt to gradually fall until it converges to a level that would not be reached otherwise.
- Intuition: BBR raises the expected cost of taxation in the future. This induces legislators to save more.
- Welfare is higher in the long run. But transition cost can be high. How much?
- We evaluate the dynamics and welfare calibrating the model to the US economy:
- In the steady state, debt/GDP is reduced by $89 \%$ and welfare is higher by $2.88 \%$.
- Net of transition cost, ex ante benefit is negative.


## Plan for the talk

I. Review of the BC model
II. The impact of the BBR
III. Computation and calibration
IV. The effect of overrides.

## I. The model

## The economy

- A continuum of infinitely-lived citizens live in $n$ identical districts. The size of the population in each district is normalized to be one.
- There are three goods - a public good $g$, private consumption $z$, and labor $l$.
- Linear technology: $z=w l$ and $g=z / p$.
- Each citizen's per period utility function is:

$$
z+A \log g-\frac{l^{\left(1+\frac{1}{\varepsilon}\right)}}{\varepsilon+1}
$$

- The value of public goods fluctuates: A is stochastic, reflecting shocks such as wars and natural disasters.
- Discount factor: $\bar{\delta}$.
- In a competitive equilibrium:
- price of the public good is $p$,
- the wage rate is $w$,
- and the interest rate is $\rho=1 / \delta-1$.


## Politics and policies

- Public decisions are made by a legislature consisting of representatives from each of the $n$ districts.
- A policy choice in described by an $n+3$-tuple: $\left\{T, g, b^{\prime}, s_{1}, . . s_{n}\right\}$
- Marginal tax rate on income $T$
- Public good $g$
- Risk free, one period debt, $b^{\prime}\left(b^{\prime}>\right.$ or $\left.<0\right)$
- Pork transfer to district i: $s_{i}$
- $B(r, g, x ; b)=R(r)+b^{\prime}-p g-(1+\rho) b \geq \sum_{i} S_{i}$
- There is also an upper bound $x \leq \bar{x}$


## Legislative policy-making

- One of the legislators is randomly selected to make the first policy proposal:
- If the proposal is accepted by $\boldsymbol{q}$ legislators, then the plan is implemented.
- At $t+1$, the legislature meets again with the only difference being that $b_{t+1}$, and (maybe) $A$ is different.
- If the first proposal is not accepted, another proposer is selected.
- There are $T$ such proposal rounds, each of which takes a negligible amount of time.
- If at $T$ there is no agreement, a legislator is selected to choose a default policy that treats districts uniformly.


## Equilibrium

- We look for a symmetric Markov-perfect equilibrium (in weakly stage undominated strategies).
- The problem has a recursive structure with the state variables being the current debt level $b$ and the current state of the economy A.
- We show that such an equilibrium exists and is unique.


## I. 1 The planner's solution

- The planner's problem can be written in the recursive form:
$v(b ; A)=\max _{\left(\tau, g, b^{\prime}\right)}\left\{\begin{array}{c}u(\tau, g ; A)+\frac{B\left(\tau, g, b^{\prime} ; b\right)}{n}+\delta E v\left(b^{\prime}, A^{\prime}\right) \\ B\left(\tau, g, b^{\prime} ; b\right) \geq 0, b^{\prime} \leq \bar{b}\end{array}\right\}$
where $u(T, g ; A)$ is the indirect utility function in state A and $v(b, A)$ is the continuation value.
- The problem is one of "tax smoothing" (Barro 1979)
- Planner's solution:

- Policy converges to a lower bound, the government does not distort the economy in the long run because it accumulates enough assets.
- Counterfactual. Intuitively we would expect the equilibrium to generate too high T , too little g and too much debt.


## I. 2 The political equilibrium without BBR

- The proposer is effectively making decisions to maximize the collective utility of $q$ legislators.
$\max _{\tau, g, b^{\prime}}\left\{\begin{array}{c}u(\tau, g ; A)+\frac{B\left(\tau, g, b^{\prime} ; b\right)}{q}+\delta E v\left(b^{\prime}, A^{\prime}\right): \\ B\left(\tau, g, b^{\prime} ; b\right) \geq 0, b^{\prime} \leq \bar{b}\end{array}\right\}$
- When $b$ is high and/or $A$ is high, pork is too expensive, so:
$B(T, g, b ; b)=0$. Proposer's policy $=$ Planner's policy
- When $b$ is low and/or $A$ is low, the opportunity cost of revenues is lower: $B\left(T, g, b^{\prime} ; b\right)>0$. There is pork.
- This diversion of resources, effectively creates lower bounds on $T, b$, and an upper bound on $g$.

Proposition 1. There are bounds $\left\{T^{*}, g^{*}, b^{*}\right\}$, such that the equilibrium solves $a$ constrained planner's problem:

$$
\max _{\left(\tau, g, b^{\prime}\right)}\left\{\begin{array}{c}
u(\tau, g ; A)+\frac{B\left(\tau, g, b^{\prime} ; b\right)}{n}+\delta E v\left(b^{\prime}, A^{\prime}\right) \\
B\left(\tau, g, b^{\prime} ; b\right) \geq 0, \tau \geq \tau^{*} \\
g \leq g^{*}(A), \& b^{\prime} \in\left[b^{*}, \bar{b}\right]
\end{array}\right\}
$$

- So the equilibrium can be interpreted as a constrained planner's problem, subject to a set of political distortions.


## Dynamics of debt

- In the long run, debt converges to a unique, non-degenerate stationary distribution.

- Theory predicts perpetual indebtedness as long as $b^{*}>0$.


## II. The case with a BBR

- The MWC will maximize the coalition aggregate utility:

$$
\max _{\left(\tau, g, b^{\prime}\right)}\left\{\begin{array}{c}
u(\tau, g ; A)+\frac{B\left(\tau, g, b^{\prime} ; b\right)}{q}+\delta E v\left(b^{\prime}, A^{\prime}\right) \\
\left.B\left(\tau, g, b^{\prime} ; b\right) \geq 0, b^{\prime} \leq b\right]
\end{array}\right\}
$$

- As before, the level of debt chosen when there is pork $(B(T, g, b ’ ; b)>0)$, will define a lower bound on b :

$$
b_{c}^{*}(b) \in \arg \max \left\{\frac{b^{\prime}}{q}+\delta E v_{c}\left(b^{\prime}, A^{\prime}\right): b^{\prime} \leq b\right\}
$$

- This corresponds to $b^{*}$ but now it depends on $b$.

Proposition 2. Under a BBR, the equilibrium solves:

$$
v_{c}(b, A)=\max _{\left(\tau, g, b^{\prime}\right)}\left\{\begin{array}{c}
u(\tau, g ; A)+\frac{B\left(\tau, g, b^{\prime} ; b\right)}{n}+\delta E v_{c}\left(b^{\prime}, A^{\prime}\right) \\
B\left(\tau, g, b^{\prime} ; b\right) \geq 0, \tau \geq \tau^{*} \\
g \leq g^{*}(A), \& b^{\prime} \in\left[b_{c}^{*}(b), b\right]
\end{array}\right\}
$$

- Two key differences from before:
- First, there is an additional upper bound, $b$.
- Second, the endogenous lower bound on debt $b_{c}{ }^{*}(b)$.
- Determining the shape of the function $\boldsymbol{b}_{c}^{*}(b)$ will be crucial to the analysis.

Proposition 3. There exists a unique well-behaved equilibrium under a strict BBR. The associated function $b_{c}{ }^{*}(b)$ is given by:

where: $b_{0}<b^{*}$ and $b^{*}(b)<b$ for $b>b_{0}$.

Proposition 4. When there is a constraint on deficits, debt gradually declines until $b_{0}<b^{*}$ is reached:


The long run distribution of debt is thus degenerate with all the mass at $b_{0}<b^{*}$.

Why does the political constraint $b_{c}^{*}(b)$ have this shape?

- Is useful to start from what $b_{c}^{*}(b)$ can not be...
- Assume $E v_{c}\left(b^{\prime}, A^{\prime}\right)$ is strictly concave, as in the benchmark.

- Then $b^{*}(b)=\min \left\{b, b^{\prime}\right\}$
- However, we would have a contradiction:
- On the left of $\mathrm{b}^{\prime}, b_{c}{ }^{*}(b)=b$ so $\frac{\partial b^{*}{ }_{c}(b)}{\partial b}=1$ : a marginal reduction on debt is permanent
- On the right, $b_{c}^{*}(b)=b^{\prime}$, so $\frac{\partial b_{c}^{*}(b)}{\partial b}=0$ : we would have no effect.
- So concavity would fail.
- We need to "flatten" the value function:

- If $b_{c}^{*}(b)<b_{0}, b_{c}^{*}(b)=b$. But we can choose $b_{c}^{*}(b)$ in $\left[b_{o}, b_{1}\right]$.
- $\exists$ a unique way to select $b_{c}^{*}(b)$ to keep $E v_{c}$ weakly concave:

$$
\frac{1}{q}=-\delta \frac{\partial E v_{c}\left(b ; A^{\prime}, b_{c}^{*}(\square), \frac{\partial b_{c}^{*}(\square)}{\partial b}\right)}{\partial b}
$$

## III. Computation and Calibration: no BBR

- The characterization gives us a simple way of finding the equilibrium:
- Step 1. Guess $b^{*}=z$. The operator is a contraction. Find $v_{z}$ by iteration:

$$
v_{z}(b, A)=\max _{\left(\tau, g, b^{\prime}\right)}\left\{\begin{array}{c}
u(\tau, g)+A \ln g+\frac{B\left(\tau, g, b^{\prime} ; b\right)}{n}+\delta E v_{z}\left(b^{\prime}, A^{\prime}\right) \\
B\left(\tau, g, b^{\prime} ; b\right) \geq 0, g \leq g^{*}(A), \tau \geq \tau^{*} \\
\& b^{\prime} \in[z, \bar{b}]
\end{array}\right\}
$$

- Step 2. Find the fix point of:

$$
\Phi(z)=\arg \max \left\{b^{\prime} / q+\delta E v_{z}\left(b^{\prime}, A^{\prime}\right)\right\}
$$

## III. 1 Calibration

- Preferences: $\delta=0.95$ and $\varepsilon=2$ from Aiyagari et al.
- Shocks:
- Peace (95.5 \% of the time): $\log (A) \sim N\left(\mu, \theta^{2}\right)$.
- $\operatorname{War}(4.5 \%$ of the time $): \log (\mathrm{A})=\mu_{\mathrm{w}}$.
- Free parameters are: $q, \mu, \theta, \mu_{W}, \bar{b}$
- The key moments are (US, for the period 1940-2005)
- Average debt/GDP.
- Conditional mean of GovExp/GDP (peacetime).
- Conditional variance of GovExp/GDP (peacetime).
- Average GovExp/GDP during WWII.
- Maximum ratio of Debt/GDP.
- The free parameters are jointly determined to match these moments under the stationary distribution.


## III. 2 Model fit: no BBR

- Model simulation vs. data:

|  |  | $\mathrm{g} / \mathrm{y}$ | $\mathrm{g} / \mathrm{y}$ | $\mathrm{b} / \mathrm{y}$ | Rev/y | Pork/TotExp |
| ---: | :--- | ---: | ---: | ---: | ---: | ---: |
| Model | Mean | $17.4 \%$ | $18.4 \%$ | $57 \%$ | $21 \%$ | $0.015 \%$ |
|  | Stdev | $2 \%$ | $5.4 \%$ | $18 \%$ | $1 \%$ | $0.375 \%$ |
| Data | Mean | $17.4 \%$ | $18.2 \%$ | $56 \%$ | $17 \%$ | n.a |
|  | Stdev | $2 \%$ | $5.5 \%$ | $20 \%$ | $3 \%$ | n.a. |

- $\operatorname{Maximum}($ Debt/GDP)=121\% (data), 120.6\% (model).
- GovExp/GDP in WWII: $40.5 \%$ vs $40 \%$ (model).
- $b^{*}=29.4 \%$ in the model, in the data is $31.5 \%$.
- The q rule that best fitted the data was $q=55.2 \%$.

Note: target values are in red.

- Stationary distribution debt/GDP:



The dynamics of debt after WWII


The dynamics of tax revenues and spending after WWII

## III. 3 The case with a BBR

- The characterization gives us a simple way of finding the equilibrium:
- Step 1. $b_{c}^{*}$ (b) can be found by solving a differential equation.
- Step 2. Given $b^{*}(b), \mathrm{v}$ is the fixpoint of:

$$
v_{c}(b, A)=\max _{\left(\tau, g, b^{\prime}\right)}\left\{\begin{array}{c}
u(\tau)+A \ln g+\frac{B\left(\tau, g, b^{\prime} ; b\right)}{n}+\delta E v_{c}\left(b^{\prime}, A^{\prime}\right) \\
B\left(\tau, g, b^{\prime} ; b\right) \geq 0, \tau \geq \tau^{*}, \\
g \leq g^{*}(A), \& b^{\prime} \in\left[b_{c}^{*}(b), b\right]
\end{array}\right\}
$$

Imposing a BBR to the US economy

- Long run impact on fiscal policy:

|  |  | $\mathrm{g} / \mathrm{y}$ | $\mathrm{b} / \mathrm{y}$ | $\mathrm{Rev} / \mathrm{y}=\tau$ | Pork/TotExp |
| ---: | :--- | ---: | ---: | ---: | ---: |
| Benchmark | Mean | $18.4 \%$ | $56.9 \%$ | $21 \%$ | $0.015 \%$ |
|  | Stdev | $5.4 \%$ | $18 \%$ | $1 \%$ | $0.375 \%$ |
| BBR | Mean | $19.05 \%$ | $6.2 \%$ | $19.78 \%$ | $2.237 \%$ |
|  | Stdev | $2.4 \%$ | $0.34 \%$ | $2 \%$ | $4.814 \%$ |

- The average level of debt decreases by $89 \%$
- Short-run impact on fiscal policy:



## Welfare

- In the long-run, the welfare gain of the constraint is 2.85 .
- In the short run, the flexibility cost.
- A BBR would reduce welfare if imposed from any $b$ in the support of the long run distribution.
- Would BBR be good at foundation? (i.e. when $b_{0}=0$ ).
- Battaglini and Coate (2008) showed that when the size of the tax base is large enough relative to the spending needs, imposing the constraint will improve welfare.
- For our calibrated economy, we find that introducing a BBR at foundation reports a welfare gain of $0.017 \%$.


## IV. Supermajority overrides

- What happens when a super majority can override the BBR?
- Assume $b \in\left(b^{*}, \bar{b}\right)$. There are two cases:
- $B\left(T, g, b^{\prime} ; b\right)=0$ : all agree, the override always passes.
- $B\left(T, g, b^{\prime} ; b\right)>0: b^{\prime}(b)=b^{*} \leq b, \mathrm{BBR}$ is irrelevant.
- So when there are overrides, a BBR is irrelevant in the steady state. A BBR matters only in the transition.
- This result, however, should be taken with caution:
- With growth debt would increase even when there is pork.
- In this case a BBR may be useful even in the steady state.


## Conclusion

- We presented a dynamic political economy model where legislators bargain over policy (revenues, spending and debt).
- The introduction of a BBR induces a gradual reduction in $b$.
- We calibrated the model to the US economy: it fits relevant moments in the data, as well as the dynamics of expenditures and debt.
- We used the model to evaluate the welfare gains of a BBR.
- Future work: more general utilities, endogenous interest rates, growth.

